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A PIEBALD FAMILY.

By E. A. COCKAYNE, M.D., M.R.C.P.

In spite of the great interest, which they have always excited, well authenticated examples of piebalds in the dark races have been found to be rare. In the white races they are much less conspicuous, in part owing to the presence of clothing and in part owing to the lack of contrast between the pigmented and unpigmented skin, but the likelihood of their coming under the notice of a skilled observer is much greater. The scarcity of records shows that piebalds in the white races also must be very uncommon. Last year I met with a case in a baby, and found that the child belonged to a family, many of whose members showed a similar defect of pigmentation. The family, belonging to a farming stock, originally came from the neighbourhood of Bury St Edmunds in Suffolk, and the anomaly is known to have descended directly through six generations. The oldest member, with whom I have talked, is fairly certain that it was present in one generation at least before this.

Of the first two generations in the pedigree (see Plate XI), I could obtain no definite information except the statement as to the existence of the piebaldism in I. 1 and II. 1, but of the third, III. 2 is said to have had a frontal blaze of white hair and white skin on the neck and forearms, which was very conspicuous owing to its marked contrast with the neighbouring weather-stained normal skin.

III. 4 appears to have been the only member of the family who showed a marked dislike to the condition and always wore a wig to hide the frontal blaze. III. 2, whose family name was C——*, had fifteen children. The first, IV. 2, a male, with dark hair, married twice, and had eight normal children, five by the first wife and three by the second. The second child, IV. 5, was a piebald, with a large frontal blaze, white skin on the front of the neck and arms, and blue eyes. He transmitted the condition to all his three children. V. 3, the eldest boy, aged 22 and unmarried, possesses dark hair, with a V-shaped frontal blaze of white or cream coloured hair, the apex of the V commencing near the coronal suture and spreading out to a width of $3\frac{1}{2}$ inches, as it reaches the forehead. The eyebrows and some of the eyelashes are white. The next boy, V. 4, is aged 18. He has light hair and a very large blaze of unpigmented hair, which covers the whole of the top of the head. His eyebrows and eyelashes are white, and the eyes are blue. Both boys have white patches on the front of the neck and on the arms (see Plate XII).

* Names preserved in the confidential register of the Galton Laboratory.

Next in the fourth generation were twins, IV. 6 and 7, both piebalds. They were evidently not uniovular, because one had dark hair, and one light, and the white blazes were dissimilar in extent, but it is uncertain which had the larger. Both died at an early age.

The next, IV. 8, a girl, was normal with dark hair and eyes and remained unmarried. Next came a woman, IV. 10, who was a piebald with a large frontal blaze, white eyebrows and eyelashes, and white skin on the front of the neck and forearms. *The right eye was blue, and the left brown* (see Plate XIII (B)). Her child, aged 13, is quite normal with light hair and dark eyes.

The next child, IV. 12, Mrs W——, has a large frontal blaze and dark brown irides. There is a large irregular patch of white skin extending from just below the chin to the heads of the clavicles, and round it the skin appears to be more deeply pigmented than the rest of the skin of the neck. There are a few small islands of pigmented skin near the edge of the unpigmented area. The skin of the anterior aspect of the forearms is unpigmented from the elbows to the wrists, and here also, there are some small islands of pigmented skin in marked contrast to the unpigmented area, in which they lie (see Plate XIV).

The first two children of this individual were daughters, V. 8 and V. 9, both piebald, the third a normal son, V. 10, and then three more piebald daughters, V. 11—13. The first of the daughters, Mrs G——, V. 8 (see Plate XIV), is very fair with a very large frontal blaze covering the whole of the top of the head, and her eyebrows and eyelashes are white. Her normal hair has pale creamy diffused pigment and, according to the individual hair, some to a decided number of granules*. The hair of the blaze has no diffused pigment and no granules. The irides are light brown, but the outer segments on both sides are paler and greenish in colour. The skin of the forehead and base of the nose is very pale in colour. She has a large white patch on the skin of the front of the neck, beginning just below the chin and widening out so as to embrace that over the inner ends of both clavicles. As in her mother there appears to be some concentration of pigment round this white area, and there are small isolated areas of pigmented skin near its edge. She has unpigmented skin on the anterior aspect of both forearms.

Of her two children the first, VI. 1, a boy aged 3, is normal, the second, VI. 2, a boy aged $1\frac{1}{2}$, is a piebald (see Plate XIV). This child, VI. 2, was nine months old when first seen. He had a very large frontal blaze, resembling that of his mother and covering all the top of the head, the eyebrows and eyelashes were white with the exception of some of the outer hairs. Hair, pale cream in colour, said to be from the light area, has pale creamy diffused pigment and some granules (β), the granules being very small. It was obvious, even at this age, that heterochromia iridis was present. The right iris was pale except for a sector of dark grey occupying the upper and outer quadrant, the left iris was entirely dark grey. No difference in colour of the skin of the neck or forearm could be made out.

* β to γ on the Galton Laboratory scale of granular pigmentation.

When the baby was seen after the summer of 1913, the grey portions of the irides were becoming brown, the pale portion was still light blue. The face and arms were sunburnt, and it was noticed that the forehead was paler than the rest of the face. There was a pale area on the front of the neck, and the whole anterior surfaces of the forearms were white, the edges being very irregular in contour. There was also a white streak running obliquely right across the posterior or extensor aspect of the left forearm, and this offered a marked contrast with the rest of the surface, which was very brown. When the sunburn had died away the difference between the pigmented and unpigmented skin could no longer be made out.

IV. 12's second daughter, V. 9, aged 23, has only a small cream coloured frontal blaze, and the rest of her hair is light brown (see Plates XV and XVI). The eyebrows are composed of an even mixture of brown and white hairs, and the eyelashes are similar, with brown and white hairs alternating. The irides are grey and uniformly pigmented. There is a large irregular area of white skin at the base of the neck.

The whole of the anterior aspect of the right forearm is unpigmented, and there are similar small areas scattered over the posterior aspect (see Plate XVII). The left forearm is white only on the anterior aspect.

The next girl, V. 11, is aged 9. She has a very small frontal blaze, but the skin of the forehead is pale (see Plate XV). The eyebrows show a division into two parts, on the inner halves grow white hairs only, and on the outer brown hairs. The eyelashes on the contrary consist of alternate brown and white hairs. The irides are grey and uniformly coloured (see Plate XVIII). There is only a small white area in the middle of the front of the neck, but there are well differentiated white areas on the anterior aspects of both forearms (see Plate XIII (A)), and on the inner aspects of both upper arms. Her hair was examined and the first sample showed very pale diffused pigment and some granules (β). Two more samples were then examined, one from the blaze and one from the neighbouring part of the scalp. The first showed no diffused pigment and no granules, the second showed the majority of hairs with yellow-brown diffused pigment and a decided number to plenty of small granules (γ — δ), but a few had no diffused pigment and no granules.

The next piebald child, V. 12, died young. She had a frontal blaze and blue irides. Some of her hair showed very pale diffused pigment, and some granules (β).

The next child, V. 13, also died young. She was a piebald nearer to the classical type than any of the others. She had a large frontal blaze, white skin on the forehead, and large areas of white skin on the front of the neck and chest, and in addition a very extensive area on the abdomen.

Of the fourth generation the next child, IV. 13, was a male with dark hair and eyes, who had 5 normal children; the next, IV. 15, had fair hair and died young. Twins, IV. 16 and 17, came next and died in infancy*. They were heterogeneous,

* The tendency to twin in this family is worth noting.

a dark-haired boy and a light-haired girl. A girl, IV. 19, was born next and she had twin sons, V. 15 and 16, who were also normal. The last three children, IV. 20—22, a girl, a boy, and one whose sex I am unable to ascertain, were all normally pigmented and all died at a very early age.

The pedigree confirms the strongly hereditary nature of piebaldism, and in this as in other published cases the character can affect either sex, but has only been transmitted by those affected. Unless we are to assume that in the case of such a rare anomaly as piebaldism, I. 2, II. 2 or III. 1, were really unnoticed piebalds, then III. 2 could only be heterozygous, or since piebaldism is dominant a (*DR*). We must take IV. 4, IV. 9, IV. 11 and V. 7 for pure recessives (*RR*). Thus the number of piebalds in the five sibships of generations IV, V and VI should be one quarter, i.e. $\frac{1}{4}(15 + 3 + 1 + 6 + 2) = 7$ nearly. We have actually 14 out of 27, thus piebaldism does not seem to act numerically as a pure dominant.

The areas of unpigmented skin are less than in the classical piebalds, but it is probable that in some, at least, they are larger and more numerous than I have stated. On the covered parts of the body and legs, which I was unable to examine except in the baby, they would not be very noticeable. It was not until I had noticed the white skin on the neck and arms of one of them that I was told anything about the existence of similar patches on the others. If true, it is remarkable that none have had white patches on the legs.

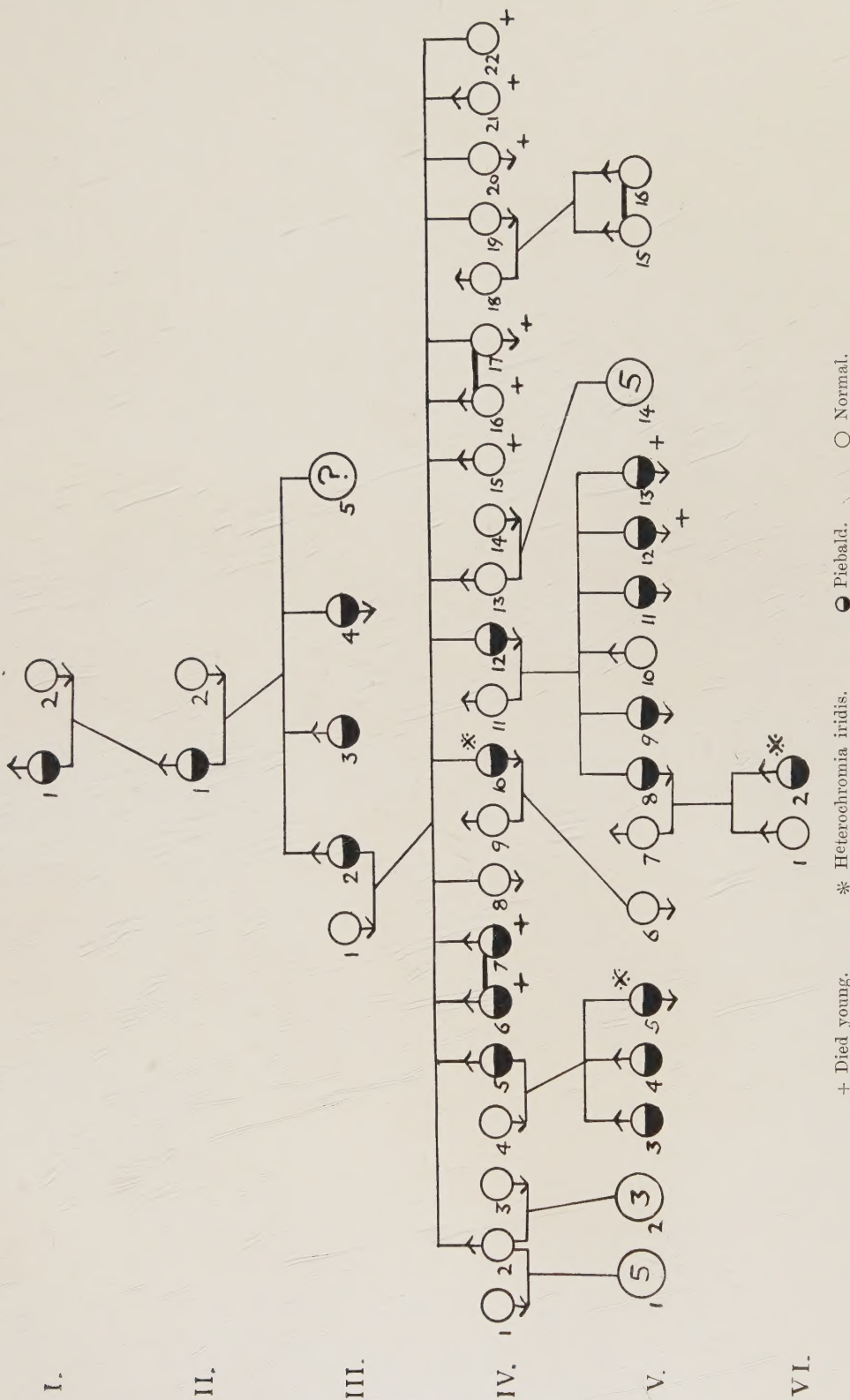
With regard to the local distribution of the pigment, there appears to be an excess at the edges of some of the unpigmented areas, as has been noted in other cases. In the case of other pale areas, the demarcation between them and the normal skin is very slight, and is probably due to the fact that they are not wholly unpigmented. This remark applies especially to the forehead, which in some of them looks paler than natural, but not wholly devoid of pigment.

In some the eyelashes are alternately white and brown, and in others the eyebrows are similar, and in one at least hairs growing on the scalp near the blaze are in some instances entirely without either diffused or granular pigment. This suggests that the skin beneath may show a deficient and irregular distribution of pigment.

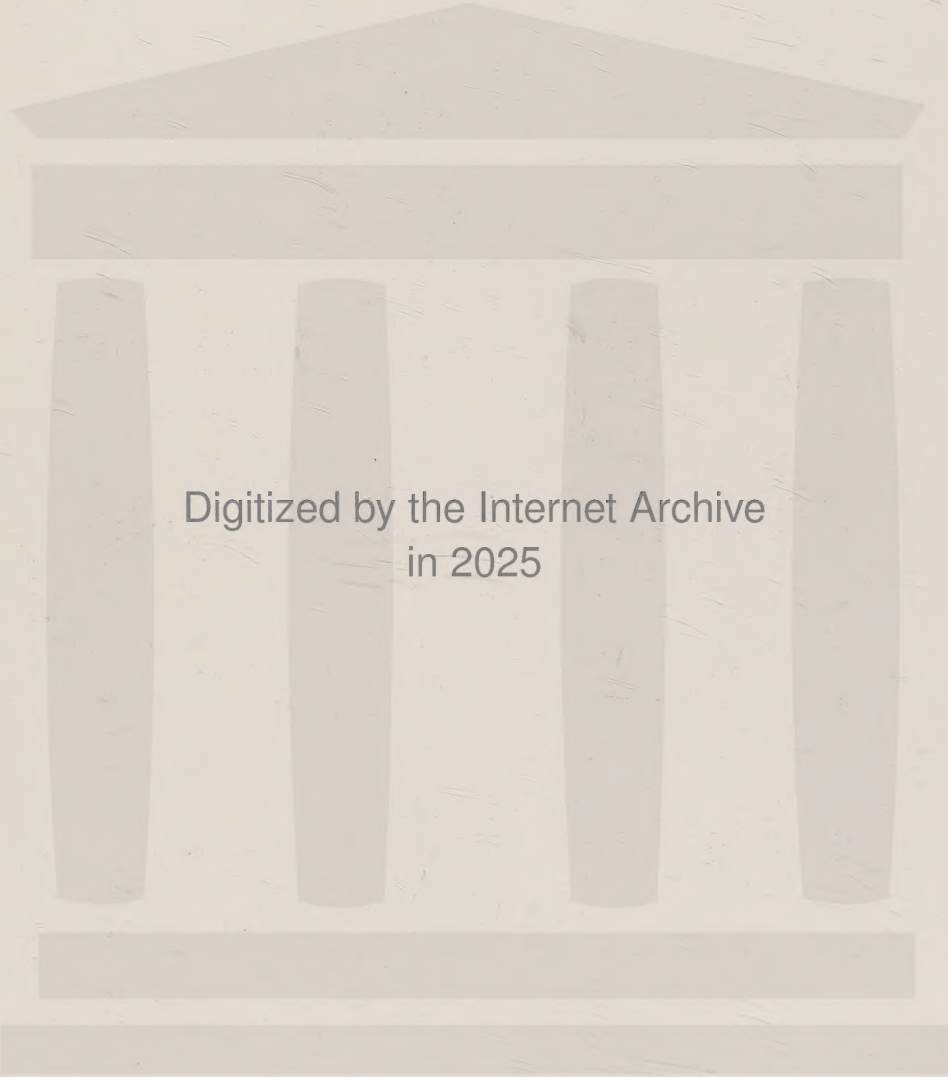
The most interesting feature is the occurrence in three members of the family of well-marked heterochromia iridis, a character which has been met with in members of a piebald family, but always independently of their piebaldism, never, as in this case, in true association with it. It proves conclusively that these cases are not congenital leucoderma.

There seems to be no association of piebaldism and general lack of pigmentation of hair and irides. Affected and unaffected members have been both fair and dark, but the fairest piebalds seem to have the most extensive frontal blazes.

In the cases photographed the individuals were blonds and there has been great difficulty in getting a good photographic contrast of differences of pigmentation very noticeable in the living subject.



Pedigree of Piebald Family.



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V. 3 and V. 4 as children showing their marked V-shaped frontal blazes.



A

V. 11. Showing a well differentiated leucotic area on the anterior aspect of the left arm.



B

IV. 10. The blaze may be seen with difficulty under a lens, but the heterochromia iridis is easily distinguishable.



Piebaldism in three generations, V. 8, mother, IV. 12, grandmother, and VI. 2, grandson. The white frontal hair is visible in all three and the darker section of the right eye of VI. 2 with the use of a lens.



V. 11 and V. 9. Two sisters with white frontal blazes.



V. 9. Showing white forelock or blaze.



Right forearm of V. 9 showing white patches on posterior aspect. The photograph is untouched and it is difficult to bring out by photography the grades of pigmentation when the arm is untanned by the sun, although they are quite clear on actual inspection.



Large photograph of V. 11 to show paleness of forehead and white hairs on inner half of eyebrows.

CLYPEAL MARKINGS OF QUEENS, DRONES AND WORKERS OF *VESPA VULGARIS*.

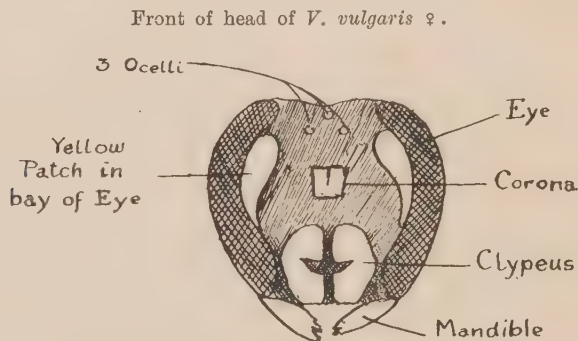
By OSWALD H. LATTER, M.A.

Upon the front of the head of *Vespa vulgaris* certain yellow markings stand out conspicuously upon the otherwise black surface. Below the three ocelli and between the upper portions of the two compound eyes there is a median four-sided yellow patch, the "corona"; to the right and left of this, separated from it by a fairly wide interval, and occupying the bay of each of the compound eyes is a pair of elongated yellow blotches; while straight below the corona and between the lower portions of the compound eyes is a very conspicuous yellow area which extends over the clypeus and down to the labrum or upper lip which lies between the two mandibles. This clypeal patch of yellow bears upon it a black mark which is subject to considerable variation. I distinguish in the queens and workers five chief types of this black mark: see diagrams on p. 202. In Type I a broad vertical black band extends right through the yellow patch from the top to the bottom; a little below its middle the band bears to right and left a pair of bluntly pointed and slightly upturned arms: the portion of the median band below these arms is somewhat narrower than that above. Type II is derived from I by suppression of the black portion below the transverse arms. In Type III the extent of the black colouring is yet further reduced by the absence of the upper half (or thereabouts) of the vertical band. In Type IV the lower part of the vertical band re-appears, but the width of all the components is very much less than in any of the preceding types. In Type V the component parts of the black marking cease to be in contact; the upper portion of the vertical band is interrupted by a broad belt of yellow; the two "arms" are separated from the lower part of what remains and from one another; while there is no black at all below these remnants of the "arms"—a feature recalling Types II and III. Types IV and V are however represented only by single individuals in the series examined.

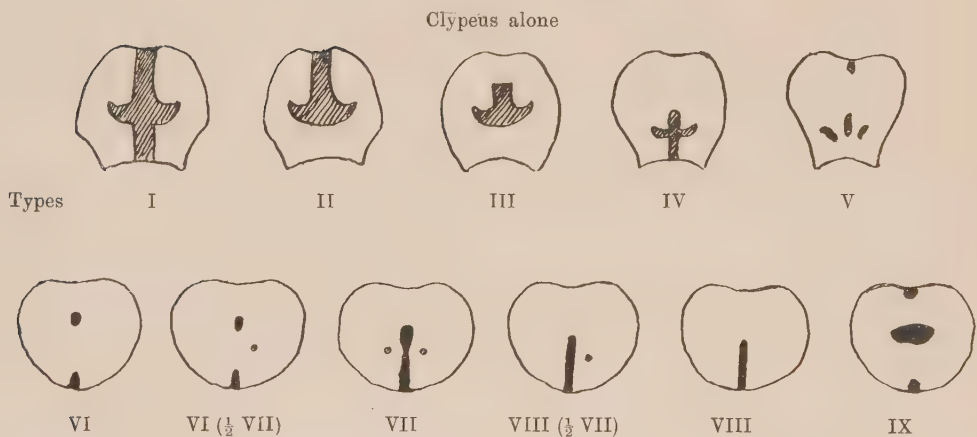
Between these main types certain intermediates occur: Thus some individuals have the black piece below the "arms" very narrow, approximating therefore

Clypeal Markings of Vespa vulgaris

to II, but conforming to I if we take extension of the black right through the yellow as the criterion of I; such individuals are distinguished as I + II. Others again conform to II but possess a slightly darker stain on the yellow in the line where the distinctive lower black portion of I might occur; these are called II + I. Similarly, intermediates between II and III are recognisable: in II + III the top of the upper portion of the vertical black band is very narrow; while in III + II there is a mere stain on the yellow of this region. A single instance occurs of an intermediate between I and III (I + III), where the vertical black band extends right through the yellow, but is much narrowed at its upper extremity.



(All parts left white are actually yellow.)



Drawn by K. W. Merrylies.

My first examination consisted of about 200 tubes containing queens of *Vespa vulgaris* from different nests. In the case of some queens the heads were missing, and in the course of transit the contents of some of the tubes had got loose in the jar. I have numbered these 199 to 208. The results are given in Table I (p. 204) and the summary below:

Type I	Pure	50	}	62
”	I + II	11		
”	I + III	1		
Type II	II + I	22	}	120
”	Pure	94		
”	II + III	4		
Type III	III + II	1	}	4
”	Pure	3		
”	III + IV	0		
Type IV	IV + III	0	}	1
”	Pure	1		
”	IV + V	0		
Type V	V + IV	0	}	1
”	Pure	1		
		Total: 188		

It will be seen that transitional cases undoubtedly occur. The bulk of the queens, however, fall into Types I and II, or queens are very little variable.

To test: (i) whether this variability was still further lessened by taking only the queens from a single nest, and (ii) the relative variability of queens, drones and workers, I now examined all the queens, workers and drones of a single nest of *V. vulgaris*.

In this case *all* the 127 queens were of Type II*.

The classes of the workers are given in Table II (p. 205) and may be summarised as follows:

Type I	Pure	5	}	10
„	v. s or (I + II) ?	5		
Type II	II + I	6	}	162
„	Pure	156		
		Total: 172		

It will be seen that they are somewhat more variable than the queens of the same nest, but not so variable as queens from different nests.

I now turn to the drones of this same nest. I had 150 at my disposal.

The drones exhibit a very wide range of facial markings. In the material examined comparatively few fall into the scheme of classification adopted for the queens and workers, and it thus becomes necessary to resort to six types of face which appear to be peculiar to the male sex. These are numbered VI,

* There were 129 queens in this nest, but No. 34 was missing and No. 98 had its head damaged too badly for classing.

VI ($\frac{1}{2}$ VII), VII, VII ($\frac{1}{2}$ VIII), VIII and IX, see diagrams, p. 202. In Type VI there are two somewhat elongated black dots upon the yellow clypeus, one being sub-central, the other on the ventral margin; in VI ($\frac{1}{2}$ VII) the ventral dot is longer dorso-ventrally and a third dot appears upon the left side (right side, in figure seen from in front) opposite the gap between the two previous dots;

TABLE I.

Types of Clypeal Markings in V. vulgaris Queens.

No.		No.		No.		No.		No.	
1	—	44	III	87	I+II	130	—	173	III
2	II	45	II	88	II	131	II	174	I+II
3	II+I	46	II	89	I	132	II	175	II
4	I+II	47	—	90	II+I	133	II	176	II+I
5	II	48	II+I	91	I	134	—	177	II
6	II	49	II	92	II	135	I	178	II
7	II	50	II	93	III+II	136	II	179	II+I
8	II	51	I	94	—	137	II	180	II+I
9	II	52	II	95	II	138	I	181	II
10	I	53	I	96	II	139	II	182	—
11	II+I	54	II+I	97	I	140	I	183	II+III
12	II+I	55	II	98	II+I	141	I	184	—
13	I	56	—	99	II+III	142	I+II	185	I
14	I	57	II	100	I	143	I	186	II
15	II	58	—	101	—	144	II	187	II
16	II	59	I	102	II	145	II	188	I
17	II	60	II+I	103	I	146	I	189	I+II
18	II	61	II	104	II	147	II	190	II
19	II+I	62	II	105	II	148	—	191	I
20	II	63	II	106	—	149	I	192	I
21	I	64	II	107	I	150	II	193	III
22	I	65	II	108	II	151	II	193 ^a	II
23	I	66	II	109	I	152	—	194	—
24	I+II	67	—	110	I+II	153	II	195	I+III
25	II+I	68	II	111	II+I	154	II	196	II
26	I	69	II+I	112	II	155	II	197	I
27	II	70	I	113	I	156	II	198	II+I
28	I	71	II	114	II	157	II		
29	I+II	72	I	115	II+III	158	II+I		
30	I	73	I	116	II	159	II		
31	II	73 ^a	I+II	117	II	160	I	Loose	
32	II	74	I	118	I	161	I		
33	V	75	II	119	II	162	II		I
34	II	76	II	120	II+III	163	I		I
35	—	77	II	121	II	164	II		I
36	—	78	—	122	II	165	II		II
37	I	79	—	123	—	166	II		II
38	II	80	—	124	I	167	I		II
39	II+I	81	II	125	II	168	—		II
40	II+I	82	—	126	II+I	169	—		II
41 ^a	II+I	83	II	127	II	170	II+I		II
41 ^b	I	84	II	128 ^a	II	171	I		II
42	II	85	I	128 ^b	II	172	—		IV
43	I	86	I+II	129	I+II				

TABLE II.

Types of Clypeal Marking of Workers of a single Nest of V. vulgaris.

No.		No.		No.		No.		No.	
1	—	37	II	74	II	112	II	150	II
2	—	38	II	75	II	113	II	151	II+I
3	—	39	II	76	II	114	II	152	II
4	—	40	II	77	II	115	II	153	II
5	—	41	II	78	—	116	II	154	II
6	—	42	II	79	I	117	II	155	II
7	—	43	II	80	II	118	II	156	II
8	—	44	II	81	II	119	II	157	II
9	II	45	I v. v. s.	82	II	120	II	158	II
10	II		[I+II]	83	II	121	II	159	II
11	II	46	II	84	II	122	II	160	II
12	I v. s.	47	II	85	II	123	II	161	II
	[I+II]	48	II	86	II	124	II	162	II
13	II	49	—	87	II	125	II	163	II
14	II	50	II	88	II	126	II	164	II+I
15	I	51	II	89	II	127	II+I	165	II
16	II	52	II	90	II	128	II	166	II
17	II	53	II	91	II	129	II	167	II
18	II	54	II	92	II	130	II	168	II
19	II	55	II	93	II	131	II	169	II
20	II	56	II	94	II	132	II	170	II
21	II	57	II	95	II	133	II	171	II
22	II	58	I v. s.	96	II	134	II	172	II
23	II		[I+II]	97	—	135	II	173	II
24	II	59	II	98	II	136	II+I	174	II
25	I	60	II	99	II	137	II	175	II
26	II	61	II	100	II	138	II	176	II
27	II	62	II	101	II	139	II	177	II
28	II	63	II	102	II	140	II	178	II
29	II	64	II	103	II	141	II	179	II
30	I v. s.	65	I	104	II	142	II	180	II
	[I+II]	66	II	105	II	143	II	181	—
31	II	67	II	106	II	144	II	182	II
32	II+I	68	II	107	II	145	II	183	II
33	II	69	I	108	II	146	II	184	II
34	—	70	II	109	II+I	147	II	185	—
35	I v. s.	71	II	110	II	148	II	186	—
	[I+II]	72	II	111	II	149	II	187	II
36	II	73	II						

in VII the two median dots are united by a slender black line, and there is a pair of lateral dots, right and left; in VII ($\frac{1}{2}$ VIII) the median line is of uniform width, extending from about the centre to the lower margin, and to its left side there is a single dot; in VIII the median line alone is visible, both lateral dots having disappeared; while in IX there is no continuous median line, but merely two black spots, one at the extreme dorsal and the other at the extreme ventral side of the clypeus. It will be noticed that Types VII—VIII approximate to Type IV in so far as the black stripe begins at about the middle of the clypeus and extends right down to the ventral margin.

The data are given in Table III (p. 207) and are summarised below :

Type I	{very narrow (I + VII)	6 1}	7
Type II	no horns 1, very narrow	1	2
Type III			0
Type IV			0
Type V			0
Type VI	Pure	59	60
"	VI + VII	1	
Type VII			22
Type VIII	(VIII near VI)	8	58
"	(VIII + $\frac{1}{2}$ VII)	2	
"	Pure	48	
Type IX			2
Total :			151

It will be realised at once how far more variable the drones, of even one nest, are than the workers or queens for this character. But their variability is rather of a negative than a positive character, appearing to consist in more or less extensive absence of the fuller markings of queen and worker.

The results here deduced for variability of non-measured characters do not wholly agree with those found by Wright, Lee and Pearson on the wing measurements of the same nest of *V. vulgaris*. They found that for *absolute* measurements the variability as determined by the coefficient of variation was in every case such that the worker was more variable than the drone and the drone than the queen. On the other hand they found when they dealt with *indices* that the drone for wing measurements was slightly more variable than the worker and the queen less variable than either*. Possibly the divergence apparent here may be explicable in the sense of the drone's variability lying in the present case in an absence of marking rather than in any positive variation. The drone's variation is about a centre of much diminished marking. If we could measure the variation in the total area of marking in queen and worker we might find it as great as the variations in the smaller markings of the drone.

It would be of much interest to investigate a series of drones from different nests. It is clear that the clypeal markings form a secondary sexual character and they would probably provide classifications for hereditary purposes.

* *Biometrika*, Vol. v. pp. 414 and 421.

TABLE III.

Types of Clypeal Markings in Drones of a single Nest of V. vulgaris.

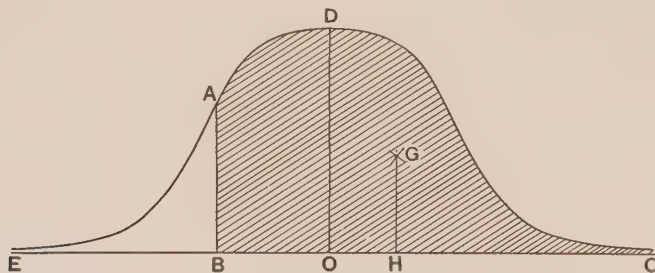
No.		No.		No.		No.		No.	
1	VIII	33	VII	63	VII	93	VI	123	VIII
	near VI	34	VI	64	VI	94	VI	124	I
2	VI	35	VIII	65	VII	95	VI		narrow
3	VIII	36	VI	66	VIII	96	VII	125	VI
4	VIII	37	VI	67	I	97	I	126	VIII
5	VI	38	VII		very		very	127	VII
6	VI	39	VI		narrow		narrow	128	VII
7	VIII	40	VII	68	VI	98	VIII	129	VIII
8	VIII	41	VI	69	VIII	99	VIII	130	VIII
9	VIII	42	VI	70	VI	100	VI	131	VI
10	IX	43	VIII	71	II	101	VI	132	VIII
11	VI	44	VI		but no	102	VIII	133	VI
12	VIII	45	VIII		horns	103	VII	134	VI
13	VI	46	VIII	72	VIII		near VI	135	VI
14	VI	47	VI	73	VI	104	VI	136	VIII
15	VIII	48	VIII	74	VIII	105	VIII	137	VI
	near VI	49	VII	75	VIII	106	VII	138	I
16	VIII	50	VIII	76	VIII	107	VIII		dots of
17	VI	51	VII	77	VI	108	VIII		VII
18	VI	52	VII	78	VI	109	VI	139	VII
19	VIII	53	VIII	79	I	110	VI	140	VI
20	VII	54	VI		narrow	111	VIII	141	VI
21	VII	55	I	80	VI	112	VIII	142	VIII
22	VIII		very	81	VI	113	VI	143	VIII
23	VIII		narrow	82	VIII	114	VIII		near VI
	near VI	56	IX		near VI	115	VI	144	VI
24	VII	57	I	83	VIII	116	VI	145	VIII
25	VII		very	84	VII	117	II		near VI
26	VI		narrow	85	VI		very	146	VIII
27	VI	58	VIII	86	VI		narrow	147	VI
28	VI		near VI	87	VIII	118	—	148	VIII
	$\frac{1}{3}$ VII	59	VIII	88	VII	119	VII	149	VI
29	VIII	60	VIII	89	VI	120	VI	150	VI
30	VIII	61	VIII	90	VIII	121	VI	151	VI
31	VI	62	VIII	91	VII	122	VI	152	VI
32	VIII		$\frac{1}{2}$ VII	92	VII				
	$\frac{1}{2}$ VII								

TABLE OF THE GAUSSIAN "TAIL" FUNCTIONS; WHEN THE "TAIL" IS LARGER THAN THE BODY.

BY ALICE LEE, D.Sc.

In a paper published in *Biometrika*, Vol. VI. pp. 59—68, tables for the incomplete normal moment functions were printed, and they have since been reproduced in *Tables for Statisticians and Biometricians* recently issued from the Cambridge University Press. From these tables values of the Gaussian "Tail" functions were deduced and a short table of ψ_1 and ψ_2 appeared in *Biometrika*, Vol. VI. p. 68. The value of these functions being demonstrated in practice during the last few years, a more complete table of ψ_1, ψ_2, ψ_3 has appeared in the *Tables for Statisticians and Biometricians*.

In the introduction to those tables, however, Professor Pearson indicated that it was important to have a similar table when the "tail" forms more than half the entire curve, and gave the fundamental formulae for obtaining the numerical values of the functions. The present table has been calculated to supply the want thus indicated.



Let the figure represent a Gaussian curve of total population N and standard deviation σ . Let AB be the ordinate at which it is truncated and let

$$OB = h = k' \times \sigma.$$

Let GH be the ordinate through the mean G of the truncated portion and $BH = d$, the distance of the mean from the line of truncation, let Σ be the standard deviation of the truncated portion about GH , and n = the area of the truncated portion, or of the population observed. Then if any material be supposed to

form a truncated portion of a normal curve, d , n and Σ can be found (see *Tables*, pp. xxvii and 25).

We have

$$\psi_1 = \Sigma^2/d^2 \dots\dots\dots(i),$$

$$\psi_2 = \sigma/d \dots\dots\dots(ii),$$

$$\psi_3 = N/n \dots\dots\dots(iii).$$

These are tabled for each value of h' , at first proceeding by .01 and then by .10 as unit. Now ψ_1 being known we find h' from the table, and hence deduce ψ_2 and ψ_3 . ψ_2 gives us the value of σ from known d . Hence $h = h' \times \sigma$ can be found, lastly (iii) gives us the total population from which n is drawn. Thus the constants N , σ and h which fix the total Gaussian are determined.

It will be sufficient to illustrate the method of using the tables on certain data as to the English thigh-bone, recently published by Parsons*.

Dwight† has adopted a method of sexing human femora on the basis of a markedly bimodal distribution obtained by him for American bones. He terms female any femur with diameter of head less than 45 mm., and male any femur with diameter of head over 47 mm. Parsons follows this rule and sexes by other points femora with heads from 45 to 47. As unsettled remainder he has 20 femora of 45 mm. and he gives 12 to ♀ and 8 to ♂; of 46 mm. and 47 mm. he has 41 femora and he gives 4 to ♀ and 37 to ♂. As a result of this process he obtains a female frequency curve which rises very abruptly at high values of the diameter, and a male frequency curve which rises very abruptly for low values of the femora. But, if there really be any marked skewness in frequency of the parts of the human skeleton, which is very unusual, we should anticipate that it would be of the same sense. Parsons' distributions are as follows (*loc. cit.* p. 256):

	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
♀	1	1	—	3	8	14	12	18	12	12	3	—	1	—	—	—	—	—	—	—
♂	—	—	—	—	—	—	—	—	—	8	8	29	17	31	19	13	10	6	8	2

The ♀ 48 mm. femur according to the rule should have been treated as a male but presumably it had marked female characters. Were there no marked male characters in any bone below 45 mm.? It will be seen that there is a remarkable dip in the total material at 46 mm. which corresponds to Dwight's division. In material measured six years ago in the Biometric Laboratory, where every bone in a relatively large series was measured, no such dip occurs and there is in those data no justification for Dwight's method of sexing‡. The group of 29 ♂ bones at 47 mm. and the sudden cut off at 45 mm. seems to condemn this method of sexing, at any rate from the statistical standpoint.

* *Journal of Anatomy and Physiology*, Vol. XLVIII. pp. 238—267.

† *American Journal of Anatomy*, Vol. IV. p. 19.

‡ This material has been statistically reduced and will shortly be published.

Without arguing this point out here, we may illustrate the use of the Table (p. 214) of ψ 's by taking two of Parsons' frequency distributions for females; we will cut them off at the points suggested, and then investigate the total populations of females which result. Our author pools for these distributions right and left bones.

Taking the diameter of "head of femur" for the females, we have

Diameter in mm. ...	36	37	38	39	40	41	42	43	44
Frequency ...	1	1	—	3	8	14	12	18	12

These are exactly the bones the Dwight process gives as female. We find

$$\Sigma^2 = 2.8851,$$

$$d = 2.6159 \text{ (measured from 44.5).}$$

$$\text{Hence} \quad \psi_1 = \Sigma^2/d^2 = .4216.$$

Whence by interpolation from the table

$$h' = .782, \quad \psi_2 = .864, \quad \psi_3 = 1.278,$$

$$\text{leading to} \quad \sigma = 2.260, \quad h = 1.767,$$

$$\text{and} \quad \text{Mean} = 42.73 \text{ mm.}, \quad N = 88.2.$$

Parsons gives for R. femur, Mean = 43,

L. femur, Mean = 42,

and the total number of bones dealt with $55 + 48 = 103$ (Tables, *loc. cit.* pp. 249—251). In his frequency distribution (p. 256) he only records 85 female bones, which give a mean of 42.54 and a standard deviation of 2.078 mm. These values are clearly not widely divergent from those we have found above by supposing all bones under 45 to be female.

To test the matter further the 105* female bones of which the head was measured by Parsons were taken out. They provide the distribution:

Diameter in mm. ...	36	37	38	39	40	41	42	43	44	45	46	47	48
Frequency ...	1	1	—	4	10	18	16	24	13	14	3	—	1

These give
$$\left. \begin{array}{l} \text{Mean} = 42.47 \\ \text{s.d.} = 1.996 \end{array} \right\} N = 105.$$

* It is not possible to say whether he has omitted two queried measurements. He has not omitted bones he queries in breadth of lower articulation.

Cutting off all bones over 45.5 we find

$$\Sigma^2 = 2.6392, \quad d = 2.6264,$$

leading to $\psi_1 = .3826$.

Hence $h' = .984$, $\psi_2 = .783$ and $\psi_3 = 1.195$.

These provide for the non-truncated population,

$$\text{Mean } 42.48, \quad \text{s.d.} = 2.056, \quad N = 104,$$

which are in still better agreement with Parsons' constants for the 105 bones than the constants for the 85 bones were for their series. It would appear therefore that, if we suppose all bones under 45 female and use our Tables, we get results in reasonable accordance with Parsons', and possibly by a theoretically more justifiable method than endeavouring to sex the bones above 44 and below 48 from other characters.

We have considered from the same aspect the character breadth of lower articular end of femur. Parsons' distribution of 89 female femora is as follows (p. 257):

Breadth in mm. ...	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Frequency ...	1	—	—	5	6	8	6	17	13	13	8	6	3	1	1	1

In this Table he has only one bone in excess of the numbers on which he bases his means on pp. 250—1. If we truncate at 69.5, i.e. reject all bones over 69 mm., we find

$$\Sigma^2 = 3.9803, \quad d = 2.9058,$$

and $\psi_1 = .4714$.

Hence we deduce

$$h' = .5295, \quad \psi_2 = .977, \quad \psi_3 = 1.426.$$

These lead to

$$h = 1.503, \quad \sigma = 2.839, \quad N = 98.4, \quad \text{Mean} = 68.00 \text{ mm.}$$

The actual values given by Parsons' distribution above are

$$\sigma = 2.571, \quad N = 89, \quad \text{Mean} = 67.54 \text{ mm.}$$

Thus the agreement is not nearly so good as for the diameter of the head of the femur, being about 10% wrong in σ and N . It should give as good a result if the method were quite satisfactory, for the bones have been sexed by the diameter of the head, and the limit 44 mm. for diameter of the head corresponds fairly closely to 69 mm. for the breadth of lower articulation.

As this paper is not intended as a discussion of Parsons' data, to which we hope again to return, we will only deal with one more illustration of the use of

the Table. We take out from his Tables, pp. 244—248, the diameter of head of femur for 174 male bones.

Diameter of Head in mm. ...	45	46	47	48	49	50	51	52	53	54	55
Frequency	9	10	33	17	38	20	18	12	6	8	3

The constants of this distribution are

Mean = 49·14, σ = 2·377, N = 174.

Truncating at 47·5 we have

Σ^2 = 3·4341, d = 2·7869,

whence ψ_1 = ·4422, and from the Table

h' = ·679, ψ_2 = ·908, ψ_3 = 1·331.

These lead to

h = 1·7182

and

Mean = 49·22, σ = 2·530, N = 162,

i.e. to a "tail" of 40 not one of 52 below 47·5. Actually this tail distributes itself as follows:

	Under 45	45	46	47
Gaussian tail	5	6	12	17
Against Parsons'	0	9	10	33

This confirms our previously expressed view that probably a considerable number of the bones classed as 47 mm. are really female femora, and that the male distribution runs considerably beyond 45 mm. into the range treated as purely female.

Finally let us try the result of pooling male and female bones and breaking up the composite frequency by the method of *Phil. Trans.* Vol. 185 A, p. 84.

We have now 279 bones distributed as follows:

Diameter of Head { in mm.	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	Total
Frequency ...	1	1	—	4	10	18	16	24	13	23	13	33	18	38	20	18	12	6	8	3	279

The constants are

Mean = 46·63, S.D. = 3·93,

μ_2 = 15·4040, μ_3 = - 6·7791,

μ_4 = 541·5162, μ_5 = - 530·5339.

The nonic is

$q_2^9 - 5·9618q_2^7 + ·0689q_2^6 + 9·8357q_2^5 - 3·4275q_2^4 - 8·2041q_2^3 - ·3020q_2^2$
 $+ ·0144q_2 - ·0097 = 0,$

giving the root q_2 = - ·934 and p_2 = - 9·34,

and ultimately the two components :

	<i>Male</i>	<i>Female</i>
Mean	49·83	43·72
Population	133·25	145·75
Standard Deviation	2·231	2·662
Max. Ordinate %	23·83	21·84

While the means agree roughly with those obtained by Parsons' sexing (49 and 43), we see that this analysis much more nearly equalises the number of male and female bones, and indeed makes the female population rather larger than the male, while Parsons has 79 % more males. The "truncated tail" method would probably give results in better accordance with the present had we not truncated at the quite arbitrary Dwight-Parsons' divisions.

These examples may suffice to illustrate the application of the Tables to anthropometric measurements on man, where we can feel fairly confident that the material, if sufficient in quantity, would be adequately described by a Gaussian or normal distribution. Such cases may arise when material for the two sexes, or for two races, is commingled and we can be fairly certain that one or other or both "tails" of the material present homogeneous parts of the mixture.

Another illustration drawn from Galton's data for American trotters will be found in the *Tables for Statisticians*, p. xxvi. The chief weakness of the method, besides the assumption of the Gaussian, often quite legitimate, is the absence as yet of the values of the probable errors, which values must be very considerable for slender material such as that used above.

See following page for Table of Gaussian "Tail" Functions.

Table of the Gaussian "Tail" Functions

Table of Gaussian "Tail" Functions, "Tail" larger than "Body."

h'	ψ_1	$(-)\Delta\psi_1$	ψ_2	$(-)\Delta\psi_2$	ψ_3	$(-)\Delta\psi_3$	h'
.00	.571	.002	1.253	.006	2.000	.016	.00
.01	.569	.002	1.248	.006	1.984	.016	.01
.02	.567	.002	1.242	.006	1.969	.015	.02
.03	.565	.002	1.236	.006	1.953	.015	.03
.04	.564	.002	1.231	.006	1.938	.015	.04
.05	.562	.002	1.225	.006	1.923	.015	.05
.06	.560	.002	1.219	.006	1.909	.014	.06
.07	.558	.002	1.214	.006	1.894	.014	.07
.08	.557	.002	1.208	.006	1.880	.014	.08
.09	.555	.002	1.203	.005	1.866	.014	.09
.1	.553	.018	1.197	.054	1.852	.126	.1
.2	.535	.018	1.143	.053	1.726	.108	.2
.3	.516	.019	1.090	.051	1.618	.093	.3
.4	.497	.019	1.040	.049	1.526	.080	.4
.5	.477	.019	.991	.047	1.446	.068	.5
.6	.458	.020	.944	.045	1.378	.059	.6
.7	.438	.020	.899	.043	1.319	.050	.7
.8	.419	.020	.857	.041	1.269	.043	.8
.9	.399	.019	.816	.039	1.226	.037	.9
1.0	.380	.019	.777	.037	1.189	.032	1.0
1.1	.361	.019	.740	.035	1.157	.027	1.1
1.2	.342	.018	.704	.033	1.130	.023	1.2
1.3	.323	.018	.671	.033	1.107	.019	1.3
1.4	.305	.017	.640	.030	1.088	.016	1.4
1.5	.288	.017	.610	.028	1.072	.014	1.5
1.6	.271	.016	.582	.026	1.058	.011	1.6
1.7	.254	.016	.556	.025	1.047	.009	1.7
1.8	.239	.015	.531	.023	1.037	.008	1.8
1.9	.224	.014	.508	.022	1.030	.006	1.9
2.0	.210	.013	.487	.020	1.023	.005	2.0
2.1	.197	.013	.466	.019	1.018	.004	2.1
2.2	.184	.012	.447	.018	1.014	.003	2.2
2.3	.172	.011	.429	.017	1.011	.003	2.3
2.4	.161	.010	.413	.016	1.008	.002	2.4
2.5	.151	.010	.397	.015	1.006	.002	2.5
2.6	.141	.009	.383	.014	1.005	.001	2.6
2.7	.132	.008	.369	.013	1.003	.001	2.7
2.8	.124	.008	.356	.012	1.003	.001	2.8
2.9	.116	.007	.344	.011	1.002	.001	2.9
3.0	.109		.333		1.001		3.0

See *Tables for Statisticians and Biometricians*, Introduction, p. xxvii.

CONTRIBUTION TO A STATISTICAL STUDY OF THE CRUCIFERÆ.

VARIATION IN THE FLOWERS OF *LEPIDIUM DRABA* LINNÆUS.

By JAMES J. SIMPSON, M.A., D.Sc.

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I. INTRODUCTORY*.

In the summer of 1905 Professor J. W. H. Trail drew my attention to an extraordinary example of variation which occurred in the several organs of the flowers of *Lepidium Draba* Linnæus. At that time I examined in detail 1832 individual flowers taken from a single plant growing in a piece of uncultivated

* I am pleased to have this opportunity of expressing my great indebtedness to Dr J. F. Tocher for invaluable assistance in the biometric part of this paper. The correlation and other constants were calculated in his laboratory, and without his assistance the publication of this paper would have been greatly delayed. I must also thank Professor Karl Pearson, in whose department in University College, London, the statistical study was originally undertaken, for reviewing this paper for publication and also for much kindly criticism and advice. To Professor Trail my thanks are also due for many botanical hints.

ground in his garden, and the results of these observations form the basis of the present contribution to the study of the variation in the Cruciferae.

Botanical problems, which have been hitherto attacked from the biometric standpoint, have been comparatively easily handled, because the material has been more or less homogeneous in character. For example, variations in the number of sepals of *Anemone nemorosa** or in the number of ray-florets of *Chrysanthemum leucanthemum*†, and the consequent distribution of these are capable of direct treatment by Pearson's well-known method of fitting frequency curves.

The only work comparable to the one in hand occurs in *Biometrika*, Vol. II. p. 145 (Variation and Correlation in the Lesser Celandine), but in this case the numbers of members in the calyx, corolla and andrœcium have been examined as a basis for a study of homotypic correlation and in this flower each of these organs consists of a single constituent with numerous members.

The problems studied in this paper, however, are more complex inasmuch as they deal *not* with *one* organ of the flower but with all the organs, their constituents and members both separately and collectively.

It is also, I believe, the first biometric work of its kind on a cruciferous flower and embodies a study of chorisis, that is, "the splitting up or division of one or more components of a flower into two or more equal or unequal parts"—a factor which is supposed to have been of the utmost importance in the evolution of the natural order—Cruciferae. A complete discussion of this phenomenon is reserved until the flower is studied in detail.

It would be well here to emphasise the fact that the flowers examined for this study were not taken from different plants but, on the contrary, were obtained from several inflorescences growing on stems which had arisen from buds on the roots of a single parent plant. This mode of reproduction is rather unusual, but, in the present instance, is of particular interest inasmuch as it gives greater homogeneity to the material.

The parts of the flower which have been considered are (*a*) the perianth, which consists of (1) the calyx and (2) the corolla, (*b*) the andrœcium and (*c*) the gynœcium.

The functional differentiation of these organs is of great importance in the interpretation of results so that it might be well to recall the particular rôles which these play in plant economics.

The gynœcium and the andrœcium are respectively the female and male organs of reproduction and consist of carpels and stamens, while the perianth forms a protective covering for these delicate structures. The calyx or outer organ of

* Yule, *Biometrika*, Vol. I. p. 307.

† *Biometrika*, Vol. II. p. 309 et seq.

the perianth is concerned solely in the protection of the flower in the bud, but the corolla, in the open flower, also serves, along with the honey-secreting sacs at the base of the stamens, as an attraction for insects.

The *characters* which have been taken as a basis for this study are *numerical*, e.g. the number of petals in the corolla, but no measurable characters, e.g. the length and breadth of the petals, have been considered, although, as will be pointed out later in connection with possible future studies in this flower, these characters might also with advantage be taken.

The Cruciferae, as an order, are usually regarded by botanists as being very definite in type and no observations have been recorded to show to what extent, if any, deviation from the recognised botanical floral formula exists, so that the main object of this paper was to determine the frequency of the variability of the parts of the various organs and constituents, and also the degrees of correlation existing between the organs themselves.

The mode of observation is worthy of remark, however, as it might well be argued that if the flowers used for examination were fully "blown" deficiency in the number of parts *might* be due to post-developmental fracture, but in all the cases here recorded the observations were made on flowers in bud or only half open so that the influence of wind or other external agency is altogether discounted. The material was also examined microscopically in all cases so that there should be no possible doubt as to the exact origin of any member. The importance of this will be seen in the details of the analysis.

II. BOTANICAL.

1. *Specific characters.*

The generic and specific characters of *Lepidium Draba* may be obtained in any complete systematic botanical work so that it is unnecessary to repeat them here, but a few notes bearing especially on the study in hand may be of value.

It is a perennial about a foot in height and is covered by a minute down from which its popular name, the *hoary cress*, is derived. The inflorescence is a raceme not much lengthened and so forms a broad, almost flat, corymb-like termination. The individual flowers are small, white and numerous. The constituents of calyx, namely the *sepals*, are green; they are short, nearly equal and bear no pouch at the base. The *petals* are small and white; they are equal in size, obovate, undivided and generally stalked. The *stamens* are six in number; the filament is simple, i.e. it bears no appendages, and is shorter than the petals; the anther consists of two roundish lobes. The *Pods* are "broader than long"; they are compressed laterally at right angles to the narrow partition. The thick valves are boat-shaped and sharply keeled but not winged; each valve contains a single seed.

2. *Morphology of the flower.*

The typical flower consists of six whorls, made up in the following manner:

- (a) Calyx 2.
- (b) Corolla 1.
- (c) Andræcium 2.
- (d) Gynæcium 1. (Plate I, fig. 1.)

(a) *Calyx*. This organ is composed of two whorls each consisting of two sepals. The outer pair arise at one level on opposite sides of the flower and are inserted on a slightly lower plane than the inner pair; they are parallel to the plane of compression of the gynæcium. The inner pair are also situated opposite one another but in a plane perpendicular to that of the outer pair; they are thus at right angles to the plane of compression of the gynæcium. These whorls are denoted on Plate I, fig. 1, by the Roman numerals I and II respectively.

(b) *Corolla*. This organ consists of four petals all inserted at *one level* and alternating with the position of the sepals; they thus constitute a single whorl. (See III, Plate I, fig. 1.)

(c) *Andræcium*. Six stamens form the andræcium; they arise at two different levels and thus constitute two separate whorls. The outer whorl, which is lower down, consists of two stamens which are shorter than the others and correspond in position to the inner sepals. The inner whorl consists of four stamens, arranged in pairs which correspond in position to the outer sepals. (A reference to the figure (Plate I, fig. 1) in which the two whorls are marked IV and V respectively will make this clear.)

(d) *Gynæcium*. This organ consists of two carpels forming the sixth or innermost whorl. (See VI, Plate I, fig. 1.)

It will be seen from the foregoing description that the order of the six whorls here detailed is that in which they would be found were we to strip the flower of its components at the different levels consecutively from below upwards. It is also the order in which we would find them, passing from the outside to the centre, were we to cut a transverse section through the flower.

Another point, however, which is not so obvious but one which has special interest in our study, is the fact that this is also the order in time of development.

The actual sequence in which these constituents of the flower appear in the bud is therefore:

- I. Outer whorl of Calyx (Sepals).
- II. Inner whorl of Calyx (Sepals).
- III. Corolla (Petals).
- IV. Outer whorl of Andræcium (Stamens).
- V. Inner whorl of Andræcium (Stamens).
- VI. Gynæcium (Carpels).

3. *Conception of Choris.*

Choris or reduplication is generally looked upon by botanists as a means of multiplication of the parts of a flower. It consists in the division or splitting of an organ in the course of its development by which two or more organs are produced in place of one. Choris may take place in two ways:

(1) *transversely*—when the increased parts are placed one before the other, that is, the resulting components are on the same radius; this is known as *vertical*, *parallel* or *transverse* choris;

(2) *collaterally*—when the increased parts stand side by side, that is, on the same circumference.

Transverse choris is supposed to be of frequent occurrence; thus the pistils of *Lychnis* and many other caryophyllaceous plants exhibit a small scale on the inner surface at the point where the limb of the petal is united to the claw. The formation of these scales is supposed by many to be due to the choris or unlining of an inner portion of the petal from the outer.

Collateral choris is seen in different natural orders. In *Strephanthus*, in place of two stamens there is sometimes a single filament forked at the top and each division bears an anther. This is usually supposed to be due to collateral choris arrested in its progress.

The flowers of the Fumitory are also generally considered to afford another example of this type of choris. In these we have two sepals, four petals in two rows and six stamens, two of which are perfect and four more or less imperfect. The latter are said to arise by collateral choris, one stamen being divided into three parts.

Collateral choris may be compared, according to Bentley, to a compound leaf which is composed of two or more distinct and similar parts.

Let us now consider choris in its bearing to the flower under consideration. In the description of the morphology of the flower we noted that in the inner whorl of the andrœcium there were four stamens arranged in pairs while in the outer whorl there were only two stamens situated singly. Various opinions have from time to time been advanced to explain this anomalous structure so that it might be well to briefly review these. Of the andrœcium of the Cruciferae Oliver says: "The two pairs of long stamens are generally thought to be due to choris or the division in the course of development of single antero-posterior stamens. Others have thought that the six glands represent abortive stamens and that these with the six stamens make up a normal series of twelve in three whorls."

De Candolle held the view that the stamens formed a single, originally tetramerous whorl alternating with the petals in which the median members, i.e. the anterior and posterior, were cleft (chorised) in two. Since however the lateral stamens are inserted lower down than the median stamens and are also,

as already pointed out, formed earlier in the bud, this view is clearly untenable. Two whorls must be taken into consideration owing to the difference in the levels of insertion, the single stamens being lower down. Kunth, Wydler, Chatin and others regard these two whorls as typically four-membered (tetramerous), those of the outer whorl corresponding in position to the sepals, those of the inner whorl corresponding in position to the petals. To arrive at a typical cruciferous flower from this, two stamens in the outer whorl abort, while the individuals of the two pairs of the inner whorl come together. (Plate I, fig. 3.)

Others (Krause, Wretschko and Duchartre) regard the outer whorl as typically dimerous (i.e. with two constituents) and the inner whorl as typically tetramerous (i.e. four-membered).

The more modern view, however, regards both whorls as dimerous but the inner one chorised collaterally thus giving the typical cruciferous flower.

The reasons put forward to support this theory are as follows :

(1) The upper long stamens are usually *paired* in the median line, also sometimes *coherent*. Further, in place of one or both of the pairs, there occurs sometimes a single stamen—a hint at reversion, or one or both pairs may be replaced by three or more—a suggestion of further chorisism.

(2) In the earliest visible stage of development in the bud it may be seen that each pair of stamens arises from a single wart-like projection and that division is therefore a secondary result. This is not very easily demonstrable in the Cruciferae but is more evident in a closely allied family, the Capparidaceae.

Since the present study includes numerical variation in the different constituents and positions of the androecium it will be interesting to note to what extent any one of these theories is borne out by the variations in this flower.

4. *Orientation of the flower.*

Having defined the positions of the various stamens relative to one another, in what is usually regarded as a normal cruciferous flower, let us now consider the different possibilities when the flower is abnormal.

Suppose that one of the pairs of stamens of the inner whorl is represented by a single stamen, that is, suppose that chorisism had not taken place. Now with regard to the peduncle of the inflorescence this stamen might be placed in two diametrically opposite positions, namely (1) it might be adjacent to the peduncle (Plate I, fig. 4) or (2) it might be on the distal half of the flower with reference to the peduncle (Plate I, fig. 5).

Two questions now arise, (1) do non-chorised stamens occur as frequently as chorised stamens on the side of the flower next to the peduncle? or (2) do either of these occur with greater frequency in this adjacent position?

According to which of these questions is answered in the affirmative must we conclude whether there is any connection or correlation between the proximity of

the chorised stamens to the peduncle and choris. The former would suggest no correlation, whereas the degree of correlation hinted at by the latter would depend on the frequency of the occurrence.

We have so far considered only two possible positions, viz. a non-chorised stamen adjacent to the peduncle, i.e. in the proximal half of the flower, and a non-chorised stamen opposite to the peduncle (i.e. in the distal half of the flower with reference to the peduncle), but the question naturally arises "Are these the only two possible relative positions which might occur?" Might the petiole not twist so as to bring the hypothetical non-chorised stamen into any position varying from 0° to 180° with reference to the original plane?

Let us illustrate this by means of the Figure 6, Plate I.

Taking the position of the peduncle as our fixed point the non-chorised stamen might occupy the "adjacent" position a or the "opposite" position a1. A rotation of the petiole, however, might cause this stamen to occupy any of the positions marked a2, a3 or a4 or even any intermediate position between a and a1 on *either side* of the vertical plane $A-B$, in the horizontal plane a, a4, a2, a3, a1.

In a study of the variations in this flower, this is precisely what was found to occur, i.e. the distribution was equal round a fixed point so that we are unable to say whether there is any connection between the proximity of the non-chorised stamen to the peduncle and choris or not.

But the full bearing of this consideration does not end here. The orientation of the flower is of practical importance in fixing a basis on which to establish a grouping of the different variations. Any analysis of the data is impossible unless some definite part of the flower be agreed upon as a starting point.

Now we have seen that the position of the peduncle with respect to any definite stamen does not require to be taken into consideration. Consequently we may take either of the two stamens of the *outer* whorl, which correspond in position to the outer sepals and which are "normally" non-chorised, as our fixed point and call it 1; the stamen opposite, i.e. in the same whorl, we shall call 2; the chorised pair of the *inner* whorl to the left (or in the floral diagram above) may be termed 3 and 4; while the corresponding pair to the right (or in the floral diagram below) would thus be 5 and 6 (Plate I, fig. 7).

Where variations occur in any of these stamens we shall hereafter refer to those as occurring in "position" 1, 2, 3, 4 and 5, 6 respectively.

On this basis of symmetry, it will simplify matters considerably if we regard as 1, in flowers in which either of the two outer stamens is modified, that one which still maintains its original character while, on the other hand, if both are modified, that one which retains the greatest approximation to normality, e.g. if one be chorised while the other is not, the latter would be in position 1; or if one

were chorised while the other was only partially chorised* the latter would again be in position 1.

Following on this it is at once seen that where both are normal or where both are equally abnormal it makes absolutely no difference which position we choose as 1.

III. EXAMINATION OF THE DATA.

1. *Classification.*

Considerable difficulty was experienced in classifying the variations owing to these occurring in so many different forms yet with so few characteristics in common as to warrant their inclusion in definite classes.

The total number of flowers examined was 1832, of which 1062 had the accepted normal structure (see page 218). The remaining 770 showed variation in different degrees of advance or regression, i.e. there was an excess or deficiency in the number and structure of the members of the various organs. Thus we see that there was a deviation from the accepted normal structure in over 42 per cent. of the individuals examined.

The perianth has been selected as a basis for classification and Table A shows the sub-divisions which have been adopted. Amongst those flowers in which the

TABLE A.

	Number of Variations	Number in Group	Number in Sub-class	Number in Class	Variations in the Class
Class I. Perianth normal	—	—	—	1687	—
Sub-Class A. Gynæcium normal	—	—	1680	—	—
Group (a). Androecium normal	1	1062	—	—	—
Group (b). Androecium abnormal	57	618	—	—	—
Sub-Class B. Gynæcium abnormal	—	—	7	—	—
Group (a). Gynæcium one carpel	2	4	—	—	—
Group (b). Gynæcium reduplicated	2	3	—	—	62
Class II. Perianth abnormal	—	—	—	115	—
Sub-Class A. Calyx normal, corolla abnormal	—	—	55	—	—
Group (a). Gynæcium normal	11	54	—	—	—
Group (b). Gynæcium a single carpel	1	1	—	—	—
Sub-Class B. Both calyx and corolla abnormal	—	—	60	—	—
Group (a). Gynæcium normal	11	46	—	—	—
Group (b). Gynæcium a single carpel	6	14	—	—	29
Totals	91	1802	1802	1802	91

* For the present we use the terms "chorised" and "chorisis" in the sense of the definition already given.

perianth was normal there were no fewer than 62 different types of variation, and amongst those in which the perianth showed a departure from the accepted normal structure there were 29 types of variation. Thus of 1802 flowers examined, 1062 had the typical cruciferous structure, 625 had the perianth normal but the andrœcium and gynœcium modified in 62 different ways and 115 had all three organs modified in 29 different types of variation.

The remaining 30 individuals are not capable of classification under the foregoing scheme but have been grouped into three classes as shown in Table B.

TABLE B.

								Number of Variations	Number of individuals in the Class
Class III.	Reduplication of parts but flowers not separate	...						10	11
Class IV.	Reduplication of parts with flowers separate	...						6	17
Class V.	Part of a flower replaced by a flower	...	...					2	2
Totals	...	...	...	...	...	...	...	18	30

Altogether, therefore, there are five separate classes which give a total of 109 different modes of variation.

2. *Analysis.*

In the further reduction of the data it is essential that we consider the variations in the stamens, and for this purpose we must naturally commence with Class I, Sub-class A.

To avoid describing each of these in detail, it is necessary to have recourse to a graphic method of representation. Several such methods suggested themselves and although none are ideal we have chosen one which may help to give a true impression of the various modifications assumed by the andrœcium. We shall also give a few examples by another method which might have been adopted but which seems to us to be even more complicated.

Let us, in the first place, consider in what directions abnormalities have occurred. A typical stamen consists of two parts, (1) the filament and (2) the anther.

(1) Filament. This may be of its normal length or less than its normal length or altogether absent.

(2) Anther. This may be present or absent.

But other complications arise. As already explained, in the accepted typical cruciferous flower, chorisis has taken place in positions 3.4 and 5.6 so as to give

rise to two stamens in each of these positions. Now, we find that, in certain flowers chorisis has only partially taken place and in others it has not occurred at all so that we have thus another three possibilities to consider.

In describing the andrœcium, therefore, we must (1) define the position of each stamen to which we refer, (2) state the nature of the filament, (3) note the presence or absence of the anther and (4) emphasise the nature of the chorisis.

Let us use the following symbols 1, $\frac{1}{2}$ and 0.

1 with reference to	$\left\{ \begin{array}{l} \text{Filament, indicates that it is present and complete.} \\ \text{Chorisis, indicates that it is total or complete.} \\ \text{Anther, indicates that it is present.} \end{array} \right.$
$\frac{1}{2}$ with reference to	$\left\{ \begin{array}{l} \text{Filament, indicates that it is only half-length.} \\ \text{Chorisis, indicates that it is only partial.} \end{array} \right.$
0 with reference to	$\left\{ \begin{array}{l} \text{Filament, indicates that it is absent.} \\ \text{Chorisis, indicates that it has not taken place.} \\ \text{Anther, indicates that it is absent.} \end{array} \right.$

We have already fixed upon our nomenclature for the various positions; these are 1; 2; 3.4; and 5.6. To avoid descriptions and at the same time give a graphic representation of the floral formula of the andrœcium the following system might be adopted:

- (1) Place the whole floral formula within square brackets thus [].
- (2) Place positions 1; 2; 3.4; and 5.6 within curled brackets thus { }; and
- (3) Place individuals, i.e. 1, 2, 3, 4, 5 and 6, within rounded brackets thus ().

Expanding this with reference to a normal flower we would have for the andrœcium only

$$[\{ 2 \} \{ 2 \} \{ 2 \}],$$

or still further in the order of Filament, Chorisis, Anther, Stamen

$$[\{ (1.0.1)(1.0.1) \} \{ (1.1.1)(1.1.1) \} \{ (1.1.1)(1.1.1) \}].$$

Or, taking an actual example from our data :

Stamen number 1 is normal and complete, and there is no chorisis; stamen number 2 has a filament only half-length but the anther is present and complete, and there is no chorisis; stamen number 3 is normal and complete; stamen number 4 is only half-length but with a complete anther—chorisis between 3 and 4 is complete; stamens 5 and 6 are only half the normal length but have complete anthers—chorisis between 5 and 6 is complete.

This would be represented thus :

$$[\{ (1.0.1)(\frac{1}{2}.0.1) \} \{ (1.1.1)(\frac{1}{2}.1.1) \} \{ (\frac{1}{2}.1.1)(\frac{1}{2}.1.1) \}].$$

Another graphic method and the one which we have adopted is as follows. Each flower is represented in a table similar to the following:

Frequency	Number of Diagram	Stamen	Filament	Chorisis	Anther

The first vertical column gives the frequency of the variation or the number of individuals examined with this structure. The second vertical column gives the number of the corresponding diagram in the plates. The third vertical column gives the individual stamens in the positions already defined while the other three columns denote the various factors to be considered. The different possibilities of variation in these may be shown by the symbols 1, $\frac{1}{2}$ and 0 as already defined. It should be noted, however, that in positions 1 and 2 a dash (—) will be placed in the chorisis column to indicate that these are typically non-chorised stamens and that absence of chorisis does not therefore indicate abnormality. Representing the same example as before, by this method, we would have:

Frequency	Number of Diagram	Stamen	Filament	Chorisis	Anther
		1	1	---	1
		2	$\frac{1}{2}$	—	1
		3	1	1	1
		4	$\frac{1}{2}$	1	1
		5	$\frac{1}{2}$	1	1
		6	$\frac{1}{2}$	1	1

The following table shows graphically the types of variations illustrated in Figs. I—LVIII, i.e. Class 1, Sub-class A, flowers in which the perianth and gynæcium are both normal.

It will be seen that in the flowers illustrated in Figs. XLVIII—LVIII another complication has crept in. Stamens 3, 4, 5 and 6 have themselves sometimes undergone partial or total secondary chorisis. In the tables, therefore, by subdividing the squares containing the details we can thus adhere to our initial nomenclature. Let us take the three most difficult examples to illustrate this.

(1) Fig. XLVIII. The division corresponding to stamen 3 is sub-divided. This would indicate that in this position there were actually two stamens. The nature of each of these individual stamens is, as before, given in the sub-divisions. In

TABLE C.

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
1062	I	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	2	II	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	130	III	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 0 0	1 1 1 1 1 1
38	IV	1 2 3 4 5 6	1 1 1 1 1 $\frac{1}{2}$	— — 1 1 1 1	1 1 1 1 1 1	1	V	1 2 3 4 5 6	1 1 1 1 1 $\frac{1}{2}$	— — 1 1 1 1	1 1 1 1 1 0	8	VI	1 2 3 4 5 6	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	— — 1 1 1 1	1 1 1 1 1 1
6	VII	1 2 3 4 5 6	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	— — 1 0 0 0	1 1 1 1 1 1	227	VIII	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 0 0 0	1 1 1 1 1 0	7	IX	1 2 3 4 5 6	1 1 1 1 $\frac{1}{2}$ 0	— — 1 1 0 0	1 1 1 1 1 0
5	X	1 2 3 4 5 6	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	— — 1 1 1 1	1 1 1 1 1 1	1	XI	1 2 3 4 5 6	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	— — 1 1 1 1	1 1 1 1 1 1	8	XII	1 2 3 4 5 6	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1	— — 1 1 0 0	1 1 1 1 1 1
8	XIII	1 2 3 4 5 6	1 1 1 $\frac{1}{2}$ 1 0	— — 1 1 0 0	1 1 1 1 1 0	3	XIV	1 2 3 4 5 6	1 1 1 $\frac{1}{2}$ $\frac{1}{2}$ 0	— — 1 1 0 0	1 1 1 1 1 0	3	XV	1 2 3 4 5 6	1 1 1 1 1 1	— 0 0 0 0 0	1 1 1 1 1 1
1	XVI	1 2 3 4 5 6	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	— 0 0 1 1 1	1 1 1 1 1 1	1	XVII	1 2 3 4 5 6	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$	— 0 0 0 0 0	1 1 1 1 1 1	4	XVIII	1 2 3 4 5 6	1 1 1 1 1 0	— 0 0 0 0 0	1 1 1 1 1 0
2	XIX	1 2 3 4 5 6	1 1 1 0 $\frac{1}{2}$ $\frac{1}{2}$	— — 0 0 1 1	1 1 1 0 1 1	21	XX	1 2 3 4 5 6	1 1 1 0 1 0	— — 0 0 0 0	1 1 1 0 1 0	2	XXI	1 2 3 4 5 6	1 1 1 0 $\frac{1}{2}$ 0	— — 0 0 0 0	1 1 1 0 1 0
2	XXII	1 2 3 4 5 6	1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	— — 1 1 0 0	1 1 1 1 1 1	15	XXIII	1 2 3 4 5 6	1 $\frac{1}{2}$ 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	1	XXIV	1 2 3 4 5 6	1 $\frac{1}{2}$ 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 1

TABLE C—(continued).

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
7	XXV	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	2	XXVI	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	3	XXVII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 0 0	1 1 1 1 1 1
11	XXVIII	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0	3	XXIX	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0	1	XXX	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1
3	XXXI	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 0 0	1 1 1 1 1 1	1	XXXII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	3	XXXIII	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0
3	XXXIV	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0	1	XXXV	1 2 3 4 5 6	1 1 1 0 1 0	— — 0 0 0 0	1 1 1 0 1 0	9	XXXVI	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1
2	XXXVII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	3	XXXVIII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	2	XXXIX	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0
18	XL	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 1	3	XLI	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 1	1	XLII	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 0
5	XLIII	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 0 0	1 0 1 1 1 1	2	XLIV	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 1	2	XLV	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 1 1	1 0 1 1 1 1
1	XLVI	1 2 3 4 5 6	1 0 1 1 1 0	— — 1 1 0 0	1 0 1 1 1 0	3	XLVII	1 2 3 4 5 6	1 0 1 1 1 0	— — 1 1 0 0	1 0 1 1 1 0	3	XLVIII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1

TABLE C—(continued).

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
8	XLIX	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	3	L	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 0	1 1 1 1 1 1	5	LI	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 0 1	1 1 1 1 1 1
2	LII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	2	LIII	1 2 3 4 5 6	1 1 1 1 1 1	— — 0 0 1 1	1 1 1 1 1 1	1	LIV	1 2 3 4 5 6	1 1 1 1 1 1	— — 0 0 0 0	1 1 1 1 1 1

TABLE C—(continued).

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
2	LV	1 2 3 4 5 6	1 1 1 1 1 1	— — 0 0 0 1	1 1 1 1 1 1	4	LVI	1 2 3 4 5 6	1 1 1 0 1 1	— — 0 0 1 1	1 1 1 0 1 1
1	LVII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	2	LVIII	1 2 3 4 5 6	1 0 1 1 1 1	— — 1 1 0 1	1 0 1 1 1 1

this example what really occurs is: There are two individual stamens each equal to the original length and bearing a complete anther and separated from one another by secondary choris. In the former, choris has been complete but has resulted in one being full-length and with a functioning anther while the other is only half-length with a functioning anther. In the latter, choris has not been complete inasmuch as only the anther has been chorised.

(2) Fig. L. Divisions 5 and 6 are sub-divided to show that there has been secondary choris in both of these stamens. In the former, choris has been complete but has resulted in one being full-length and with a functioning anther while the other is only half-length with a functioning anther. In the latter, choris has not been complete inasmuch as only the anther has been chorised.

(3) Fig. LII. Divisions 5 and 6 are both sub-divided, consequently we may infer that both of these stamens have undergone some stage of choris. In the first column we see that all are full-length, in the third that all have functioning anthers, but the second tells us that choris has been only partial in each case. The symbol $\frac{1}{2}$ between 5 and 6 indicates that between these two choris has also been partial. Thus we conclude a state of affairs as follows: In position 5.6 (1) there arises a single filament which divides into two at some distance from the base and (2) that each of these again sub-divides and (3) that on the end of each of these four sub-filaments there arises a functioning anther. The others may be worked out in a similar manner but a reference to the diagrams will at once obviate any misrepresentation.

From the foregoing table and illustrations it is evident that further classification is possible but it would be well to point out here certain difficulties which arise. As an example let us consider such a case as (using our original terminology) that in which, in any of the positions (1; 2; 3.4; or 5.6), the stamens are represented thus (1.1.0)(0.0.0), thus $(\frac{1}{2}.1.1)(\frac{1}{2}.1.1)$ or thus $(\frac{1}{2}.0.1)(\frac{1}{2}.0.1)$. Which shall have precedence? If we are to consider these variations as deviations from the usually accepted normal cruciferous flower, then we may safely assume that that flower which has the greatest number of functioning parts in a certain position is less aberrant than one in which any or all of the parts are altogether wanting; while, on the other hand, if in a position in which choris normally takes place, we have defective groups like those in cases 2 and 3 cited above, in one of which choris has taken place but not in the other, we must consider *that* group in which choris has occurred as being the one less removed from normal. On this basis then the above examples would be placed in the following order with regard to normality:

(1) $(\frac{1}{2}.1.1)(\frac{1}{2}.1.1)$; (2) $(\frac{1}{2}.0.1)(\frac{1}{2}.0.1)$; (3) (1.1.0)(0.0.0).

Similarly for any of the others.

Consequently we are now in a position to classify the actual cases under observation.

So far we have considered only those flowers in which there was the typical number of stamens, with their manifold variations in size and structure, but now

we must classify those in which secondary chorisis has given rise to more than the accepted number.

Let us take stamens 5 and 6 as our basis, i.e. those individuals in which position 5.6 is occupied by more than two stamens.

The relative frequencies of the different types of variation in the andrœcium in the 618 specimens so far considered (see Table C) are very interesting. The number 1062 in Table F refers to 1062 flowers in which the andrœcium was

TABLE D.

	Number of Variations in		Number of Individuals in	
	Section	Sub-group	Section	Sub-group
Group <i>a</i> .				
Whole of the andrœcium normal. Fig. I	—	1	—	1062
Group <i>b</i> .				
Andrœcium variously modified	—	—	—	—
Sub-group <i>a</i> .		21	—	480
Outer whorl normal (5 and 6 variously modified)	—	—	—	—
Section i.				
Stamens 3 and 4 normal. Figs. II—IX	8	—	419	—
Section ii.				
Stamen 3 normal, 4 represented thus ($\frac{1}{2}$ ·1·1). Figs. X—XIV	5	—	25	—
Section iii.				
Stamens 3 and 4 thus {(1·0·1) (1·0·1)}. Figs. XV—XVIII	4	—	9	—
Section iv.				
Stamens 3 and 4 thus {($\frac{1}{2}$ ·1·1) ($\frac{1}{2}$ ·1·1)}. Fig. XXII	1	—	2	—
Section v.				
Stamens 3 and 4 replaced by one. Figs. XIX—XXI	3	—	25	—
Sub-group <i>β</i> .		13	—	54
Outer whorl represented thus {(1·—·1) ($\frac{1}{2}$ ·—·1)}	—	—	—	—
Section i.				
Stamens 3 and 4 normal. Figs. XXIII—XXIX	7	—	42	—
Section ii.				
Stamens 3 and 4 thus {(1·1·1) ($\frac{1}{2}$ ·1·1)}. Figs. XXX—XXXIV	5	—	11	—
Section iii.				
Stamens 3 and 4 thus {($\frac{1}{2}$ ·0·1) (0·0·0)}. Fig. XXXV	1	—	1	—
Sub-group <i>γ</i> .				
Stamens 1 and 2 represented thus {($\frac{1}{2}$ ·—·1) ($\frac{1}{2}$ ·—·1)}	—	4	—	16
Section i.				
Stamens 3 and 4 normal. Figs. XXXVI and XXXVII	2	—	11	—
Section ii.				
Stamens 3 and 4 thus {(1·1·1) ($\frac{1}{2}$ ·1·1)}. Figs. XXXVIII and XXXIX	2	—	5	—
Sub-group <i>δ</i> .				
Stamen 1 normal, 2 absent	—	5	—	29
Section i.				
Stamens 3 and 4 normal. Figs. XL—XLIII	4	—	27	—
Section ii.				
Stamen 3 normal, 4 thus ($\frac{1}{2}$ ·1·1). Fig. XLIV	1	—	2	—
Sub-group <i>ε</i> .				
Stamen 1 thus ($\frac{1}{2}$ ·—·1), 2 absent. Figs. XLV—XLVII	—	3	—	6

TABLE E.

	Number of Variations in		Number of Individuals in	
	Section	Sub-group	Section	Sub-group
Sub-group η .				
Stamens 1 and 2 are normal	—	9	—	30
Section i.				
Stamens 3 and 4 are represented by three. Fig. XLVIII ...	1	—	3	—
Section ii.				
Stamens 3 and 4 are normal. Figs. XLIX—LI	3	—	16	—
Section iii.				
Stamens 3 and 4 represented thus $\{(1 \cdot 1 \cdot 1) (\frac{1}{2} \cdot 1 \cdot 1)\}$. Fig. LII	1	—	2	—
Section iv.				
Stamens 3 and 4 thus $\{(1 \cdot 0 \cdot 1) (1 \cdot 0 \cdot 1)\}$. Figs. LIII—LV ...	3	—	5	—
Section v.				
Stamens 3 and 4 represented by one. Fig. LVI	1	—	4	—
Sub-group θ .				
Stamens 1 and 2 thus $\{(\frac{1}{2} \cdot \text{—} \cdot 1) (\frac{1}{2} \cdot \text{—} \cdot 1)\}$. Fig. LVII ...	—	1	—	1
Sub-group κ .				
Stamen 1 normal, 2 absent. Fig. LVIII	—	1	—	2

TABLE F.

Frequencies more than 3 in order of magnitude.

(References have been made to the figures.)

Figure	Frequency	Figure	Frequency	Figure	Frequency
I	1062	XXVIII	11	XXV	7
VIII	227	XXXVI	9	VII	6
III	130	VI	8	X	5
IV	38	XII	8	XLIII	5
XX	21	XIII	8	LI	5
XL	18	XLIX	8	XVIII	4
XXIII	15	IX	7	LVI	4

normal. Where variation occurs, the greatest frequency, namely 227, occurs in flowers in which one of the pairs in the inner whorl is replaced by a single stamen while the next highest frequency, namely 130, occurs in those flowers in which partial chorisis has taken place in the inner whorl of the andrœcium. Following this the magnitude of the frequencies diminishes rapidly. The next, namely 38, occurs in flowers in which nearly all the parts of the andrœcium are modified while, near this, is the frequency 21 which exists in flowers having only one stamen in each position. In the next two frequencies, namely 18 and 15, we find that stamens 1 and 2 are involved.

From these raw data we can see that the inner whorl of the andrœcium is the whorl most subject to variation and further that this variation is in the direction of a decrease in number.

TABLE G.

Class I, Sub-class B: Perianth normal, Gynœcium abnormal (see Table A).

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
3	LIX	1	1	—	1	1	LX	1	1	—	1
		2	1	—	1			2	1	—	1
		3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1
		5	1	0	1			5	1	1	1
		6	0	0	0			6	1	1	1
1	LXII	1	1	1	1	2	LXIII	1	1	—	1
		2	1	1	1			2	1	—	1
		3	1	1	1			3	1	1	1
		4	1	0	1			4	1	1	1
		5	1	0	1			5	1	1	1
		6	1	1	1			6	1	1	1

Class II: Perianth abnormal.

Variations in the members of the perianth (calyx and corolla) have necessitated the introduction of new symbols in the diagrams. These are shown in the composite diagram Plate I, fig. 10, and are explained on p. 257.

It will be evident from Table H, p. 233, that the same type of variation in the andrœcium occurs with different types of variation in the perianth, e.g. in the second and fifth figures no fewer than six different variations in the perianth accompany a single type of variation in the andrœcium. Reference to the diagrams in the plates will show what these variations are and will render a detailed explanation unnecessary. The asterisk in Table H under LXXVIII indicates that there has been *adhesion* between stamen 1 and one of the stamens in position 3.4, in other words between one of the members of the outer whorl and one of the members in the inner whorl.

Class III.

The members of this class are characterised by a reduplication of the various organs but without separation into two distinct flowers. There are in all 11 individuals with 10 different types of variation. A word of explanation is necessary with regard to the interpretation of the position of the various stamens

TABLE H.

Class II.

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
1	XC	1 2 3 4 5 6	1 1 1 0 0 0	— 1 0 0 0 0	1 1 1 0 0 0	1 33 2 2 2 1	LXXXVII LXIV LXVIII LXIX LXX LXXIV	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1	1	LXVI	1 2 3 4 5 6	1 1 1 1 1* 1	— — 1 1 1 1	1 1 1 1 1 1
6 2 1	LXV LXXII LXXIII	1 2 3 4 5 6	1 1 1 1 1 0	— — 1 1 0 0	1 1 1 1 1 0	2 1 1 31 5 1	XCI LXXI LXXVI LXXXVII LXXXI XCII	1 2 3 4 5 6	1 0 1 1 1 1	— 0 1 1 1 1	1 0 1 1 1 1	1 1	LXXIX LXXX	1 2 3 4 5 6	1 0 1 1 1 1	— 0 1 1 1 1	1 0 1 1 1 1
1	LXXXIII	1 2 3 4 5 6	1 0 1 1 1 2	— 0 1 1 1 1	1 0 1 1 1 1	1	LXXXII	1 2 3 4 5 6	1 0 1 1 1 0	— 0 1 1 0 0	1 0 1 1 1 0	2	LXXXIV	1 2 3 4 5 6	1 2 1 1 1 1	1 0 1 1 1 1	1 0 1 1 1 1
1	LXXXV	1 2 3 4 5 6	0 0 1 1 1 1	— — 1 1 1 1	0 0 1 1 1 1	1 1	LXXV LXXXVI	1 2 3 4 5 6	0 0 1 1 1 1	— — 1 1 1 1	0 0 1 1 1 1	1	LXXVIII	1 2 3 4 5 6	1 1 1 1* 1 0	— — 1 1 0 0	1 1 1 1 1 0
3	LXVII	1 2 3 4 5 6	1 1 1 1 2 0	— — 1 1 0 0	1 1 1 1 1 0	6	LXXXIX	1 2 3 4 5 6	1 1 1 0 1 0	— — 0 0 0 0	1 1 0 0 1 0	3	LXXXVIII	1 2 3 4 5 6	1 1 1 1 1 1	— — 1 1 1 1	1 1 1 1 1 1

in these flowers. The stamens in the outer whorl are longer than those in the inner whorl. Consequently if these were reduced in length they might easily be mistaken for members of the outer whorl. In all cases of difficulty, however, the crucial test, the point of origin, was applied and the positions assigned to the various members as shown in the diagrams were determined microscopically in this manner. See Table I, p. 234.

TABLE I.
Class III.

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
2	XCVIII	1	1	1	1	1	XCIX	1	1	0	1	1	XCIV	1	1	1	1
		2	1	1	1			2	1	0	1			2	1	1	1
		3	1	1	1			3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1			4	1	1	1
		5	1	1	1			5	1	1	1			5	1	1	1
		6	1	1	1			6	1	1	1			6	1	1	1
	XCVIII	1	1	1	1	1	XCV	1	0	0	0	1	CI	1	1	1	1
		2	1	1	1			2	1	0	0			2	1	1	1
		3	1	1	1			3	1	0	0			3	1	1	1
		4	1	1	1			4	1	0	0			4	1	1	1
		5	1	1	1			5	1	0	0			5	1	1	1
		6	1	1	1			6	1	0	0			6	1	1	1
1	XCVII	1	1	1	1	1	CII	1	1	1	1	1	CIII	1	1	1	1
		2	1	1	1			2	1	1	1			2	1	1	1
		3	1	1	1			3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1			4	1	1	1
		5	1	1	1			5	1	1	1			5	1	1	1
		6	1	1	1			6	1	1	1			6	1	1	1
	XCVII	1	1	1	1	1	CIV	1	1	1	1	1	XCVI	1	1	1	1
		2	1	1	1			2	1	1	1			2	1	1	1
		3	1	1	1			3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1			4	1	1	1
		5	1	1	1			5	1	1	1			5	1	1	1
		6	1	1	1			6	1	1	1			6	1	1	1
1	XCVII	1	1	1	1	1	CV	1	1	1	1	1	XCVIII	1	1	1	1
		2	1	1	1			2	1	1	1			2	1	1	1
		3	1	1	1			3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1			4	1	1	1
		5	1	1	1			5	1	1	1			5	1	1	1
		6	1	1	1			6	1	1	1			6	1	1	1
	XCVII	1	1	1	1	1	CVI	1	1	1	1	1	XCVI	1	1	1	1
		2	1	1	1			2	1	1	1			2	1	1	1
		3	1	1	1			3	1	1	1			3	1	1	1
		4	1	1	1			4	1	1	1			4	1	1	1
		5	1	1	1			5	1	1	1			5	1	1	1
		6	1	1	1			6	1	1	1			6	1	1	1

TABLE J.
Class IV.

Frequency	5	1	3
Number of Diagram	CHH A	CV A	CVII A
Number of Stamen	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6
Filament	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Chorisis	— —	— —	— —
Anther	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Number of Diagram	CHH B	CV B	CVII B
Number of Stamen	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6
Filament	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Chorisis	— —	— —	— —
Anther	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Frequency	1	6	1
Number of Diagram	CIV A	CVI A	CVIII A
Number of Stamen	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6
Filament	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Chorisis	— —	— —	— —
Anther	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Number of Diagram	CIV B	CVI B	CVIII B
Number of Stamen	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6
Filament	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Chorisis	— —	— —	— —
Anther	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Number of Diagram	CIV B	CVI B	CVIII B
Number of Stamen	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6
Filament	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1
Chorisis	— —	— —	— —
Anther	1 1 1 1 1 1	1 1 1 1 1 1	1 1 1 1 1 1

Class IV.

In this class there are 17 individuals giving six different modes of variation. Reduplication has taken place to such an extent as to give rise to two separate flowers on one pedicel. Each of the flowers was diminutive in size. Two tables are thus necessary for each "flower," *A* and *B*: see Table J, p. 235.

Class V.

This class has been formed to include two very aberrant flowers showing two distinct variations. In both cases part of the flower has been replaced by another flower, in one case normal in the other slightly divergent. CIX is one of these in which the original flower is normal except that one of the carpels has been replaced by a small flower (see Table K and diagram, Plate X). CX is the other. In the original flower stamen 1 has been chorised and one of the chorised parts has given origin to a separate flower (see Table K and diagram, Plate X).

TABLE K.

Class V.

Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther	Frequency	Number of Diagram	Number of Stamen	Filament	Chorisis	Anther
1	CIX <i>A</i>	1	1	—	1	1	CIX <i>B</i>	1	1	—	1
		2	1	—	1			2	1	—	1
		3	1	1	1			3	1	0	1
		4	1	1	1			4	0	0	0
		5	1	1	1			5	1	0	1
		6	1	1	1			6	0	0	0
1	CX <i>A</i>	1	1	0	1	1	CX <i>B</i>	1	1	—	1
			1	0	*			2	1	—	1
		2	1	—	1			3	1	1	1
		3	1	1	1			4	1	1	1
			1	1	1			5	1	1	1
		4	1	1	1			6	1	1	1
		5	1	1	1						
		6	1	1	1						

The asterisk denotes the position of the origin of the secondary flower.

This concludes our analysis of variations LIX to CX both as to perianth and andrœcium, but before proceeding to the statistical part it is desirable that certain peculiarities should be observed and that an understanding be arrived at with regard to the interpretation of these.

In this procedure 44 variations, namely LIX to CII, must be dealt with. When we study the number of parts which occur in the position of individual members of a whorl and then try to draw conclusions as to normality or abnormality of the whorl itself we find the following difficulties.

Let us take the outer whorl of the andrœcium as an example.

(1) If in the position normally occupied by stamen 1 there were two stamens and in the position normally occupied by stamen 2, no stamen occurred, then with regard to the whorl the total number of stamens would be *two*. Now this is the accepted normal number of stamens in the outer whorl, so that if number alone were considered the inference would legitimately be drawn from the table that the whorl was normal. But this is not so!

Or (2) If in position number 1, one normal stamen occurred and in position number 2, one functioning stamen, with the filament only half the normal length, occurred, then the number of functioning stamens in the whorl would be *two*, i.e. the accepted normal number. But again, on the basis of number alone, we should not be able to say whether the whorl as a whole was normal or abnormal.

Now as this state of affairs exists not only in the whorl under consideration but in all the whorls of the flower, we have thought it not only advisable but necessary to emphasise these abnormalities as a safeguard in the interest of systematic statistical treatment.

For this purpose, therefore, small diagrammatic formulæ have been drawn up, and these have been given in conjunction with the diagrams: see Plate I, figs. 11, 12.

We have already defined the positions of the various parts of the andrœcium but have hitherto refrained from naming the different constituents of the perianth.

In the cases under consideration, however, it is necessary to do so, and Plate I, fig. 11 illustrates how these are definitely determined.

The two outer sepals are named *A* and *B* (see Fig. 11). *A* corresponds in position to stamens 3.4 and *B* to stamens 5.6. *C* and *D* are the two inner sepals; *C* corresponds in position to stamen 1 and *D* to the stamen in position 2. The petals are named *A'*, *B'*, *C'* and *D'* and lie respectively between sepals *A* and *C*, *B* and *D*, *A* and *D*, and *B* and *C*.

The actual order of all the parts is summed up in Plate I, fig. 12 (1—14).

TABLE L. Class V.

Number of Whorl	Member of Whorl	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency
I	A	1	2	N			1	2	N			1	2	N		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1	4	N	LIX	2	1	4	N	LX	1	1	4	N	LXI	1
	B'	1					1					1				
	C'	1					1					1				
	D'	1					1					1				
IV	1	1	2	N			1	2	N			1	2	N		
	2	1					1					1				
V	3,4	1	3	A			2	4	A			1	3	A		
	5,6	2					2					2				
I	A	1	2	N			1	2	N			1	2	N		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1	4	N	LXII	1	1	4	N	LXIII	2	1	4	A	LXIV	33
	B'	1					1					1				
	C'	1					1					1				
	D'	1					1					1				
IV	1	2	4	A			1	2	A			1	2	N		
	2	2					1					1				
V	3,4	2	6	A			3	6	A			2	4	N		
	5,6	4					3					2				
I	A	1	2	N			1	2	N			1	2	N		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1	4	A	LXV	6	1	4	A	LXVI	1	1	4	A	LXVII	3
	B'	1					1					1				
	C'	1					1					1				
	D'	1					1					1				
IV	1	1	2	N			1	2	N			1	2	N		
	2	1					1					1				
V	3,4	2	3	A			2	4	A			2	3	A		
	5,6	1					2					1				
I	A	1	2	N			1	2	N			1	2	N		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1	4	A	LXVIII	2	1	3	A	LXIX	2	1	3	A	LXX	2
	B'	1					0					1				
	C'	1					1					1				
	D'	1					1					1				
IV	1	1	2	N			1	2	N			1	2	N		
	2	1					1					1				
V	3,4	2	4	N			2	4	N			2	4	N		
	5,6	2					2					2				

TABLE L—(continued).

Number of Whorl	Member of Whorl	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency
I	A	1	2	N			1	2	N			1	2	N		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1					1					1				
	B'	1	3	A	LXXI	1	1	3	A	LXXII	2	1	3	A	LXXIII	1
	C'	1					0					0				
	D'	0					1					0				
IV	1	1	1	A			1	2	N			1	2	N		
	2	0					1					1				
V	3.4	2	4	N			2	3	A			1	3	A		
	5.6	2					1					2				
I	A	1	2	N			1	2	N			1	2	A		
	B	1					1					1				
II	C	1	2	N			1	2	N			1	2	A		
	D	1					1					1				
III	A'	0					0					1				
	B'	0	1	A	LXXIV	1	0	5	A	LXXV	1	1	2	A	LXXVI	1
	C'	0					2					0				
	D'	1					3					0				
IV	1	1	2	N			0	0	A			1	1	A		
	2	1					0					0				
V	3.4	2	4	N			3	5	A			2	4	N		
	5.6	2					2					2				
I	A	1	1	A			1	1	A			1	1	A		
	B	0					0					0				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1					1					1				
	B'	1	3	A	LXXVII	31	1	3	A	LXXVIII	1	0	3	A	LXXIX	1
	C'	0					0					1				
	D'	1					1					1				
IV	1	1	1	A			1	2	N			1	1	A		
	2	0					1					0				
V	3.4	2	4	N			2	3	A			3	5	A		
	5.6	2					1					2				
I	A	1	1	A			1	1	A			1	1	A		
	B	0					0					0				
II	C	1	2	N			1	2	N			1	2	N		
	D	1					1					1				
III	A'	1					0					0				
	B'	1	3	A	LXXX	1	0	2	A	LXXXI	5	0	2	A	LXXXII	1
	C'	0					1					1				
	D'	1					1					1				
IV	1	1	1	A			1	1	A			1	1	A		
	2	0					0					0				
V	3.4	3	5	A			2	4	N			2	3	A		
	5.6	2					2					1				

TABLE L—(continued).

Number of Whorl	Member of Whorl	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency
I	A	1	1	A	LXXXIII	1	1	1	A	LXXXIV	2	2	2	A	LXXXV	1
II	B	0					0					0				
III	C	1	2	N			1	2	N			1	2	N		
IV	D	0	0	A			0	0	A			1	4	N		
V	A'	0					0					1				
	B'	0					0					1				
	C'	0					0					1				
	D'	0					0					1				
IV	1	1	1	A			1	1	A			0	0	A		
	2	0					0					0				
V	3.4	2	4	A			2	4	N			2	5	A		
	5.6	2					2					3				
I	A	1	2	N	LXXXVI	1	1	2	N	LXXXVII	1	1	2	N	LXXXVIII	3
II	B	1					1					1				
III	C	0	0	A			1	2	N			1	2	N		
IV	D	0					1					1				
V	A'	1	2	A			1	4	A			0	2	A		
	B'	1					1					0				
	C'	0					1					0				
	D'	0					1					1				
IV	1	0	0	A			1	2	N			1	2	A		
	2	0					1					1				
V	3.4	3	5	A			2	4	N			2	4	N		
	5.6	2					2					2				
I	A	1	1	A	LXXXIX	6	1	1	A	XC	1	0	0	A	XCI	2
II	B	1	2	N			1	2	N			1	2	N		
III	C	0					1					0				
IV	D	0	2	A			0	5	A			0	1	A		
V	A'	1					2					0				
	B'	1					2					0				
	C'	1					1					1				
	D'	1					2					0				
IV	1	1	2	N			1	3	A			1	1	A		
	2	1					2					0				
V	3.4	1	2	A			1	1	A			2	4	N		
	5.6	1					0					2				
I	A	1	2	A	XCII	1	1	3	A	XCIII	1	1	3	A	XCIV	1
II	B	1	1	A			2	2	N			1	2	N		
III	C	0					1					1				
IV	D	1	3	A			1	5	A			1	5	A		
V	A'	1					2					1				
	B'	1					1					2				
	C'	1					2					2				
	D'	0					1					2				
IV	1	1	1	A			2	3	A			2	4	A		
	2	0					1					2				
V	3.4	2	4	N			2	5	A			5	10	A		
	5.6	2					3					5				

TABLE L—(continued).

Number of Whorl	Member of Whorl	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency
I	A B	1 2	3	A			2 2	4	A			2 2	4	A		
II	C D	1 1	2	N			1 1	2	N			1 1	2	N		
III	A' B' C' D'	1 1 2 1	5	A	XCV	1	1 0 2 2	5	A	XCVI	1	2 2 1 1	6	A	XCVII	1
IV	1 2	1 0	1	A			2 2	4	A			2 2	4	A		
V	3.4 5.6	5 3	8	A			4 5	9	A			4 4	8	A		
I	A B	1 2	3	A			1 2	3	A			1 1	2	N		
II	C D	2 2	4	A			2 2 2	4	A			2 2	4	A		
III	A' B' C' D'	1 2 1 1	5	A	XCVIII	2	2 2 1 1	6	A	XCIX	1	1 2 2 1	6	A	C	1
IV	1 2	2 2	4	A			1 1	2	N			1 1	2	N		
V	3.4 5.6	7 7	14	A			5 3	8	A			3 3	6	A		

Number of Whorl	Member of Whorl	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency	Number in each position	Members in Whorl	Whorl, Normal or Abnormal	Number of Diagram	Frequency
I	A B	1 1	2	N			1 1	2	N		
II	C D	2 2	4	A			2 2	4	A		
III	A' B' C' D'	1 2 2 1	6	A	CI	1	0 0 0 0	0	A	CII	1
IV	1 2	2 2	4	A			1 2	3	A		
V	3.4 5.6	4 4	8	A			3 2	5	A		

IV. STATISTICAL.

The analysis which we have given of 1813 flowers is sufficient to show that the idea of a definite fixed number of sepals in the calyx, of petals in the corolla, of stamens in the andrœcium or of carpels in the gynœcium of cruciferous plants is not upheld by an examination of a large number of flowers of this species. In less than 1 per cent. of the flowers examined there was an increase* or decrease in the number of sepals in the calyx; in less than 1 per cent. there was also an increase or decrease in the number of petals in the corolla, but in 2 per cent. there was an increase in the number of stamens in the andrœcium, while in 22 per cent. there was a decrease in the number.

Since then the number of sepals, petals and stamens is not *absolutely* fixed for any of the organs it becomes necessary now to consider whether the number of members in one organ is related to the number in the others.

As has already been pointed out we have not only to consider organs as a whole, but, in the case of the calyx and the andrœcium, the constituents of these organs, owing to the fact that these organs are each divided into two separate whorls which are inserted at different levels and are placed in directions at right angles to one another.

Further, a special study has been made of the various positions in andrœcium to ascertain to what extent bilateral symmetry may be regarded as an inherent character of the flower under consideration.

By this means also it seems that some definite information might be obtained with regard to the perplexing and, at present, hypothetical theory of chorisis, the reasons for the existence of which have been summarised on p. 219.

The statistical part has been divided into two sections:

- (1) a study of the Means and Standard Deviations, and
- (2) a study of the Correlation Coefficients.

1. *Study of the Means and Standard Deviations.*

Although it is obvious from the analysis of the data under consideration that the numbers given for the botanical floral formula, namely, Calyx—4, Corolla—4, Stamens—6 and Gynœcium—2, are the nearest integers, it is not at all certain from a mere inspection of the tables whether the actual means deviate from this number in the direction of excess or deficiency.

* Where chorisis of a sepal or petal has resulted in two or more distinct individuals we have regarded each of these as a distinct sepal or petal in recording the numbers. This method is *natural* however inasmuch as it is the only means by which we may possibly trace reduplication of parts.

Consequently the mean and standard deviation for each of the organs and its constituents have been calculated and these are given in the following table:

TABLE M.

Means and Standard Deviations of the Number of the Organs and their Constituents.

Organ	Constituent	Member	Mean Number	Standard Deviation	Coefficient of Variation
Calyx	—	—	3·9796	·2553	6·415
„	Outer whorl	—	1·9757	·1979	10·020
„	Inner whorl	—	2·0039	·1304	6·507
Corolla	—	—	3·9520	·3523	8·914
Andrœcium	—	—	5·8092	·7567	13·025
„	Outer whorl	—	1·9570	·2704	13·817
„	—	Stamen 1	·9840	·1602	16·280
„	—	Stamen 2	·9713	·1915	19·715
„	Inner whorl	—	3·8577	·6588	17·077
„	—	Stamens 3, 4	1·9950	·2728	13·674
„	—	Stamens 5, 6	1·8627	·4863	26·107

(1) The most obvious result which is revealed by these constants is the fact that in *all* cases (except the inner whorl of calyx) the actual mean of the organs is less than the recognised typical number, thus:

The mean number for the calyx is 3·979 instead of 4.

The mean number for the corolla is 3·952 instead of 4.

The mean number for the andrœcium is 5·809 instead of 6.

(2) The inner whorl of the calyx shows the smallest departure from the accepted typical number, namely, 2·004 instead of 2.

Let us, however, test how far the differences in the character of the analogous parts are significant by ascertaining the Probable Error of the difference of the means of the characters.

TABLE N (1).

I. *Constituents of the Calyx.*

Constituent	Mean Number	Standard Deviation
Outer whorl ...	1·9757	·1979
Inner whorl ...	2·0039	·1304

The difference here is $D = \cdot 02813$ and the probable error of the difference $E_{D_m} = \cdot 0037$; thus the value $\frac{D}{E_{D_m}} = 7\cdot 6$. The difference is therefore clearly significant.

It is worthy of notice that the outer whorl of the calyx is more variable than the inner whorl and that it possesses on an average fewer sepals.

TABLE N (2).

II. *Members of the Outer Whorl of the Andræcium.*

Member	Mean Number	Standard Deviation
Position 1 ...	·9840	·1602
Position 2 ...	·9713	·1915

The difference here is $D = \cdot 01268$ and $E_{D_m} = \cdot 00391$; the value $\frac{D}{E_{D_m}} = 3\cdot 2$.

Thus when the two positions of the outer whorl of the andræcium are taken into consideration, a probably significant difference is found between the means of the distribution of the parts of this whorl. Now in position number 1 there is a greater approach to the accepted type owing to the fact that when the component of one of the positions of the outer whorl was found to depart from the accepted type, the other position was selected as the starting point for the orientation of the flower and was called position number 1. It is all the more noteworthy that the deviation for position 2 is not in the direction of greater but of lesser frequency and the variability of position 2 is greater. We have thus again a reduction in the value of the type with greater variability.

TABLE N (3).

III. *Members of the Inner Whorl of the Andræcium.*

Member	Mean Number	Standard Deviation
Position 3, 4 ...	1·9950	·2728
Position 5, 6 ...	1·8627	·4863

Here the ratio $\frac{m_1 - m_2}{\sum (m_1 - m_2)}$ is nearly 15 and therefore there is quite a significant difference between the means of the distributions of the two members of the inner whorl of the andræcium. In both cases the tendency is towards a suppression of functioning stamens rather than an increase, together with greater variability in the case where the reduction from the accepted type is more marked.

This difference in variability is, in the main, real and is not due to the arbitrary selection of the 3.4 position. This will be evident from a study of Table XIX. It will there be seen that there were 1754 cases where two stamens occurred in

one of the positions of the inner whorl of the andrœcium. This is the number in the accepted type, and thus there is no variability. What is the nature of the distribution of the stamens in the other position (Table XVIII)? It is as follows:

1	2	3	4	
287	1436	26	5	1754

The mean for this array is 1·8569 stamens, with a variability of ·4116. Thus when there is no variability in one position of the inner whorl of the andrœcium there is a large variability in the other position.

Similarly we find the following distribution for position 2 in the outer whorl of the andrœcium when position 1 is of the accepted type, i.e. shows no variability.

0	1	2	
57	1708	1	1766

The mean for this array is ·9683 with a variability of ·1784. Again therefore, when there is no variability in position 1, there is a reduction of type in position 2 with great variability.

2. *Study of the Correlation Coefficients.*

For the purposes of this study a number of correlation tables have been prepared and as the results of these will have to be considered under different groupings it seems advisable to tabulate them, and insert them consecutively. The system which has been adopted to facilitate reference is to commence with the outer whorl of the calyx and consider all *its* relations with the other whorls of the flower passing from the outside inwards; following this comes the inner whorl of the calyx and *its* relations with the other constituents of the flower from the outside inwards and so on.

The following table shows the characters studied and the correlation coefficients found.

In order to make the comparison of the various correlations as complete as possible it will be necessary to consider each constituent or organ with all the other constituents or organs and to avoid overlapping as far as possible. The most natural method would be to commence either with the outermost constituent, namely, the outer whorl of the calyx, or with the innermost constituent, namely, the inner whorl of the andrœcium. For reasons of a morphological character, which will be seen later, the inner whorl of the andrœcium has been chosen as the starting point.

TABLE O.

Correlation Coefficients between the Number of Various Organs and their Constituents.

	Table	r.
The outer whorl of the calyx and the inner whorl of the calyx	I	.1957
The outer whorl of the calyx and the corolla	II	.7275
The outer whorl of the calyx and the outer whorl of the andræcium	III	.5886
The outer whorl of the calyx and the inner whorl of the andræcium	IV	.2613
The outer whorl of the calyx and the andræcium	V	.4371
The inner whorl of the calyx and the corolla	VI	.2476
The inner whorl of the calyx and the outer whorl of the andræcium	VII	.3229
The inner whorl of the calyx and the inner whorl of the andræcium	VIII	.3905
The inner whorl of the calyx and the andræcium	IX	.4592
The calyx and the corolla	X	.6926
The calyx and the outer whorl of the andræcium	XI	.6245
The calyx and the inner whorl of the andræcium	XII	.4014
The calyx and the andræcium	XIII	.5721
The corolla and the outer whorl of the andræcium	XIV	.4762
The corolla and the inner whorl of the andræcium	XV	.1773
The corolla and the andræcium	XVI	.3174
The outer whorl of the andræcium and the inner whorl of the andræcium	XVII	.1984
The inner whorl of the andræcium, position 3.4 and the inner whorl of the andræcium, position 5.6	XVIII	.4305
The outer whorl of the calyx and the inner whorl of the andræcium, position 3.4	XIX	.4646
The outer whorl of the calyx and the inner whorl of the andræcium, position 5.6	XX	.2134
The inner whorl of the calyx and the inner whorl of the andræcium, position 3.4	XXI	.4634
The inner whorl of the calyx and the inner whorl of the andræcium, position 5.6	XXII	.2519
The corolla and the inner whorl of the andræcium 3.4	XXIII	.2558
The corolla and the inner whorl of the andræcium 5.6	XXIV	.0903
The outer whorl of the andræcium and the inner whorl of the andræcium, position 3.4	XXV	.2661
The outer whorl of the andræcium and the inner whorl of the andræcium, position 5.6	XXVI	.1539

(a) The inner whorl of the andræcium.

From the standpoint of the systematic botanist the most anomalous constituent of the cruciferous flower is the inner whorl of the andræcium, inasmuch as in each of the positions where one stamen would naturally be expected, the presence of two is regarded as typical. It has been explained in a previous section that botanists now usually regard this anomaly as having arisen by collateral chorisis from what was originally a single stamen in ancestral forms. For the sake of conciseness and in order to avoid unnecessary repetition the following abbreviations have been used in Tables P—X.

O. W. Ca. = Outer whorl of the calyx.

I. W. Ca. = Inner whorl of the calyx.

Ca. = Calyx.

Co. = Corolla.

O. W. A. = Outer whorl of the andræcium.

I. W. A. = Inner whorl of the andræcium.

A. = Andræcium.

The following Table, P, gives the correlation coefficients between the I. W. A. and the other constituents or organs of the flower in order of position.

TABLE P.
I. W. A. and the other Constituents.

Constituent or Organ	Table	Correlation
O. W. Ca.	IV	·2613
I. W. Ca.	VIII	·3905
Ca.	XII	·4014
Co.	XV	·1773
O. W. A.	XVII	·1984

The highest correlation between the inner whorl of the andrœcium and the other constituents or organs is that with the calyx; next in order come the inner whorl of the calyx, the outer whorl of the calyx, the outer whorl of the andrœcium, and lastly the corolla. In other words, we should be better able to predict the number of stamens in the inner whorl of the andrœcium from the number of members in the calyx than from the number of members in any other constituent or organ.

(b) *Relations between the organs themselves.*

Having thus discussed the inner whorl of the andrœcium with the other organs and constituents it might lead to some useful result if we proceed to determine the "organic correlation" existing between the various organs themselves. In this connection we have to consider the calyx, the corolla and the andrœcium, and for this purpose the correlation Tables X, XIII and XVI have been prepared. The character which has been selected for this study is the number of members in each organ.

The following Table (Q) shows the results obtained :

TABLE Q.
Correlation Coefficients between

Ca. and Co. ...	·6926
Ca. and A. ...	·5721
Co. and A. ...	·3174

(1) The calyx and corolla are much more highly correlated to one another than is either of these with the andrœcium. In other words, the two protective organs of the perianth are more highly correlated to one another than is either protective organ with the male reproductive organ. It is further evident that (2) the calyx is much more highly correlated to both the corolla and the andrœcium than are the two last named to one another. From (1) it may be concluded that, on an average,

organs. For this purpose it will be necessary to tabulate the results in series and consequently it might be well to start with the outermost constituent of the flower, namely, the outer whorl of the calyx, and tabulate the correlation coefficients passing inwards to the andræcium. The inner whorl of the calyx will next be taken in relation to the other constituents and so on.

From the above table it will be seen that the outer whorl of the calyx is most highly correlated with the corolla; it is also highly correlated with the outer whorl of the andræcium but much less so with the inner whorl of the andræcium.

TABLE S.

(e) *Correlation Coefficients between the Inner Whorl of the Calyx and*

2nd Component	Table	r.
Co.	VI	·2476
O. W. A. ...	VII	·3229
I. W. A. ...	VIII	·3905
A.	IX	·4592

The low correlation between the inner whorl of the calyx and the corolla is due to the close adherence of the former to type, that is, there is very small variability.

TABLE T.

(f) *Correlation Coefficients between the Calyx and*

2nd Component	Table	r.
Co.	X	·6926
O. W. A. ...	XI	·6245
I. W. A. ...	XII	·4014
A.	XIII	·5721

There is a higher degree of correlation between the two organs of the perianth than between the calyx and the andræcium. The high correlation between the calyx and the outer whorl of the andræcium is mainly due to the high value obtained for the correlation between the outer whorl of the calyx and the outer whorl of the andræcium.

TABLE U.

(g) *Correlation Coefficients between the Corolla and*

2nd Component	Table	r.
O. W. A. ...	XIV	·4762
I. W. A. ...	XV	·1773
A.	XVI	·3174

The corolla is much more highly correlated with the outer whorl than with the inner whorl of the andræcium, and the correlation between the corolla and the andræcium as a whole is not very great.

A comparison of Tables T and U shows that there is a much greater correlation between the calyx and the andræcium and its two whorls, than between the corolla and the same constituents.

So far we have considered the relationships between the different parts of the flower from the outside inwards, but when we examine these relationships, taking the inner whorls as our starting point, some new aspects of the problem become manifest and, as these have been of great value in the interpretation of the results, it has been considered advisable to tabulate them thus :

TABLE V.

(h) *Correlation Coefficients between the Inner Whorl
of the Andræcium and*

2nd Component	Table	r.
Ca.	XII	·4014
I. W. Ca. ...	VIII	·3905
O. W. Ca. ...	IV	·2613
O. W. A. ...	XVII	·1984
Co.	XV	·1773

TABLE W.

(i) *Correlation Coefficients between the Outer Whorl
of the Andræcium and*

2nd Component	Table	r.
Ca.	XI	·6245
O. W. Ca. ...	III	·5886
Co.	XIV	·4762
I. W. Ca. ...	VII	·3229

A comparison of Tables V and W shows that the correlations between the outer whorl of the andræcium and the other components are higher than for the inner whorl of the andræcium, except in the case of the inner whorl of the calyx.

TABLE X.

(j) *Correlation Coefficients between the
Andræcium and*

2nd Component	Table	r.
Ca.	XIII	·5721
I. W. Ca. ...	IX	·4592
O. W. Ca. ...	V	·4371
Co.	XVI	·3174

This table shows that when the andræcium is considered as a whole it is most highly correlated with the calyx and least correlated with the corolla.

V. MORPHOLOGICAL SIGNIFICANCE OF THE STATISTICAL RESULTS.

It is quite clear from the tabulated results that there is a definite departure from the usually accepted cruciferous structure in a very large number of the flowers of *Lepidium Draba* which have been examined for this study. This does not obtain merely in any one organ or constituent but in *all* the organs and constituents, although not to the same degree in each.

The statistical results will now be examined from the standpoint of the botanist in order (*a*) to note their morphological or genetic significance and (*b*) in order to see whether these figures throw any light on the evolution of this cruciferous plant.

It is almost axiomatic to state that the "purpose" of a flower is a purely reproductive one and that therefore its existence is justified only in so far as it serves to reproduce its kind. But not all the parts of a flower are solely reproductive in function. Each individual consists of two parts, (1) Reproductive, (2) Protective. (1) The reproductive organs are the gynæcium (♀) and the andræcium (♂), while (2) The protective organs (perianth) are the corolla and the calyx.

One of the organs of the perianth, namely the corolla, is still further specialised. The calyx consists of four sepals, green in colour, whose sole function is to protect the flower when in the bud, and in many cases these are reflexed immediately after the flower has opened up, and are of no further importance to it. On the other hand the petals though essentially sepal-like in structure, in this as in the great majority of flowers, are not green but of some other colour. In the species under consideration they are white. Now although the petals are of great importance in protecting the reproductive organs while in the bud their utility does not cease

when the flower opens but, along with small nectaries at the base of the stamens, serve as an attraction for insects whose visits are essential for cross-fertilisation.

The reproductive organs of what is regarded as the typical cruciferous flower consist of (1) the gynæcium which is composed of two carpels and (2) the androecium which is composed of six stamens. The stamens are delicate structures and do not hold an isolated position in the flower. When in the bud and immature they are subject to external influences, for example, (1) they might be shrivelled up by the heat of the sun, (2) they might be blasted by rain or wind or (3) they might be attacked by herbivorous insects, so that the protective perianth plays an important part in flower economics. Now what does an increase in the number of stamens imply? It is obvious that if the number of stamens is increased the total volume occupied by the reproductive organs is increased and consequently a tax is put upon the protective organs if they are to fulfil their function adequately. If the perianth does not respond to this tax from space considerations, the reproductive organs stand a small chance of ever fulfilling their function, so that one would naturally expect that variation of some kind in the perianth would follow variation in the reproductive organs.

Another important point which must never be lost sight of when interpreting the statistical results is the symmetry of the cruciferous flower. The calyx consists of two whorls each with two sepals; the corolla of one whorl of four petals and the androecium of two whorls of stamens, the outer having two members and the inner four members (see Plate I, fig. 7). Consequently a cruciferous flower is bilaterally symmetrical only on that vertical plane which passes *through* the division wall of the carpels, *between* each of the pairs of stamens in the inner whorl, *between* two petals on either side and *through* the middle of the outer pair of sepals. This plane may be referred to as the "plane of symmetrical division." Owing to the fact that the corolla consists of only one whorl, the outer whorl of the calyx corresponds in position to the inner whorl of the androecium, and the inner whorl of the calyx to the outer whorl of the androecium.

From a study of the Means and Standard Deviations of the various organs and constituents we arrive at the following conclusions:

Calyx. (1) The greatest approach to constancy in number in the whole flower is in the inner whorl of the calyx.

(2) There is a significant difference between the means of the two whorls.

(3) There is much greater variability in the outer than in the inner whorl of the calyx and on an average it possesses fewer sepals.

(4) There is a tendency towards a reduction from type in the number of sepals in the calyx.

Corolla. (5) There is a tendency towards a reduction from the accepted typical number in the number of petals in the corolla.

Andrœcium. (6) There is a significant difference between the means of the distributions of

- (a) the members of the two whorls of the andrœcium,
- (b) the members of the two positions in the inner whorl,
- and (c) the members of the two positions in the outer whorl.

(7) From whatever axis we view the andrœcium as an organ it is distinctly asymmetrical in the distribution of its functioning stamens.

(8) There is much greater variability in the inner whorl than in the outer whorl of the andrœcium.

(9) In both positions in the inner whorl of the andrœcium there is a tendency towards a reduction from the accepted typical number of stamens and in the position where this is most marked there is the greatest variability.

The interpretation of these results is not at first sight very evident.

Why should there be a tendency towards a reduction in the number of members in the different organs of the flower and why should this tendency be most marked in the inner whorl of the andrœcium? As has already been pointed out all the flowers examined were taken from a single plant which gave rise to new stems by means of buds on the roots. May this tendency to reduction in the parts of the flower whose function is sexual reproduction not be an expression of a tendency towards an elimination of sexual in favour of vegetative reproduction? Another phenomenon which lends support to this hypothesis is the fact that in this plant the percentage of "pods" which attain maturity is extremely small.

Whether there is or is not a tendency towards vegetative reproduction, may we not also have here a harking back towards an ancestral form in which the number was less than the at present accepted typical number? In fact one would expect that if the present constitution of the inner whorl of the andrœcium had been most recent in development, reversion would first take place in it, and conversely one might reasonably conclude that since this whorl shows greatest variability, and most marked tendency to reduction in the number of members, it is more than probable that its present constitution was arrived at by an increase in number from a more primitive type.

Let us now examine the deductions made from a study of the correlation coefficients and see if they have any morphological interpretation.

(1) The calyx and corolla are more highly correlated with one another than is either of these with the andrœcium.

(2) The calyx is more highly correlated with the andrœcium than is the corolla. In other words, the two protective parts are more intimately associated in increase or decrease with one another than is either of these with the male reproductive organs, and further the calyx which is solely protective in function is more

intimately correlated with the male reproductive organs than is the corolla which serves as an attraction for insects as well as a protective covering of the bud.

(3) The two whorls of the calyx are not highly correlated, i.e. they vary independently of one another.

(4) The two whorls of the andrœcium also are not highly correlated. Morphologically this means that when there are two constituents in one organ, each having the same function, they may vary independently of one another, so that although an increase or decrease in the number in either *may* be correlated with an increase or decrease in the number in any other constituent of the flower, the same does not hold true with regard to the two constituents.

(5) The outer whorl of the calyx is most highly correlated with the corolla, next with the outer whorl of the andrœcium and lastly with the inner whorl of the andrœcium. The reason why the outer whorl of the calyx is more highly correlated with the outer whorl than with the inner whorl of the andrœcium is not at first sight very evident, but may be explained on the basis of its protective power. The members of the outer whorl of the andrœcium lie in a plane parallel to that of the outer whorl of the calyx, and are much more widely separated in this plane than are the members of the inner whorl of the andrœcium. Consequently any increase in the number of stamens in the outer whorl would involve a much greater increase in volume within the flower than a corresponding increase in the number of stamens in the inner whorl. Thus we are not surprised to find that such an increase in the outer whorl of the andrœcium is more intimately associated with an increase in the outer whorl of the calyx than a corresponding increase in the inner whorl of the andrœcium would be.

(6) There is very low variability in the inner whorl of the calyx and it is almost equally correlated to the two whorls of the andrœcium. The morphological explanation of these facts follows as a corollary to that given above.

(7) The calyx is much more highly correlated with the andrœcium as a whole and with its two whorls than is the corolla.

As we have already said the calyx is the predominantly protective organ and consequently this higher correlation has a physical basis. The corolla being partly attractive does not enter so closely into space economics.

(8) The outer whorl of the andrœcium is more highly correlated with the other components of the flower than is the inner whorl of the andrœcium. This again follows on the basis of space considerations. Any increase in the number of members in the inner whorl of the andrœcium does not involve so radical a change in the *volume* of the flower as does a corresponding increase in the outer whorl of the andrœcium.

VI. VARIATION IN THE GYNÆCIUM.

So far we have not considered the gynæcium on account of the small number of variations which occur in that organ and from the fact that these do not lend themselves to statistical treatment.

The gynæcium consists typically of two carpels which are flattened in a vertical plane parallel to those containing the pairs of stamens in the inner whorl of the andrœcium. The thin partition wall separating the two carpels therefore stands at right angles to this plane.

Now when we examine the different types of variations in the structure and number of the carpels we find the following: (1) a single carpel, (2) two carpels (typical), (3) three carpels, (4) four carpels, (5) two sets of two carpels within a single perianth, (6) two sets of two carpels within separate perianths but on one pedicel.

Let us now proceed to examine each of these in some detail.

(1) *The gynæcium consists of a single carpel* (see Figs. LXXXVII—XCII).

In all these cases, except LXXXVII, as will be at once seen by reference to the figures, the suppression of a carpel is accompanied by the suppression of some of the members of nearly all the other organs thus:

In LXXXVIII two petals are absent and one stamen is aborted.

In LXXXIX one sepal, two petals and two stamens are absent.

In XC, XCI and XCII all the organs are deficient in members.

A noteworthy phenomenon in this respect also is that the suppression of members which accompanies the suppression of a carpel is usually in the vertical plane which passes through the plane of separation of the carpels.

(2) *The gynæcium consists of two carpels.*

This is the accepted typical structure and the statistical study deals with these in detail.

(3) *The gynæcium consists of three carpels* (see Figs. XCIII and CII).

When three carpels occur in the gynæcium they are never found co-laterally, i.e. the additional carpel is never found with its origin at the side of a carpel, but always arising from the plane of separation, which is in the plane of greatest variability.

(4) *The gynæcium consists of four carpels* (see Fig. CI).

Just as in the previous case the increase in the number of carpels takes place in the plane of separation of the carpels—one on either side, so that a cruciate structure is found. A reference to Fig. CI will make this clear. In both of these groups it will be evident that an increase in the female reproductive organs is

associated not only with an increase in the male reproductive organs but also in an increase in the protective organs or perianth.

(5) *The gynæcium consists of two sets of carpels within a single perianth.*

This is rather an anomalous group but is extremely interesting inasmuch as it contains a series of annectant forms linking group 2 to group 6. What we actually have here is a complete reduplication of the reproductive organs encased within a single series of protective organs. In some of the flowers examined with this structure it was rather difficult to determine the orientation owing to a torsion of the thalamus, but in the types figured on Plates IX and X (Fig. XCIV and Figs. XCV et seq.) the mode of origin of these is quite evident. Several important observations on these forms may be stated.

(a) There are really two complete sets of reproductive organs and in one case (see Fig. XCVII) each of these is of the typical cruciferous structure.

(b) Increase in the number of the reproductive organs is accompanied by an increase in the number of members in the protective organs.

(c) The increase in the number of members of the reproductive organs is for the most part in the plane of division of the carpels, in other words, in the outer whorl of the calyx and its associated petals.

(d) This is also the plane along which the separation of the reproductive organs has taken place.

(e) This plane is the one which we have already shown in the statistical part to be the plane of greatest variability.

(6) *The gynæcium consists of two sets of two carpels within separate perianths but on one pedicel.*

In this group we reach the limit of variability in the material examined. In place of a single flower consisting of calyx, corolla, andrœcium and gynæcium we actually find two complete sets of all these organs, on one pedicel (see Figs. CIII—CVIII), while in one case (Fig. CIII) each of the two flowers has the typical cruciferous structure, so that were each of these separately examined it would undoubtedly be regarded as a normal flower. Yet we must bear in mind that, botanically considered, one flower and one flower only arises from a pedicel. Were this, therefore, an isolated example, and if no annectant forms existed, the departure might well be regarded as a "mutation," but a consideration of the numerous variations which we have already considered, taken in conjunction with group 5, only serves to emphasise the fact that "the vertical plane which passes through the partition wall of the two carpels and consequently separates the individuals of the pairs of stamens in the inner whorl and passes through the centres of the sepals of the outer whorl of the calyx is a plane along which this flower is in a state of flux and is the plane in which it is probable that the flower has changed, and is still changing, from some quite different ancestral form."

VII. SUGGESTIONS FOR FUTURE STUDIES IN THIS PLANT.

It must be very obvious to anyone who has perused this paper that the results which might be obtained from a study of this plant are by no means exhausted. An attempt, however, has been made to interpret the variability in its flowers, both from a morphological and an evolutionary standpoint. Studies of a different nature might be undertaken in order to test the results obtained, e.g.:

(1) What is the degree of fertility in the flowers of this plant? For this purpose it would be necessary to find the percentage of flowers which produce fertile seed.

(2) What are the variants, if any, which are associated with infertility?

(3) What are the characters of the flowers which are produced from the seeds of the different variants? If seeds selected from the different variants were grown separately and self-fertilised, one could trace the variations in the flowers of the next generation and see to what extent the different variations were transmitted. This study is capable of much elaboration and is one which would be fraught with great possibilities. It seems to involve a satisfactory method of determining how far these variations are concerned in plant economics, and also to what extent they have been instrumental in the evolution of the Order Cruciferae.

EXPLANATION OF FIGURES 8, 9 AND 10. PLATE I.

FIGURE 8.

- (a) Typical stamen (outer whorl).
- (b) Stamen with half-length filament and complete anther (outer whorl).
- (c) Typical stamen (inner whorl).
- (d) Non-chorised stamen with two complete anthers (inner whorl).
- (e) Stamen of inner whorl with two complete anthers but only chorised in the upper half.

FIGURE 9.

- (a) Aborted stamen of outer whorl, i.e. filament with no anther.
- (b) Absence of stamen in outer whorl.
- (c) Full-length filament in inner whorl with no anther.
- (d) Half-length filament in inner whorl with complete anther.
- (e) Half-length filament in inner whorl with no anther.
- (f) Non-chorised stamen in inner whorl with half-length filament but with two complete anthers.

FIGURE 10.

- (a) Normal sepal.
- (b) Sepal divided almost to the very base.
- (c) Sepal completely divided into two distinct sepals.
- (d) Sepal absent.
- (e) Normal petal.
- (f) Petal divided almost to the very base.
- (g) Aborted petal.
- (h) Petal absent.

TABLE I.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	—	1	—	—	1
1	—	2	1	—	—	3
2	2	48	1748	3	2	1803
3	—	—	—	—	—	0
4	—	—	3	3	—	6
Totals	2	50	1753	6	2	1813

Inner Whorl of Calyx.
Number of Sepals.

TABLE II.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	3	1	—	—	4
1	2	—	1	—	—	3
2	—	12	6	—	—	18
3	—	34	8	—	—	42
4	—	—	1734	—	—	1734
5	—	1	1	5	1	8
6	—	—	2	1	1	4
Totals	2	50	1753	6	2	1813

Corolla.
Number of Petals.

TABLE III.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	—	5	—	—	5
1	2	42	40	1	—	85
2	—	7	1705	1	—	1713
3	—	1	1	1	—	3
4	—	—	2	3	2	7
Totals	2	50	1753	6	2	1813

Outer Whorl of
Androecium.
Number of Stamens.

TABLE IV.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
1	—	1	—	—	—	1
2	—	6	24	—	—	30
3	—	2	292	—	—	294
4	2	39	1399	—	—	1440
5	—	2	25	1	—	28
6	—	—	12	—	—	12
8	—	—	1	2	1	4
9	—	—	—	—	1	1
10	—	—	—	1	—	1
14	—	—	—	2	—	2
Totals	2	50	1753	6	2	1813

Inner Whorl of Androecium.
Number of Stamens.

TABLE V.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
4	—	8	30	—	—	38
5	2	40	325	—	—	367
6	—	2	1365	—	—	1367
7	—	—	19	—	—	19
8	—	—	12	1	—	13
9	—	—	—	1	—	1
10	—	—	1	1	—	2
12	—	—	1	—	1	2
13	—	—	—	—	1	1
14	—	—	—	1	—	1
18	—	—	—	2	—	2
Totals	2	50	1753	6	2	1813

Androecium.
Number of Stamens.

TABLE VI.

Inner Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	2	1	—	1	4
1	—	—	3	—	—	3
2	1	—	17	—	—	18
3	—	1	41	—	—	42
4	—	—	1734	—	—	1734
5	—	—	6	—	2	8
6	—	—	1	—	3	4
Totals	1	3	1803	0	6	1813

Corolla.
Number of Petals.

TABLE VII.

Inner Whorl of Calyx.
Number of Sepals.

Outer Whorl of Androecium. Number of Stamens.		0	1	2	3	4	Totals
	0	1	—	4	—	—	5
	1	—	3	82	—	—	85
	2	—	—	1711	—	2	1713
	3	—	—	2	—	1	3
	4	—	—	4	—	3	7
	Totals	1	3	1803	0	6	1813

TABLE VIII.

Inner Whorl of Calyx.
Number of Sepals.

Inner Whorl of Androecium. Number of Stamens.		0	1	2	3	4	Totals
	1	—	—	1	—	—	1
	2	—	—	30	—	—	30
	3	—	—	294	—	—	294
	4	—	3	1437	—	—	1440
	5	1	—	26	—	1	28
	6	—	—	11	—	1	12
	8	—	—	2	—	2	4
	9	—	—	1	—	—	1
	10	—	—	1	—	—	1
	14	—	—	—	—	2	2
	Totals	1	3	1803	0	6	1813

TABLE IX.

Inner Whorl of Calyx.
Number of Sepals.

Androecium. Number of Stamens.		0	1	2	3	4	Totals
	4	—	—	38	—	—	38
	5	1	3	363	—	—	367
	6	—	—	1367	—	—	1367
	7	—	—	19	—	—	19
	8	—	—	11	—	2	13
	9	—	—	1	—	—	1
	10	—	—	1	—	1	2
	12	—	—	1	—	1	2
	13	—	—	1	—	—	1
	14	—	—	1	—	—	1
	18	—	—	—	—	2	2
	Totals	1	3	1803	0	6	1813

TABLE X.

Calyx.
Number of Sepals.

Corolla. Number of Petals.		2	3	4	5	6	7	Totals
	0	2	1	—	—	1	—	4
	1	2	—	1	—	—	—	3
	2	1	12	5	—	—	—	18
	3	—	35	7	—	—	—	42
	4	—	—	1734	—	—	—	1734
	5	—	1	1	3	1	2	8
	6	—	—	—	—	3	1	4
	Totals	5	49	1748	3	5	3	1813

TABLE XI.

Calyx.
Number of Sepals.

Outer Whorl of Androecium. Number of Stamens.		2	3	4	5	6	7	Totals
	0	1	—	4	—	—	—	5
	1	4	41	39	1	—	—	85
	2	—	7	1704	—	1	1	1713
	3	—	1	—	1	1	—	3
	4	—	—	1	1	3	2	7
	Totals	5	49	1748	3	5	3	1813

TABLE XII.

Calyx.
Number of Sepals.

Inner Whorl of Androecium. Number of Stamens.		2	3	4	5	6	7	Totals
	1	—	1	—	—	—	—	1
	2	—	6	24	—	—	—	30
	3	—	2	292	—	—	—	294
	4	4	38	1398	—	—	—	1440
	5	1	2	23	1	1	—	28
	6	—	—	11	—	1	—	12
	8	—	—	—	1	2	1	4
	9	—	—	—	—	1	—	1
	10	—	—	—	1	—	—	1
	14	—	—	—	—	—	2	2
	Totals	5	49	1748	3	5	3	1813

TABLE XIII.

Calyx.
Number of Sepals.

Andrœcium. Number of Stamens.		2	3	4	5	6	7	Totals
	4	—	8	30	—	—	—	38
	5	5	39	323	—	—	—	367
	6	—	2	1365	—	—	—	1367
	7	—	—	19	—	—	—	19
	8	—	—	10	1	2	—	13
	9	—	—	—	1	—	—	1
	10	—	—	—	—	—	1	2
	12	—	—	—	—	2	—	2
	13	—	—	—	—	1	—	1
	14	—	—	—	1	—	—	1
	18	—	—	—	—	—	2	2
	Totals	5	49	1748	3	5	3	1813

TABLE XIV.

Corolla.
Number of Petals.

	0	1	2	3	4	5	6	Totals
0	—	—	1	2	1	1	—	5
1	3	2	7	35	37	1	—	85
2	—	1	10	5	1695	—	2	1713
3	1	—	—	—	—	2	—	3
4	—	—	—	—	1	4	2	7
Totals	4	3	18	42	1734	8	4	1813

TABLE XV.

Corolla.
Number of Petals.

Inner Whorl of Andrœcium. Number of Stamens.		0	1	2	3	4	5	6	Totals
	1	—	—	—	—	—	1	—	1
	2	—	—	6	—	24	—	—	30
	3	—	—	2	3	289	—	—	294
	4	3	3	9	37	1388	—	—	1440
	5	1	—	1	2	22	2	—	28
	6	—	—	—	—	11	—	1	12
	8	—	—	—	—	—	1	3	4
	9	—	—	—	—	—	1	—	1
	10	—	—	—	—	—	1	—	1
	14	—	—	—	—	—	2	—	2
	Totals	4	3	18	42	1734	8	4	1813

TABLE XVI.

Corolla.
Number of Petals.

Andrœcium. Number of Stamens.		0	1	2	3	4	5	6	Totals
	4	—	—	7	2	28	1	—	38
	5	3	2	8	36	317	1	—	367
	6	—	1	3	4	1359	—	—	1367
	7	—	—	—	—	19	—	—	19
	8	1	—	—	—	10	1	1	13
	9	—	—	—	—	—	1	—	1
	10	—	—	—	—	1	—	1	2
	12	—	—	—	—	—	—	2	2
	13	—	—	—	—	—	1	—	1
	14	—	—	—	—	—	1	—	1
	18	—	—	—	—	—	2	—	2
	Totals	4	3	18	42	1734	8	4	1813

TABLE XVII.

Outer Whorl of Andrœcium.
Number of Stamens.

Inner Whorl of Andrœcium. Number of Stamens.		0	1	2	3	4	Totals
	1	—	—	—	1	—	1
	2	—	—	30	—	—	30
	3	—	5	289	—	—	294
	4	2	75	1363	—	—	1440
	5	3	4	19	2	—	28
	6	—	—	11	—	1	12
	8	—	1	1	—	2	4
	9	—	—	—	—	1	1
	10	—	—	—	—	1	1
	14	—	—	—	—	2	2
	Totals	5	85	1713	3	7	1813

TABLE XVIII.

Inner Whorl of Andrœcium, position 3. 4.
Number of Stamens.

Inner Whorl of Andrœcium 5. 6. Number of Stamens.		1	2	3	4	5	6	7	Totals
	0	1	—	—	—	—	—	—	1
	1	30	287	—	—	—	—	—	317
	2	7	1436	2	1	—	—	—	1446
	3	4	26	6	—	1	—	—	37
	4	—	5	—	2	—	—	—	7
	5	—	—	1	1	1	—	—	3
	6	—	—	—	—	—	—	—	0
	7	—	—	—	—	—	—	2	2
	Totals	42	1754	9	4	2	0	2	1813

TABLE XIX.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
1	—	8	34	—	—	42
2	2	42	1709	1	—	1754
3	—	—	8	1	—	9
4	—	—	2	—	2	4
5	—	—	—	2	—	2
6	—	—	—	—	—	0
7	—	—	—	2	—	2
Totals	2	50	1753	6	2	1813

Inner Whorl of Androecium,
position 3. 4.
Number of Stamens.

TABLE XX.

Outer Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	1	—	—	—	1
1	—	7	310	—	—	317
2	2	40	1404	—	—	1446
3	—	2	33	2	—	37
4	—	—	6	—	1	7
5	—	—	—	2	1	3
7	—	—	—	2	—	2
Totals	2	50	1753	6	2	1813

Inner Whorl of Androecium,
position 5. 6.
Number of Stamens.

TABLE XXI.

Inner Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
1	—	—	42	—	—	42
2	—	3	1750	—	1	1754
3	1	—	7	—	1	9
4	—	—	3	—	1	4
5	—	—	1	—	1	2
6	—	—	—	—	—	0
7	—	—	—	—	2	2
Totals	1	3	1803	0	6	1813

Inner Whorl of Androecium,
position 3. 4.
Number of Stamens.

TABLE XXII.

Inner Whorl of Calyx.
Number of Sepals.

	0	1	2	3	4	Totals
0	—	—	1	—	—	1
1	—	—	317	—	—	317
2	1	3	1442	—	—	1446
3	—	—	34	—	3	37
4	—	—	6	—	1	7
5	—	—	3	—	—	3
6	—	—	—	—	—	0
7	—	—	—	—	2	2
Totals	1	3	1803	0	6	1813

Inner Whorl of Androecium,
position 5. 6.
Number of Stamens.

TABLE XXIII.

Corolla.
Number of Petals.

	0	1	2	3	4	5	6	Totals
1	—	—	7	1	33	1	—	42
2	4	3	10	41	1695	1	—	1754
3	—	—	1	—	5	2	1	9
4	—	—	—	—	1	1	2	4
5	—	—	—	—	—	1	1	2
6	—	—	—	—	—	—	—	0
7	—	—	—	—	—	2	—	2
Totals	4	3	18	42	1734	8	4	1813

Inner Whorl of Androecium,
position 3. 4.
Number of Stamens.

TABLE XXIV.

Corolla.

Number of Petals.

Inner Whorl of Androeium, position 5. 6. Number of Stamens.		0	1	2	3	4	5	6	Totals
	0	—	—	—	—	—	1	—	1
	1	—	—	7	2	308	—	—	317
	2	3	3	11	38	1390	1	—	1446
	3	1	—	—	2	31	1	2	37
	4	—	—	—	—	5	—	2	7
	5	—	—	—	—	—	3	—	3
	6	—	—	—	—	—	—	—	0
	7	—	—	—	—	—	2	—	2
	Totals	4	3	18	42	1734	8	4	1813

TABLE XXV.

Outer Whorl of Androeium.

Number of Stamens.

Inner Whorl of Androeium, position 3. 4. Number of Stamens.		0	1	2	3	4	Totals
	1	—	—	41	1	—	42
	2	3	84	1665	2	—	1754
	3	2	1	6	—	—	9
	4	—	—	—	—	4	4
	5	—	—	1	—	1	2
	6	—	—	—	—	—	0
	7	—	—	—	—	2	2
	Totals	5	85	1713	3	7	1813

TABLE XXVI.

Outer Whorl of Androeium.

Number of Stamens.

Inner Whorl of Androeium, position 5. 6. Number of Stamens.		0	1	2	3	4	Totals
	0	—	—	—	1	—	1
	1	—	5	312	—	—	317
	2	4	75	1366	0	1	1446
	3	1	4	30	2	—	37
	4	—	—	5	—	2	7
	5	—	1	—	—	2	3
	6	—	—	—	—	—	0
	7	—	—	—	—	2	2
	Totals	5	85	1713	3	7	1813

PLATE I.

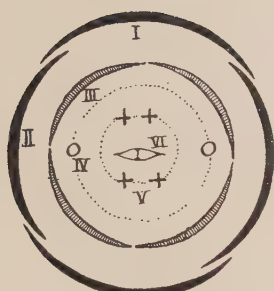


Fig 1



Fig 2

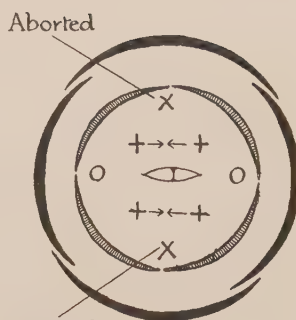


Fig 3

⊕ Peduncle.



Fig 4

⊕ Peduncle.

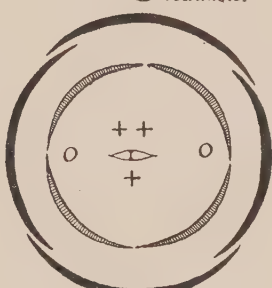
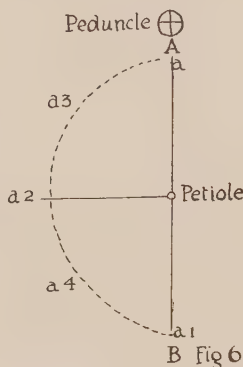


Fig 5



B Fig 6

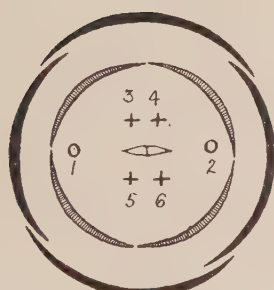


Fig 7

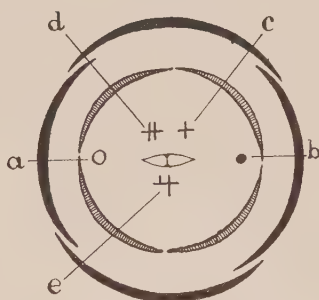


Fig 8

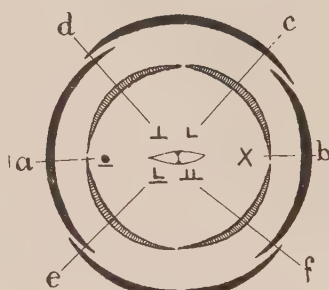


Fig 9

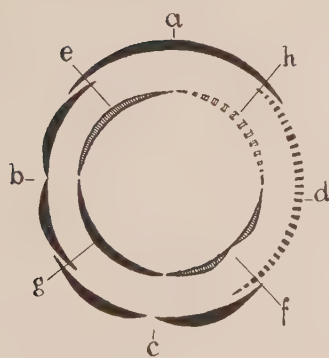


Fig 10

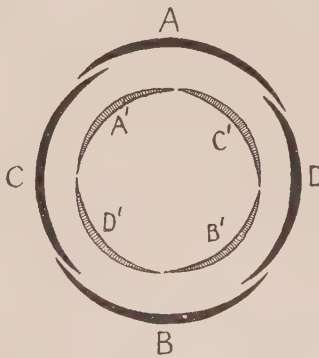


Fig 11

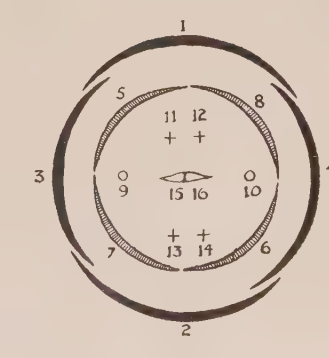


Fig 12

PLATE III.

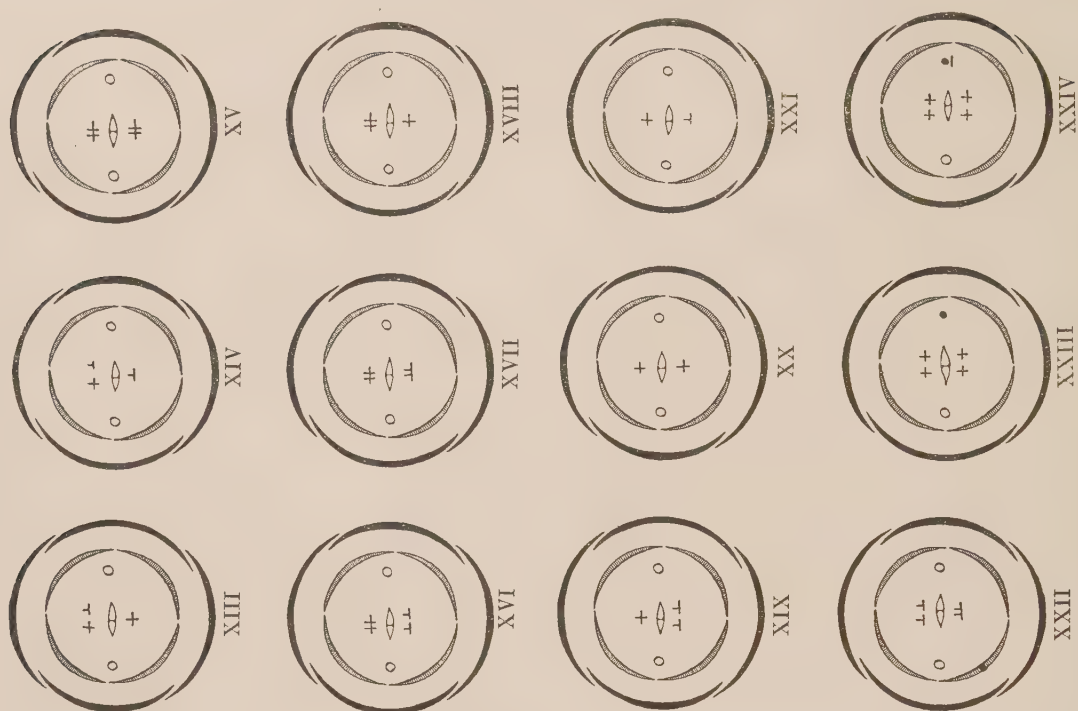


PLATE II.

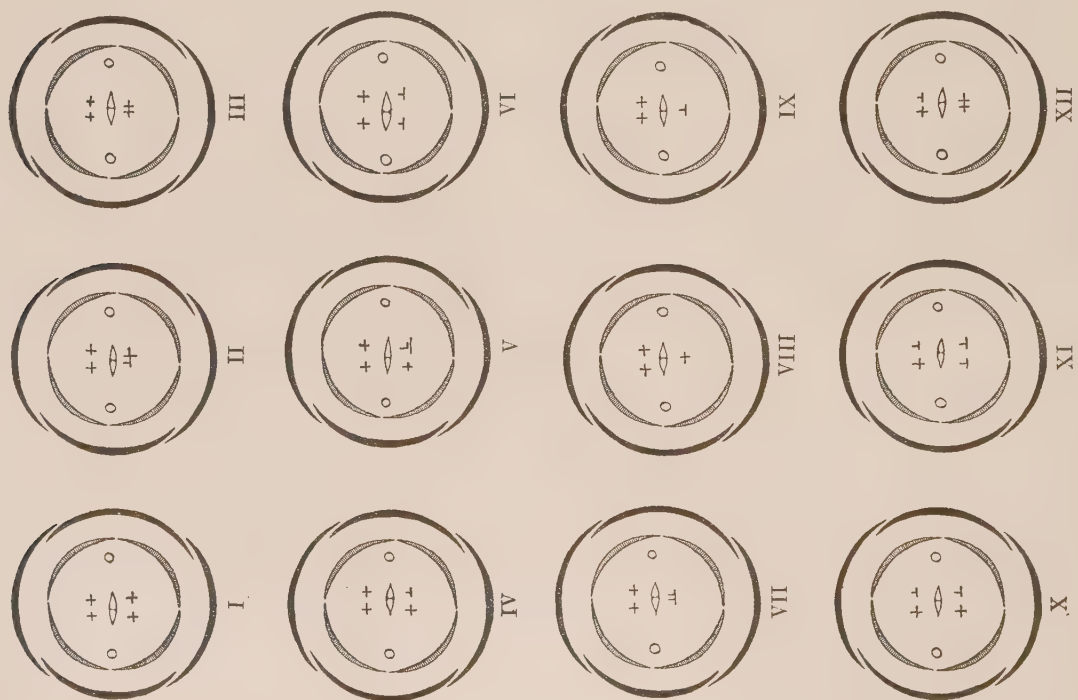


PLATE V.



XXXIX



XLII



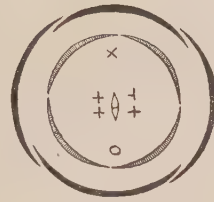
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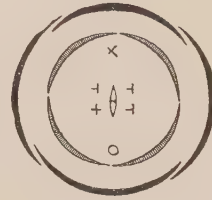
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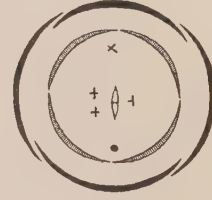
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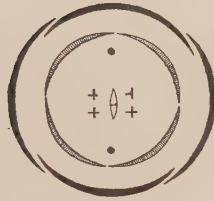
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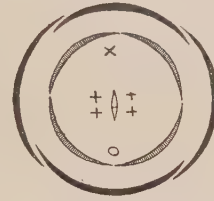
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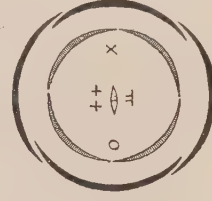
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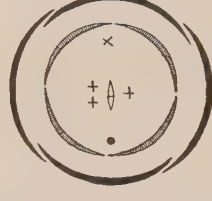
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XL

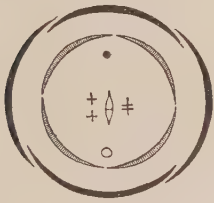


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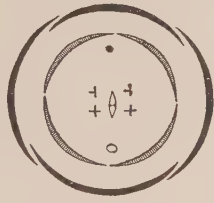


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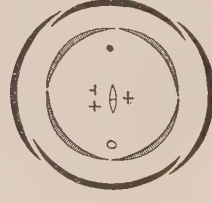
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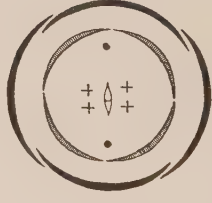
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XXX



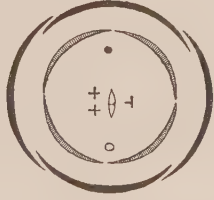
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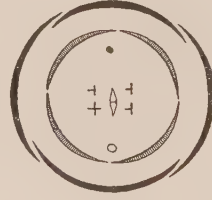
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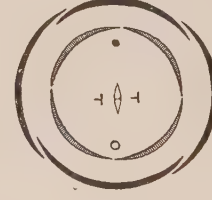
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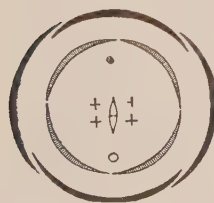
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XXXII



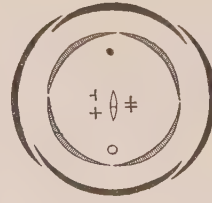
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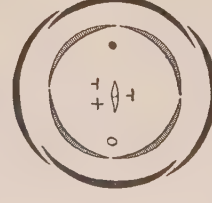
XXV



XXVIII



XXXI



XXXIV

PLATE VII.

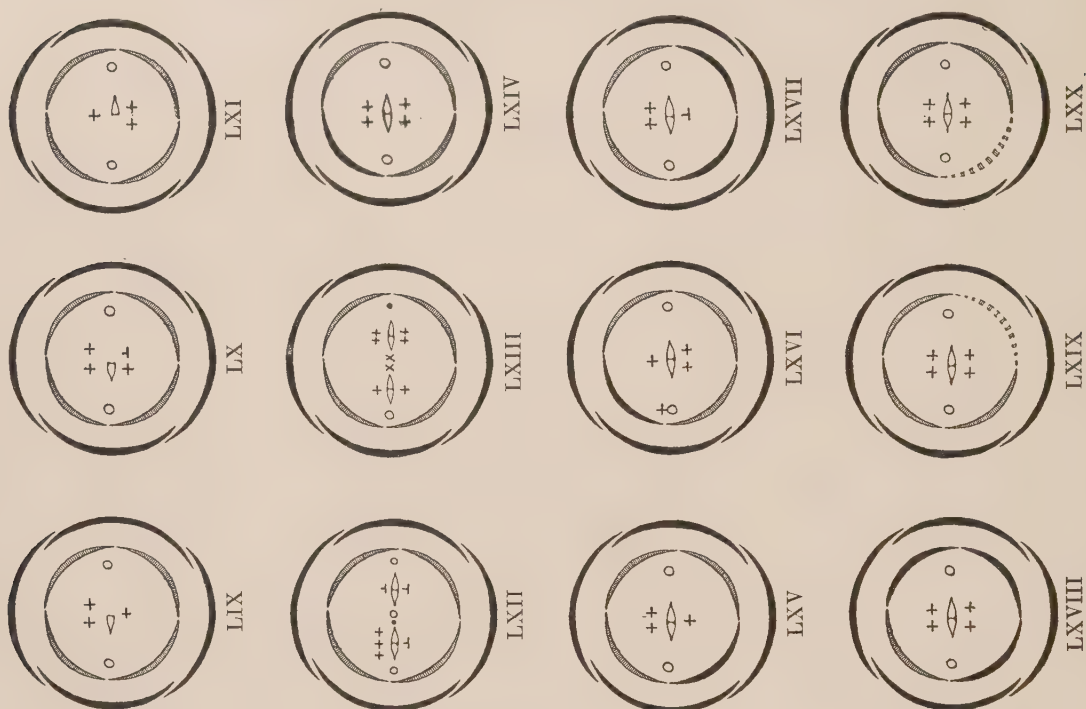


PLATE VI.

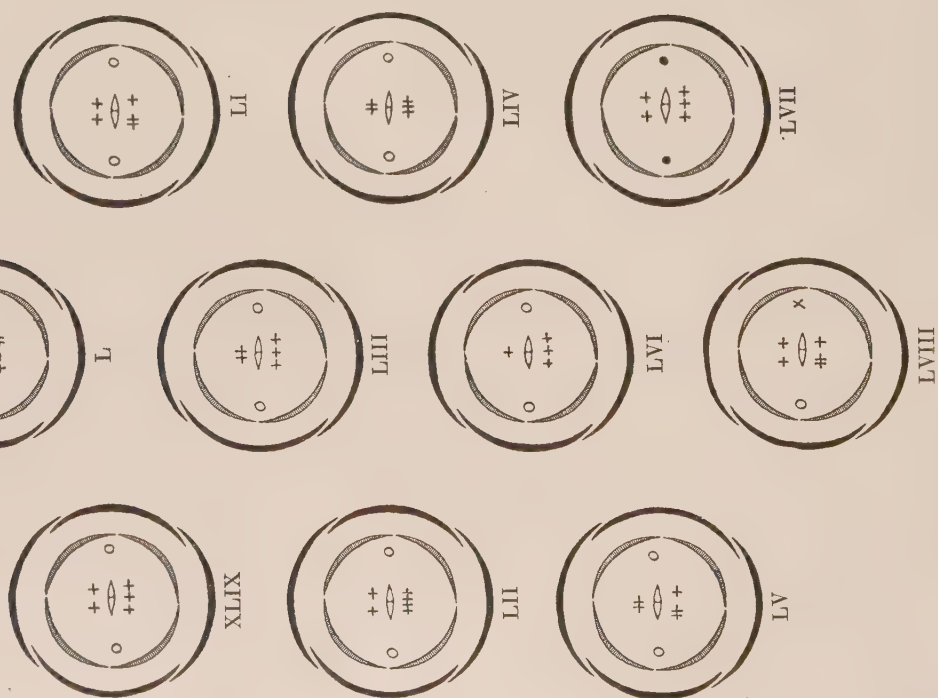
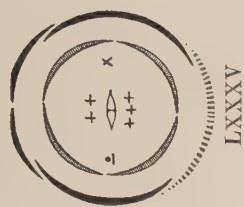
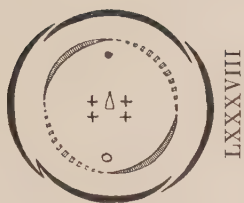


PLATE IX.



LXXXV



LXXXVIII



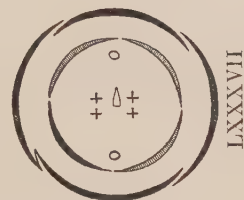
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XCIV



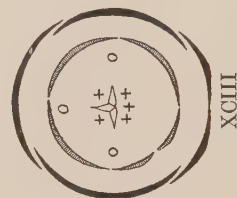
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LXXXVII



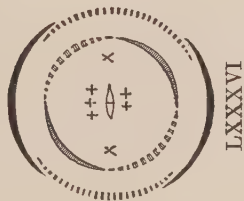
XC



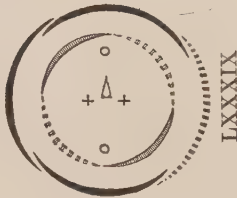
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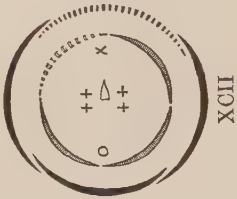
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LXXXVI



LXXXIX

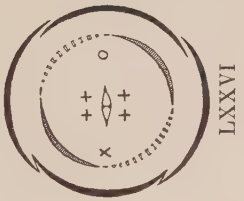


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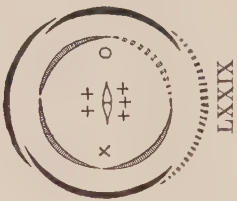
PLATE VIII.



LXXIII



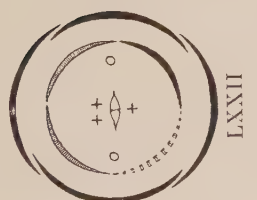
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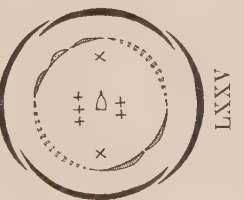
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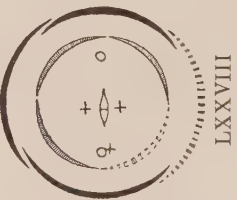
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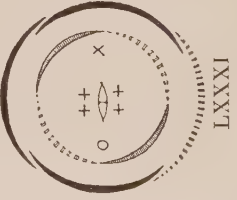
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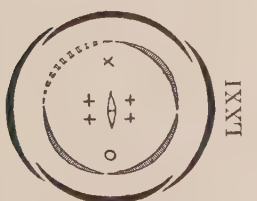
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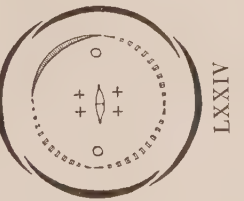
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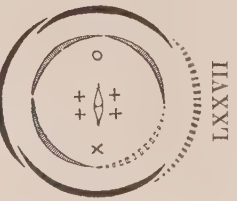
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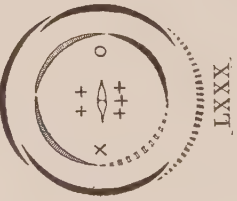
LXXI



LXXIV



LXXVII



LXXX

PLATE XI.

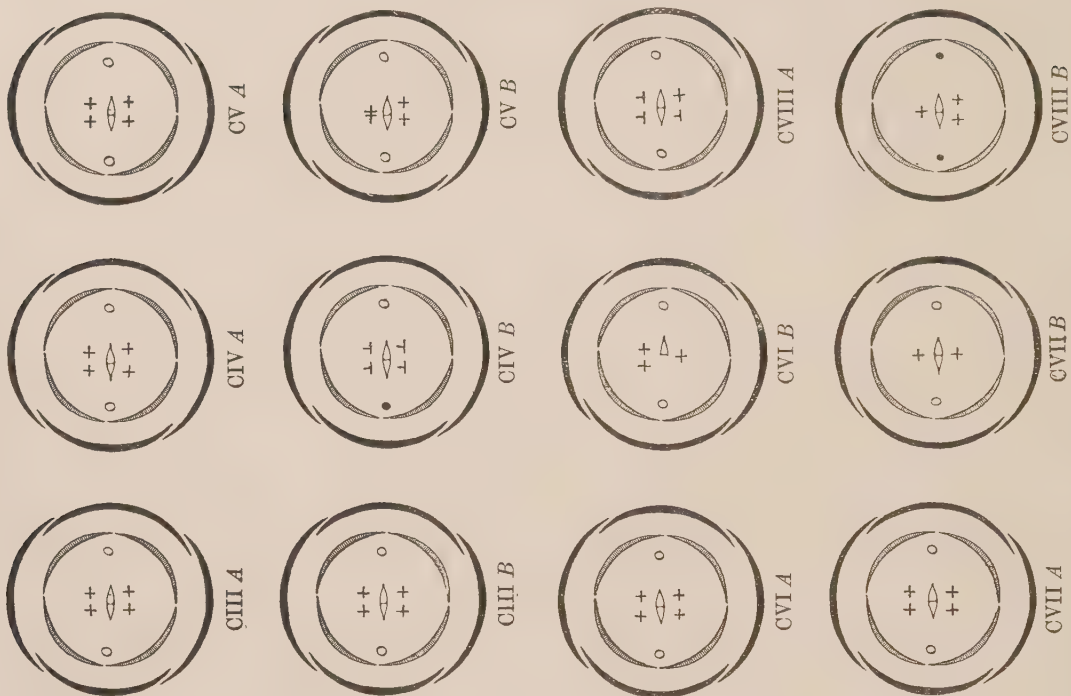
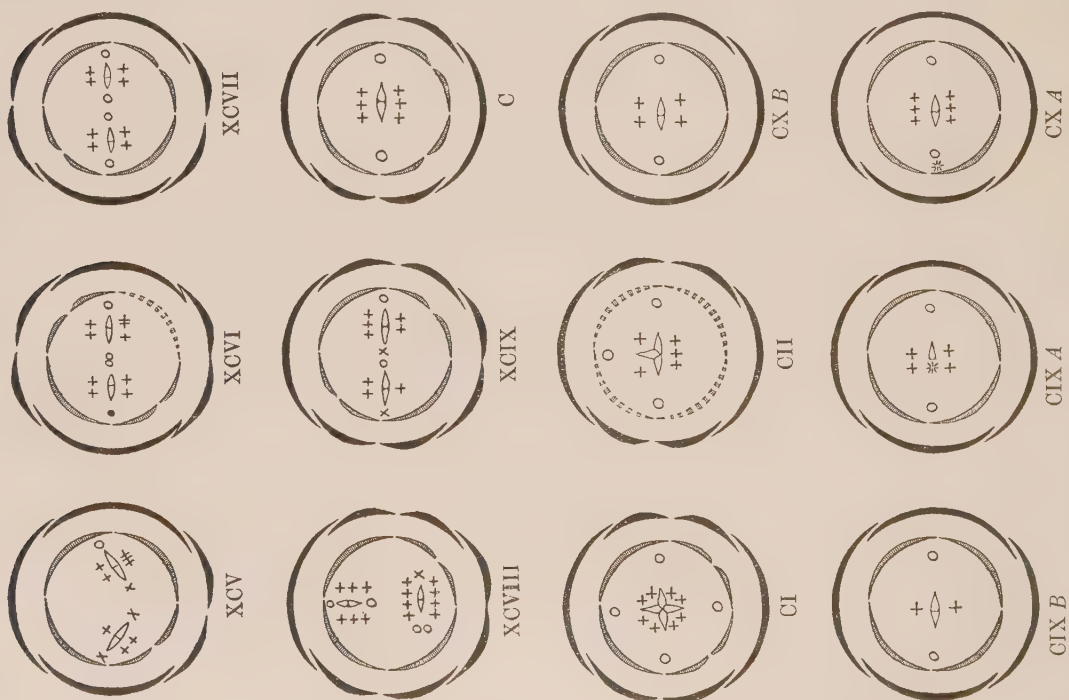


PLATE X.



NOCHMALS ÜBER "THE ELIMINATION OF SPURIOUS CORRELATION DUE TO POSITION IN TIME OR SPACE."

VON O. ANDERSON, St. Petersburg, Rußland.

1. Im Aprilheft der *Biometrika*, hat "Student" gezeigt*, daß das von Cave und Hooker vorgeschlagene Verfahren, den Korrelationskoeffizienten zweier oscillierender Variablen durch Berechnung erster Differenzen (also durch Ersetzung der Reihe $x_1, x_2, \dots, x_n$ durch die Reihe: $\Delta'x_1 = x_1 - x_2, \Delta'x_2 = x_2 - x_3, \dots, \Delta'x_{n-1} = x_{n-1} - x_n$) vom evolutorischen Element zu befreien, eine Verallgemeinerung zuläßt. Das Verfahren ist nämlich, streng genommen, nur dann richtig, wenn die evolutorische Komponente durch eine lineare Gleichung darstellbar ist. Findet letzteres nicht statt, kann also, z. B., jener nur eine parabolische Gleichung höherer Ordnung genügen, so muß man zweite, dritte u.s.w. Differenzen nehmen (also statt $\Delta'x_1, \Delta'x_2 \dots$ nehme man $\Delta''x_1 = \Delta'x_1 - \Delta'x_2, \Delta''x_2 = \Delta'x_2 - \Delta'x_3, \dots$ etc.) und danach Korrelationskoeffizienten berechnen. Letztere können bald einen konstanten Grenzwert erreichen, der das gewünschte Resultat darstellt.

Unterzeichneter ist schon vor etwa 2 Jahren zu ähnlichen Schlüssen gekommen. Durch von ihm unabhängige Gründe wurde er aber bis jetzt vom Drucke seiner diesbezüglichen Schrift abgehalten. Da er bei seiner Untersuchung Wege einschlägt, die von denen des "Students" sehr verschieden sind, und auch zu manchen Schlüssen kommt, welche letzterem unbekannt geblieben zu sein scheinen, so könnte vielleicht eine kurzgefaßte Darstellung der wichtigsten Resultate seiner Untersuchung für die Leser der *Biometrika* von einigem Interesse sein.

2. *Methode.* Die englische statistische Schule vernachlässigt in ihren Untersuchungen ein Verfahren, das von russischen und deutschen Gelehrten oft angewandt wird (Tchebycheff, Markoff, v. Bortkiewicz, u.s.w.) und neben großer Strenge und Exaktheit noch den Vorzug hat recht elementar zu sein—die Methode der mathematischen Erwartung nämlich. Mathematische Erwartung einer Größe (A) heißt bekanntlich soviel als das Produkt aus dieser Größe und ihrer Wahrscheinlichkeit (w), also Aw . Wenn eine Variable eine Reihe einander ausschließender

* *Biometrika*, Vol. x. Part 1, S. 179, "The Elimination of Spurious Correlation due to Position in Time or Space." By "Student."

Größen annehmen kann, so ist deren math. Erwartung als die Summe der Erwartungen aller dieser Größen definiert. Wir werden hier die mathem. Erwartung überall durch das Symbol $E(\)$ bezeichnen. $E(A)$ ist also, z. B., gleich Aw .

Die hauptsächlichsten Sätze über mathematische Erwartungen dürften als bekannt angenommen werden. Um aber die Nachprüfung der Formeln dieser Schrift zu erleichtern, werden wir hier die für uns wichtigsten Sätze noch kurz andeuten:

$$(1) \quad E(x + y - z + u - t \dots) = E(x) + E(y) - E(z) + E(u) - E(t) \dots$$

(2) Wenn $x, y, z, \dots$ von einander unabhängig sind, so ist

$$E(x \cdot y \cdot z \cdot \dots) = E(x) \cdot E(y) \cdot E(z) \dots$$

(3) $E(kx) = kE(x)$, wo k const. ist; und daher auch:

$$E(k) = k.$$

(4) Wenn eine Variable X die Werte $x_1, x_2, \dots, x_n$ annehmen kann, so ist die Wahrscheinlichkeit W , daß die Differenz $x_i - E(x)$ zwischen den Grenzen $-\alpha \sqrt{E(x^2) - [E(x)]^2}$ und $+\alpha \sqrt{E(x^2) - [E(x)]^2}$ enthalten sei, größer als $1 - \frac{1}{\alpha^2}$, wo α größer als 1 sein muß (ein Theorem von Tchebycheff).

In unserer Untersuchung werden wir überall statt des wahrscheinlichsten Wertes einer Größe deren mathematische Erwartung berechnen.

Bestimmen wir zuerst, wie sich der Korrelationskoeffizient zweier oscillierender Reihen verhält, wenn man deren Größen durch Differenzen $\Delta'x, \Delta''x, \Delta'''x \dots, \Delta'y, \Delta''y, \Delta'''y, \dots$ ersetzt, und darauf untersuchen wir die Frage von den Grenzen der Anwendbarkeit der verallgemeinerten Cave-Hookerschen Methode. Um Raum zu sparen, werden wir nur die endgiltigen Resultate der Berechnungen anführen, ausgenommen die 3 ersten Formeln, deren Bestimmung als Beispiel der Rechnungsmethode dienen möge.

3. *Definition.* Unter einer oscillatorischen Reihe werden wir eine solche Reihe

$$x_1, x_2, x_3, \dots, x_i, \dots, x_n$$

verstehen, bei der

$$E(x_1) = E(x_2) = \dots = E(x_i) = \dots = E(x_n) = E(x) = \text{const.}$$

und alle einzelnen Glieder von einander völlig unabhängig sind, so daß

$$E(x_i x_j) = E(x_i) \cdot E(x_j), \text{ wobei } i \neq j.$$

Solchen Bedingungen würde *zum Beispiel* eine Reihe genügen, deren Glieder die Resultate einer Versuchsreihe mit konstanter Wahrscheinlichkeit darstellen, etwa Resultate von Ziehungen aus einer Urne mit m weißen und n schwarzen Kugeln.

4. Mittleres Fehlerquadrat.

Bezeichnen wir $x_i - E(x)$ durch ξ_i , so ist

$$E(\xi_i) = E[x_i - E(x)] = E(x) - E(x) = 0.$$

Da die einzelnen ξ von einander völlig unabhängig sind, so ist

$$E(\xi_i \xi_j) = E(\xi_i) \cdot E(\xi_j) = 0.$$

Das mittlere Fehlerquadrat der Reihe x ist gleich

$$\frac{\sum_1^n [x_i - E(x)]^2}{n}.$$

Seine mathematische Erwartung wollen wir (nicht ganz in Übereinstimmung mit der üblichen Bezeichnung) σ_x^2 nennen.

$$\begin{aligned} \sigma_x^2 = E \left\{ \frac{\sum_1^n [x_i - E(x)]^2}{n} \right\} &= \frac{1}{n} \left\{ E \left[\sum_1^n x_i^2 \right] - 2E \left[\sum_1^n x_i \right] \cdot E(x) + n[E(x)]^2 \right\} \\ &= E(x^2) - [E(x)]^2, \end{aligned}$$

ein Ausdruck, der oben im Satze 4 (§ 2) unter dem Zeichen der Quadratwurzel steht.

$$\text{Andererseits ist aber } E \left\{ \frac{\sum_1^n [x_i - E(x)]^2}{n} \right\} \text{ gleich } E \left\{ \frac{\sum_1^n \xi_i^2}{n} \right\} \text{ und daher}$$

$$\sigma_x^2 = E(\xi^2).$$

Untersuchen wir den Ausdruck $E \left[\sum_1^n (x_i - M_x)^2 \right]$, wo M_x das arithmetische

Mittel der Reihe x , also $\frac{\sum_1^n x_i}{n}$ bedeutet. Da

$$x_i - M_x = E(x) + \xi_i - \frac{E(x) + \xi_1 + E(x) + \xi_2 + \dots + E(x) + \xi_n}{n} = \xi_i - M_\xi$$

(wenn man M_ξ für $\frac{\sum_1^n \xi_i}{n}$ einsetzt), so haben wir:

$$\begin{aligned} E \left[\sum_1^n (x_i - M_x)^2 \right] &= E \left[\sum_1^n (\xi_i - M_\xi)^2 \right] = E \left[\sum_1^n \xi_i^2 - nM_\xi^2 \right] \\ &= nE(\xi^2) - nE(M_\xi^2) = nE(\xi^2) - nE \left\{ \frac{\sum_1^n \xi_i^2 - 2\sum_1^n \xi_i \xi_j}{n^2} \right\} = nE(\xi^2) - E(\xi^2). \\ E \left[\sum_1^n (x_i - M_x)^2 \right] &= (n-1) E(\xi^2). \end{aligned}$$

Um $E(\xi^2)$ zu bekommen, muß man diesen Ausdruck durch $(n-1)$ dividieren.

Es ist also auch

$$\sigma_x^2 = E \left\{ \frac{\sum_{i=1}^n (x_i - M_x)^2}{n-1} \right\}.$$

Um das Fehlerquadrat $\sigma_{\Delta'x}^2$ für die erste Differenz $\Delta'x$ zu erhalten, berücksichtigen wir, daß

$$\Delta'x_i = (x_i - x_{i+1}) = (\xi_i - \xi_{i+1})$$

und

$$E(\Delta'x_i) = E(\xi_i) - E(\xi_{i+1}) = 0.$$

Daher haben wir:

$$\begin{aligned} \sigma_{\Delta'x}^2 &= E \left\{ \frac{\sum_{i=1}^{n-1} [\Delta'x_i - E(\Delta'x)]^2}{n-1} \right\} = E \left\{ \frac{\sum_{i=1}^{n-1} (\xi_i - \xi_{i+1})^2}{n-1} \right\} \\ &= \frac{1}{n-1} \left\{ E \left[\sum_{i=1}^{n-1} \xi_i^2 \right] - 2E \left[\sum_{i=1}^{n-1} \xi_i \xi_{i+1} \right] + E \left[\sum_{i=2}^n \xi_i^2 \right] \right\} \\ &= \frac{1}{n-1} \{ (n-1) E(\xi_i^2) - 0 + (n-1) E(\xi_i^2) \} = 2E(\xi^2). \end{aligned}$$

Es ist also

$$\sigma_{\Delta'x}^2 = 2\sigma_x^2.$$

Nach demselben Rechenschema ergibt sich für das mittlere Fehlerquadrat

der zweiten Differenz $\Delta''x_i$	der Ausdruck	$6\sigma_x^2$
„ dritten „ „ $\Delta'''x_i$ „ „		$20\sigma_x^2$
„ vierten „ „ $\Delta^{(iv)}x_i$ „ „		$70\sigma_x^2$
.....		
„ k-ten „ „ $\Delta^{(k)}x_i$ „ „		$\frac{2k!}{k! \cdot k!} \sigma_x^2$

Wir können daher folgende Gleichung aufstellen

$$\sigma_x^2 = \frac{\sigma_{\Delta'x}^2}{2} = \frac{\sigma_{\Delta''x}^2}{6} = \frac{\sigma_{\Delta'''x}^2}{20} = \frac{\sigma_{\Delta^{(iv)}x}^2}{70} = \dots = \frac{\sigma_{\Delta^{(k)}x}^2}{\binom{2k!}{k! \cdot k!}} = E(\xi^2), \dots\dots\dots(1)$$

welche exakt ist, und folgende

$$\frac{\sum_{i=1}^n (x_i - M_x)^2}{n-1} = \frac{\sum_{i=1}^{n-1} \Delta'x_i^2}{2(n-1)} = \frac{\sum_{i=1}^{n-2} \Delta''x_i^2}{6(n-2)} = \frac{\sum_{i=1}^{n-3} \Delta'''x_i^2}{20(n-3)} = \frac{\sum_{i=1}^{n-4} \Delta^{(iv)}x_i^2}{70(n-4)} = \dots = \frac{\sum_{i=1}^{n-k} \Delta^{(k)}x_i^2}{\frac{2k!}{k! \cdot k!} (n-k)} = \sigma_x^{2*},$$

..... (1 a)

welche nur annähernd richtig ist.

* Es ist vorteilhafter $\frac{1}{\frac{2k!}{k! \cdot k!} (n-k)}$ und nicht $\frac{1}{\frac{2k!}{k! \cdot k!} (n-k)}$ zu berechnen.

5. Das mittlere Produkt $\frac{\sum_{i=1}^n x_i y_i}{n}$.

Wenn zwei Reihen

$$x_1, x_2, x_3, \dots x_n,$$

und

$$y_1, y_2, y_3, \dots y_n,$$

beide im Sinne des § 3 oscillatorisch sind, und eine Korrelation nur zwischen Größen mit gleichen Indexen, also zwischen x_1 und y_1 , x_2 und y_2 , x_3 und y_3 , u.s.w. bestehen kann, so ist es leicht ersichtlich, daß

$$E \{ [x_i - E(x)] [y_j - E(y)] \} = 0, \text{ wenn } i \neq j.$$

Bezeichnen wir $y_i - E(y)$ durch ψ_i , $x_i - E(x)$ wieder durch ξ_i , so finden wir leicht folgende Ausdrücke:

$$p_{xy} = E \left\{ \frac{\sum_{i=1}^n \xi_i \psi_i}{n} \right\} = E(\xi_i \psi_i),$$

$$E \left\{ \frac{\sum_{i=1}^n (x_i - M_x)(y_i - M_y)}{n-1} \right\} = E(\xi_i \psi_i) = p_{xy},$$

$$p_{\Delta' x \Delta' y} = E \left\{ \frac{\sum_{i=1}^{n-1} \Delta' x_i \Delta' y_i}{n-1} \right\} = 2p_{xy},$$

$$p_{\Delta'' x \Delta'' y} = E \left\{ \frac{\sum_{i=1}^{n-2} \Delta'' x_i \Delta'' y_i}{n-2} \right\} = 6p_{xy},$$

$$\dots \dots \dots$$

$$p_{\Delta^{(k)} x, \Delta^{(k)} y} = E \left\{ \frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i}{n-k} \right\} = \frac{2k!}{k! \cdot k!} p_{xy}.$$

Wir können daher wieder zwei Gleichungssysteme:

$$p_{xy} = \frac{p_{\Delta' x \Delta' y}}{2} = \frac{p_{\Delta'' x \Delta'' y}}{6} = \frac{p_{\Delta''' x \Delta''' y}}{20} = \frac{p_{\Delta^{(iv)} x \Delta^{(iv)} y}}{70} = \dots = \frac{p_{\Delta^{(k)} x \Delta^{(k)} y}}{\frac{2k!}{k! \cdot k!}} = E(\xi_i \psi_i), \dots (2)$$

und

$$\begin{aligned} \frac{\sum_{i=1}^n [x_i - E(x)] [y_i - E(y)]}{n-1} &= \frac{\sum_{i=1}^{n-1} \Delta' x_i \Delta' y_i}{2(n-1)} = \frac{\sum_{i=1}^{n-2} \Delta'' x_i \Delta'' y_i}{6(n-2)} = \frac{\sum_{i=1}^{n-3} \Delta''' x_i \Delta''' y_i}{20(n-3)} \\ &= \frac{\sum_{i=1}^{n-4} \Delta^{(iv)} x_i \Delta^{(iv)} y_i}{70(n-4)} = \dots = \frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i}{\frac{2k!}{k! \cdot k!} \cdot (n-k)} = p_{xy}, \dots (2a) \end{aligned}$$

aufstellen, von denen das erste exakt und das zweite angenähert ist, und welche den Gleichungen für σ_x^2 (also auch für σ_y^2) des § 4 genau analog sind.

6. Das Fehlerquadrat der Fehlerquadrate.

Betrachten wir jetzt den Bereich der Schwankungen der Größen der Systeme (1a) und (2a) um deren mathematisch zu erwartenden Größen in (1) und (2). Mit anderen Worten, gehen wir (mit Rücksicht auf Satz 4 § 2) zur Darstellung der math. Erwartung der Fehlerquadrate der genannten Größen über.

$$\text{Für } \frac{\sum_{i=1}^n [x_i - E(x)]^2}{n} \text{ ergibt sich das Fehlerquadrat } \frac{E(\xi^4) - [E(\xi^2)]^2}{n}.$$

$$\text{Für } \frac{\sum_{i=1}^n [x_i - M_x]^2}{n-1} \text{ ergibt sich das Fehlerquadrat } \frac{E(\xi^4) - [E(\xi^2)]^2}{n} + \frac{2[E(\xi^2)]^2}{n(n-1)}.$$

$$\text{Für } \frac{\sum_{i=1}^{n-1} [\Delta' x_i]^2}{2(n-1)} \text{ ergibt sich das Fehlerquadrat } \frac{(2n-3) \{E(\xi^4) - [E(\xi^2)]^2\} + 2(n-1)[E(\xi^2)]^2}{2(n-1)^2}.$$

$$\text{Für } \frac{\sum_{i=1}^{n-2} [\Delta'' x_i]^2}{6(n-2)} \text{ ergibt sich das Fehlerquadrat } \frac{(9n-23) \{E(\xi^4) - [E(\xi^2)]^2\} + (17n-42)[E(\xi^2)]^2}{9(n-2)^2}.$$

$$\text{Und endlich für } \frac{\sum_{i=1}^{n-k} (\Delta^{(k)} x_i)^2}{\frac{2k!}{k! \cdot k!} (n-k)} \text{ ergibt sich der recht komplizierte Ausdruck:}$$

$$\frac{1}{A_0^2 (n-k)^2} \left\{ A_0^2 (n-2k) \{E(\xi^4) - [E(\xi^2)]^2\} + 4[E(\xi^2)]^2 [A_1^2 (n-2k+1) + A_2^2 (n-2k+2) + A_3^2 (n-2k+3) + \dots + A_k^2 (n-2k+k)] \right. \\ \left. + 2B_0^2 \{E(\xi^4) - [E(\xi^2)]^2\} + 8[E(\xi^2)]^2 \cdot (B_1^2 + B_2^2 + \dots + B_{k-1}^2) \right\}.$$

Wenn $b_0, b_1, b_2, \dots, b_k$ die Koeffizienten der Zerlegung des Binoms $(1+1)^k$ darstellen, also $b_0 = 1, b_1 = k, b_2 = \frac{k \cdot (k-1)}{1 \cdot 2}$, u.s.w., so ist hier

$$A_0^2 = (b_0^2 + b_1^2 + b_2^2 + \dots + b_k^2)^2 = \left[\frac{2k!}{k! \cdot k!} \right]^2, \\ A_1^2 = (b_0 b_1 + b_1 b_2 + b_2 b_3 + \dots + b_{k-1} b_k)^2 = \left[\frac{2k!}{k-1! \cdot k+1!} \right]^2, \\ A_2^2 = (b_0 b_2 + b_1 b_3 + \dots + b_{k-2} b_k)^2 = \left[\frac{2k!}{k-2! \cdot k+2!} \right]^2, \\ \dots \dots \dots A_j^2 = (b_0 b_j + b_1 b_{j+1} + \dots + b_{k-j} b_k)^2 = \left[\frac{2k!}{k-j! \cdot k+j!} \right]^2, \\ \dots \dots \dots A_k^2 = 1,$$

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Es ist also klar, daß zusammen mit dem endlichen Differenzieren die Unsicherheit der Bestimmung von σ_x^2 stetig wächst, anfangs etwa im Verhältnis

$$\sqrt{2} : \sqrt{3} : \sqrt{4} : \sqrt{5} \dots$$

7. Das Fehlerquadrat des mittleren Produktes.

Für $\frac{1}{n} \sum_{i=1}^n \xi_i \psi_i$ ergibt sich das Fehlerquadrat $\frac{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2}{n}$.

Für $\frac{1}{n-1} \sum_{i=1}^n (x_i - M_x)(y_i - M_y)$ ergibt sich das Fehlerquadrat

$$\frac{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2}{n} + \frac{[E(\xi_i \psi_i)]^2 + \sigma_x^2 \sigma_y^2}{n(n-1)}.$$

Für $\frac{1}{2(n-1)} \sum_{i=1}^{n-1} \Delta' x_i \Delta' y_i$ ergibt sich das Fehlerquadrat

$$\frac{(2n-3) \{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2\} + (n-1) \{[E(\xi_i \psi_i)]^2 + \sigma_x^2 \sigma_y^2\}}{2(n-1)^2}.$$

Im allgemeinen Fall $\frac{1}{\frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i}{k! k! (n-k)}}$ erhalten wir für das Fehlerquadrat folgenden

Ausdruck:

$$\frac{1}{A_0^2 (n-k)^2} \left\{ A_0^2 (n-2k) \{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2\} + 2 \{[E(\xi_i \psi_i)]^2 + \sigma_x^2 \sigma_y^2\} [A_1^2 (n-2k+1) + A_2^2 (n-2k+2) + \dots + A_k^2 (n-2k+k)] + 2B_0^2 \{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2\} + 4 \{[E(\xi_i \psi_i)]^2 + \sigma_x^2 \sigma_y^2\} (B_1^2 + B_2^2 + \dots + B_{k-1}^2) \right\},$$

wo die Koeffizienten $A_0^2, A_1^2, \dots, B_0^2, B_1^2, \dots$ dieselbe Bedeutung haben, wie in § 6.

Wenn x_i und y_i einander vollkommen gleich sind, so ist

$$[E(\xi_i \psi_i)]^2 = \{E(\xi^2)\}^2 = \sigma_x^4; \quad E(\xi_i^2 \psi_i^2) = E(\xi_i^4); \quad \sigma_x^2 \sigma_y^2 = \sigma_x^4,$$

und obiger Ausdruck fällt mit dem in § 6 zusammen.

Für den Fall der "normalen" Verteilung können wir auch alle diese Ausdrücke beträchtlich vereinfachen, besonders wenn wir $E(\xi_i \psi_i)$ und $E(\xi_i^2 \psi_i^2)$ als Funktionen von r_{xy} darstellen. Ohne aber hier darauf einzugehen, wollen wir jetzt über den Korrelationskoeffizienten ins Klare kommen.

8. Definition des Korrelationskoeffizienten.

Der Korrelationskoeffizient R wird gewöhnlich nach der Formel

$$R_0 = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - M_x)(y_i - M_y)}{\sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - M_x)^2 \frac{1}{n} \sum_{i=1}^n (y_i - M_y)^2}} \text{ berechnet.}$$

Zu welchem Ausdruck ist er nun als empirische Annäherung aufzufassen,

$$\text{zu } E \left\{ \frac{\sum_1^n (x_i - M_x)(y_i - M_y)}{\sqrt{\sum_1^n (x_i - M_x)^2 \sum_1^n (y_i - M_y)^2}} \right\} \text{ oder zu } \frac{E \left[\sum_1^n (x_i - M_x)(y_i - M_y) \right]}{\sqrt{E \left[\sum_1^n (x_i - M_x)^2 \right] \cdot E \left[\sum_1^n (y_i - M_y)^2 \right]}} ?$$

Beide Formeln sind durchaus nicht mit einander zu identifizieren und fallen nur in erster Annäherung zusammen. Da die zweite aber bedeutend leichter zu handhaben ist und dies auch mehr den üblichen Rechnungsmethoden der englischen Schule entspricht, so *definieren* wir r_{xy} als

$$\frac{E \left[\sum_1^n (x_i - M_x)(y_i - M_y) \right]}{\sqrt{E \left[\sum_1^n (x_i - M_x)^2 \right] \cdot E \left[\sum_1^n (y_i - M_y)^2 \right]}}.$$

Anders ausgedrückt ist $r_{xy} = \frac{p_{xy}}{\sigma_x \sigma_y}$, wo p_{xy} , σ_x , σ_y die Bedeutungen haben, welche wir ihnen oben in §§ 4 und 5 beigemessen haben.

9. *Das Verhalten des Korrelationskoeffizienten zweier oscillierender Reihen x und y , wenn man deren Größen durch Differenzen ersetzt.*

Für die k -te endliche Differenz von x und y haben wir

$$r_{\Delta^{(k)}x, \Delta^{(k)}y} = \frac{E \left(\sum_1^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i \right)}{\sqrt{E \left[\sum_1^{n-k} \Delta^{(k)} x_i^2 \right] \cdot E \left[\sum_1^{n-k} \Delta^{(k)} y_i^2 \right]}} = \frac{p_{\Delta^{(k)}x, \Delta^{(k)}y}}{\sqrt{\sigma^2_{\Delta^{(k)}x} \cdot \sigma^2_{\Delta^{(k)}y}}} = \frac{\frac{2k!}{k! \cdot k!} p_{xy}}{\sqrt{\frac{2k!}{k! \cdot k!} \sigma_x^2 \cdot \frac{2k!}{k! \cdot k!} \sigma_y^2}},$$

$$r_{\Delta^{(k)}x, \Delta^{(k)}y} = \frac{p_{xy}}{\sigma_x \sigma_y} = r_{xy}.$$

Wir haben also ganz allgemein das *genaue* Resultat :

$$r_{xy} = r_{\Delta'x, \Delta'y} = r_{\Delta''x, \Delta''y} = r_{\Delta'''x, \Delta'''y} = \dots = r_{\Delta^{(k)}x, \Delta^{(k)}y}.$$

Da aber diese r unbekannt bleiben und wir für ein beliebiges $r_{\Delta^{(i)}x, \Delta^{(i)}y}$ nur

$$\text{dessen Annäherungsformel } R_i = \frac{\sum_1^{n-i} \Delta^{(i)} x_i \Delta^{(i)} y_i}{\sqrt{\sum_1^{n-i} \Delta^{(i)} x_i^2 \cdot \sum_1^{n-i} \Delta^{(i)} y_i^2}}$$

wiederum feststellen, inwiefern man sich in der Praxis auf die Übereinstimmung der empirischen Koeffizienten mit deren mathematischen Erwartungen verlassen kann, wie groß also die Unsicherheit ihrer Bestimmung zu schätzen ist.

10. Mittleres Fehlerquadrat des Korrelationskoeffizienten der endl. Differenzen zweier Reihen.

Aus der Formel $R_k = \frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i}{\sqrt{\sum_{i=1}^{n-k} \Delta^{(k)} x_i^2 \cdot \sum_{i=1}^{n-k} \Delta^{(k)} y_i^2}}$, kann man folgenden Ausdruck

ableiten:

$$\frac{R_k - r_{xy}}{r_{xy}} = \frac{\frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i \Delta^{(k)} y_i}{n-k} - p_{\Delta^{(k)} x, \Delta^{(k)} y}}{p_{\Delta^{(k)} x, \Delta^{(k)} y}} - \frac{\frac{\sum_{i=1}^{n-k} \Delta^{(k)} x_i^2}{n} - \sigma^2_{\Delta^{(k)} x}}{2\sigma^2_{\Delta^{(k)} x}} - \frac{\frac{\sum_{i=1}^{n-k} \Delta^{(k)} y_i^2}{n} - \sigma^2_{\Delta^{(k)} y}}{2\sigma^2_{\Delta^{(k)} y}},$$

der nur in erster Annäherung und, wenn alle 4 Brüche des Ausdrucks echte sind, richtig ist; dies ist bei großem n der Fall. Und ferner ergibt sich daraus die Formel:

$$\sigma^2_{R_k} = \frac{(1 - r_{xy}^2)^2}{(n-k)^2} \left\{ (n-k) + 2(n-k-1) \left(\frac{k}{k+1} \right)^2 + 2(n-k-2) \left(\frac{k \cdot (k-1)}{(k+1) \cdot (k+2)} \right)^2 \right. \\ \left. + 2(n-k-3) \left(\frac{k \cdot (k-1) \cdot (k-2)}{(k+1) \cdot (k+2) \cdot (k+3)} \right)^2 + \dots \right. \\ \left. + 2(n-k-k) \left(\frac{k \cdot (k-1) \cdot (k-2) \dots 2 \cdot 1}{(k+1) \cdot (k+2) \cdot (k+3) \dots (k+k)} \right)^2 \right\}$$

(vergl. dazu die Formel für $\frac{\sum_{i=1}^{n-k} (\Delta^{(k)} x_i)^2}{\frac{2k!}{k! \cdot k!} (n-k)}$ in § 6).

Die Formel für $\sigma^2_{R_k}$ ist immer nur dann gültig, wenn man

$$\frac{r_{xy}^2}{n} \left\{ \frac{E(\xi_i^2 \psi_i^2) - [E(\xi_i \psi_i)]^2}{[E(\xi_i \psi_i)]^2} + \frac{E(\xi^4) - [E(\xi^2)]^2}{4[E(\xi^2)]^2} + \frac{E(\psi^4) - [E(\psi^2)]^2}{4[E(\psi^2)]^2} \right. \\ \left. - \frac{E(\xi_i^3 \psi_i) - E(\xi_i \psi_i) \cdot E(\xi^2)}{E(\xi_i \psi_i) \cdot E(\xi^2)} - \frac{E(\xi_i \psi_i^3) - E(\xi_i \psi_i) \cdot E(\psi^2)}{E(\xi_i \psi_i) \cdot E(\psi^2)} \right. \\ \left. + \frac{E(\xi_i^3 \psi_i^2) - E(\xi^2) \cdot E(\psi^2)}{2E(\xi^2) \cdot E(\psi^2)} \right\}$$

gleich $\frac{(1 - r_{xy}^2)^2}{n}$ setzen darf (vergl. *Biometrika*, Vol. IX. p. 4).

Aus der Formel für $\sigma^2_{R_k}$ erhalten wir:

$$\sigma^2_{R_0} = \frac{(1 - r_{xy}^2)^2}{n}, \\ \sigma^2_{R_1} = \frac{(1 - r_{xy}^2)^2}{n-1} \cdot \frac{3n-4}{2(n-1)}, \\ \sigma^2_{R_2} = \frac{(1 - r_{xy}^2)^2}{n-2} \cdot \frac{35n-88}{18(n-2)}, \\ \sigma^2_{R_3} = \frac{(1 - r_{xy}^2)^2}{n-3} \cdot \frac{231n-843}{100(n-3)},$$

u. s. w.

Die Fehlerquadrate der Korrelationskoeffizienten aufeinanderfolgender Differenzenordnungen verhalten sich folglich zueinander ungefähr wie $2 : 3 : 4 : 5 \dots$

Die Unsicherheit wächst also mit zunehmender Differenzenordnung etwa im Verhältnis

$$\sqrt{2} : \sqrt{3} : \sqrt{4} : \sqrt{5}, \dots$$

11. *Korrelationskoeffizient zweier zusammengesetzter Reihen, die aus oscillatorischen und evolutorischen Elementen bestehen.*

Da "Student" diese Frage treffend dargelegt hat, können wir uns kurz fassen. Wenn wir in Betracht ziehen, daß für uns die evolutorische Komponente einer Reihe schon dann in der Praxis verschwunden ist, wenn sie im Verhältnis zur oscillatorischen Komponente so klein geworden ist, daß sie nur die 3^{ten}, 4^{ten}, u.s.w. Zahlenstellen des Ausdruckes für R beeinflussen kann, so kommen wir zum Schluß, daß nicht nur Komponenten, die durch eine Parabel höherer Ordnung darstellbar sind, sondern auch solche, denen nur transzendente Gleichungen (z. B. Sinusreihen) genügen, beim endlichen Differenzieren eliminiert werden. Ja mehr noch, man kann beweisen, daß überhaupt alle mehr oder minder "glatten Reihen," alle bei denen eine genügende positive Korrelation zwischen den Nachbargliedern bemerkbar ist, für die Praxis beim endlichen Differenzieren verschwinden. Das verallgemeinerte Cave-Hookersche Verfahren ist daher augenscheinlich ein sehr universales Mittel, die Korrelation oscillatorischer Elemente aus zusammengesetzten Reihen herauszuschälen. Es hat aber einen Haken, auf den hier noch hingewiesen werden muß.

12. *Kann man aus dem Verhalten der Reihe $R_0, R_1, R_2, \dots R_k$ bestimmen, ob wir den Korrelationskoeffizienten rein oscillatorischer Reihen vor uns haben?* "Student" scheint zu glauben, daß wenn irgendein R_i seinem Vorgänger R_{i-1} gleich ist, wir es sicher mit dem Korrelationskoeffizienten oszillierender Elemente zu tun haben. Vor einem solchen Schluß ist nachdrücklich zu warnen. Wie es meine (für diesen Artikel etwas zu langwierigen) Berechnungen zeigen, können zwei Nachbarkoeffizienten R_i, R_{i-1} auch bei stark evolutorischen Reihen einander ungefähr gleich sein, und die Wahrscheinlichkeit eines solchen Zusammentreffens ist gar nicht sehr gering einzuschätzen. Nur wenn wir, von irgendeinem R_j angefangen, immer dieselbe Größe für R erhalten, also $R_j = R_{j+1} = R_{j+2} = R_{j+3} = \dots$, wird ein solcher Schluß berechtigt sein, und je länger die Reihe gleicher R , desto wahrscheinlicher wird dieser Schluß.

STATISTICAL NOTES ON THE INFLUENCE OF EDUCATION IN EGYPT.

BY M. HOSNY, M.A., B.Sc.

The statistical returns for Egypt are—as compared with European data—still in a somewhat elementary stage. Age-distributions are of very little value, and in the case of infantile mortality we have only information for certain towns. Further, in the larger towns there is a considerable cosmopolitan element, which gives them a widely different character from the often sparsely populated rural and desert districts. Education is not compulsory, and schools and literacy are largely confined to Cairo, Alexandria and the Canal Government, even when we exclude all foreign scholars. In the same way criminality* preponderates, in an inverse order it is true, in these three districts, but it is not absolutely certain whether this is due to their more efficient policing, to the presence of more foreigners, or to a real absence of crime in the rural populations. Crime does not appear to arise in Egypt from poverty or drunkenness, two of the main factors of its origin in Western Europe. The criminal, indeed, is rarely habitual; he is an amateur, rather than a professional, and criminals are more often well-to-do, their crimes arising from motives of revenge or passion:

The fact that criminality in Egypt is highly correlated with literacy and scholarship would be noteworthy and might possibly be used as an argument against education, did not the association of crime and education arise from the prevalence of both in the more populated districts, where again we find the greatest abundance of foreigners. Naturally such questions arise as:

(i) Are the foreigners—and if so, which section of them—to any extent responsible for the prevalence of crime in the districts frequented by them?

(ii) If we allow for urban conditions, will there still be found a high association of crime and education?

It is perfectly easy to obtain from the Egyptian Census—we used that of 1907—the number of foreigners of each denomination in the various Egyptian

* We understand by “criminality” in this paper, not commission of but conviction for crime.

governments. The only difficulty here was the presence of British troops in Cairo and Alexandria, which placed that nationality in an anomalous position. These were estimated approximately and subtracted. The following groups of foreigners were then dealt with: (a) Ottomans, (b) British subjects, French, Austrians, Germans and Russians*, (c) Greeks, (d) Italians. The Greeks and Italians were separated from the general European group (b), because they are largely differentiated, the Greeks being frequently small traders and the Italians often manual workers. Their large numbers also justified a separate classification.

Table I gives the foreigners per 10,000 in the 17 Egyptian districts we were able to deal with. It will be noted that the Greeks far outnumber other

TABLE I.

Foreigners per 10,000 and Population per sq. kilometre.

Governments	Ottomans	Europeans other than Greeks and Italians	Greeks	Italians	Population per sq. kilometre
Cairo	453	312	298	204	6060
Alexandria	661	514	745	482	6780
Canal†	416	583	846	445	7666
Behera†	43	23	31	11	178
Charkieh	29	5	24	1	257
Dakahlieh and Damietta	18	5	18	3	346
Gharbieh	2	0	2	0	226
Kallihieh	8	3	13	2	469
Menufieh	2	1	7	0	618
Assiut†	4	2	3	1	454
Assuan	8	5	13	4	533
Beni Suef	15	6	9	1	351
Fayoum	10	3	4	0	255
Gerga	2	0	2	0	532
Guizeh	6	6	4	3	447
Kena†	5	4	4	2	339
Minia†	10	5	7	1	458

foreigners, but that all foreigners are concentrated in the Cairo, Alexandria and Canal governments.

It was far more difficult to obtain a measure of urban conditions. We had to take very rough measures of the density of the population, because the limits of certain areas are too vaguely defined to be of any service. El Arish has been excluded from the Canal district, Suez and Sinai have also been excluded as there is no enumeration of them with respect to criminality, literacy and scholarship. These densities, with such value as they have, are given in the last column of Table I.

* The contributions from other smaller nationalities were omitted.

† Various approximations and omissions occur in these cases in obtaining density.

Table II provides the number of male criminals per 1000 of the male population, the literacy or number of male persons able to read and write per 100 of

TABLE II. *Educational and Criminal Indices.*

Governments	Male Criminals per 1000 males	Literacy per 100 males	Male Scholars 5—19, per 100 boys of those ages
Cairo	12.90	28.03	30.20
Alexandria	14.15	30.09	19.99
Canal and El Arish ...	22.30	23.39	8.54
Behera	5.30	9.29	1.01
Charkieh	4.20	9.09	1.66
Dakahlieh and Damietta	4.35	8.18	1.76
Gharbieh	5.65	8.22	3.04
Kalliuhieh	6.65	8.13	1.39
Menufieh	3.85	8.45	1.06
Assiut	5.45	7.01	4.02
Assuan	4.20	7.68	0.82
Beni Suef	5.70	8.42	2.03
Fayoum	6.85	6.54	1.85
Gerga	3.95	5.84	2.22
Guizeh	5.10	6.38	1.15
Kena	3.45	5.34	1.66
Minia	5.20	7.25	3.13

the male population*, and the number of male scholars aged 5 to 19 per 100 of the native boys of those ages†.

We shall use the following symbols to denote the factors which occur in Tables I and II:

O = Ottomans, G = Greeks, I = Italians,

E = Europeans other than Greeks and Italians.

C = Criminality, L = Literacy, S = Scholarship, D = Density of Population.

Each government was treated as of equal weight, although the populations vary from 233,000 in Assuan to 1,485,000 in Gharbieh. The standard-deviations and product-moments were found without grouping. The following results were obtained:

Means	Standard Deviations	Correlations
$m_C = 7.015,$	$\sigma_C = 4.791,$	$r_{CL} = +.8450 \pm .0468,$
$m_L = 11.019,$	$\sigma_L = 7.641,$	$r_{CS} = +.6242 \pm .0999,$
$m_S = 5.031,$	$\sigma_S = 7.735,$	$r_{LS} = +.9028 \pm .0303,$
$m_D = 1528,$	$\sigma_D = 2475.5,$	—————

Correlations:

$$r_{DC} = +.9614 \pm .0124, \quad r_{DS} = +.8097 \pm .0566, \quad r_{DL} = +.9563 \pm .0138.$$

Now at first sight these results would seem to indicate a very bad influence of education on crime. Where literacy and scholarship are greatest, there criminals

* *Egyptian Census*, 1907, p. 99.

† Foreign male scholars are excluded in the case of Cairo, Alexandria and the Canal. They have no sensible numerical existence elsewhere. Criminals and scholars are taken from the *Annuaire Statistique de l'Egypte*, 1912, pp. 95 and 135.

are most numerous! And a superficial argument might be used to condemn the character of education in Egypt, or education in general. But it will be clear on examination of the isolated values that the observed high correlations arise solely from the urban character in Egypt of both criminality and education. We have endeavoured therefore to correct this by finding the partial correlations for constant density of population.

There now result

$${}_Dr_{CS} = -.9554 \pm .0143,$$

$${}_Dr_{CL} = -.9231 \pm .0242,$$

$${}_Dr_{LS} = +.7480 \pm .0721.$$

Thus, while there still remains a quite considerable relation between the prevalence of literacy and scholars for constant density, we find that for a constant degree of urban conditions, the greater the literacy and the greater the amount of education the less will be the criminality. The negative correlations are now even higher than the uncorrected positive ones and of course are markedly significant. While admitting the slender nature of the Egyptian data, we think that this swinging over of the relation of crime and education when we correct for density is suggestive, and it would be of interest to work out similar correlations for states in which the statistics are of a more ample character. It does, however, appear reasonable to assert that there is no evidence to indicate that education leads to criminality—rather the reverse—in Egypt.

We will next consider the influence of the presence of foreigners in Egypt.

We find:

Means	Standard Deviations	Correlations
$m_O = 99.53,$	$\sigma_O = 195.60,$	$r_{OC} = +.8425 \pm .0473,$
$m_E = 86.88,$	$\sigma_E = 183.70,$	$r_{EC} = +.9546 \pm .0145,$
$m_G = 119.41,$	$\sigma_G = 256.61,$	$r_{GC} = +.9429 \pm .0181,$
$m_I = 68.24,$	$\sigma_I = 152.04,$	$r_{IC} = +.9192 \pm .0254.$

Correlations:

$$r_{DO} = +.9575 \pm .0136, \quad r_{DE} = +.9844 \pm .0050,$$

$$r_{DG} = +.9491 \pm .0162, \quad r_{DI} = +.9617 \pm .0123.$$

Here, if we judged by the raw correlations only, we must assert that the correlations of crime with the presence of foreigners are so high, that the foreigners must be corrupting the Egyptian population. But again the association only arises because the criminals and foreigners are both prevalent in the big towns. If we correct for density of population, we find the results are very different. Thus we have:

$${}_Dr_{OC} = -.9811 \pm .0061, \quad {}_Dr_{EC} = +.1692 \pm .1591,$$

$${}_Dr_{GC} = +.3524 \pm .1433, \quad {}_Dr_{IC} = -.0713 \pm .1628.$$

It is now obvious that the correlation of Europeans other than Greeks and Italians with criminality has become insignificant having regard to its probable error; the correlation of the presence of Italians and criminality is now *negative*, but less than its probable error. Thus of Christians only the presence of the

Greeks may possibly, but not certainly, be detrimental. The Ottomans have now a large negative correlation of a quite significant character, or we might assert that the presence of Ottomans tends to diminish criminality. The Greeks are frequently moneylenders and alcohol dealers, and the Ottomans, especially the Arabs, have among them a good many religious teachers.

We have, however, to note that criminality is greatest in the Canal Government, where Europeans and Greeks are most frequent, while the Ottomans are most numerous in Alexandria, where crime is almost 40% less than in the Canal Government. To test the influence of the three densely populated governments, we put the Canal proportion of the Ottomans at Cairo, that of Cairo at Alexandria and that of Alexandria at the Canal. There resulted:

$$r_{OC} = +.9707, \text{ instead of } +.8425,$$

$$r_{DO} = +.9870, \text{ instead of } +.9575,$$

leading to

$$dr_{OC} = +.4918,$$

or we may safely say, that if the proportions of Ottomans at Alexandria and along the Canal were interchanged, then no relation between the presence of Ottomans and the absence of criminality would exist, indeed the relation would probably be reversed. The prevalence of the Ottomans in Alexandria has been attributed to its more temperate climate. There is certainly a large Ottoman element in Alexandria, there being 21,827 Ottomans out of a population of 332,246, and it is larger than any other foreign element except the Greeks. In Cairo, with 29,516 out of 654,476 inhabitants, the Ottomans exceed any other single foreign element. It is conceivable, therefore, that they may be able to influence the moral tone of those towns. It must be borne in mind, however, that crime is far more frequent in the Cairo and Alexandria governments than in the more purely rural districts, and we can scarcely suppose that Cairo and Alexandria would reach the still higher criminality level of the Canal, were it not for the presence of the Ottomans. In the Canal Government there are exceptional conditions, and we can hardly assume that a transfer of the Ottomans from Alexandria to the Canal would interchange their proportions of criminality. Greeks no doubt flock to the Canal for business purposes, the other Europeans largely for control purposes; the Ottomans, relatively speaking, avoid it. Without further analysis it would not be possible to assert definitely that the presence of Ottomans reduces crime. It may be doubted whether the presence of foreigners, with the possible exception of the Greeks, is really associated with the extent of criminality in Egypt.

A further investigation was undertaken in regard to the possible influence of education on infantile mortality. The birthrate and deathrate in Egypt are both remarkably high. Thus for the years 1899–1909 inclusive the average rates were:

	Births per 1000 *	Deaths per 1000	Excess of Births over Deaths
Cairo	40.7	35.7	+ 5.0
Alexandria	38.0	31.8	+ 6.2

* Still births not included.

Many European towns with half the above birthrates have considerably greater excesses of births over deaths.

Unfortunately the infantile mortality is only recorded in Egyptian towns and not in the governments at large or in the rural districts. We are obliged, therefore, to deal with these only when considering the relation of education to infantile mortality. From p. 49 of the *Annuaire Statistique de l'Égypte*, 1912, we obtain the infantile mortality for 1911, and note at once how extraordinarily high it stands. From p. 286 of the *Census of Egypt*, 1907, we take the percentage of male literates in the total male population, and from the *Statistique Scolaire*, 1912-1913, p. 74, the percentage of scholars in the total population*. As it was possible that the density of town population might influence the results, we took the number of persons per house, which was about the only social factor available. This will probably represent fairly closely the average size of family. This was taken from the *Census*, 1907, p. 286. The mean, 5.64 persons per house, suggests that the average number of living children can hardly exceed three. The marked relationship that occurs in European towns between gross size of family and infantile mortality cannot be satisfactorily tested on the Egyptian data, because we cannot ascertain the infantile mortality in each size of family. The number of persons to the house is indeed rather a measure of net than gross family, and we only know this as an *average* value for each town. It does not follow that a town with a low number of persons per house is one with small gross families; the low number may be due to the heavy infantile mortality itself. Accordingly the correlation between persons per house and infantile mortality is not necessarily even a measure of the influence of overcrowding on infantile mortality (although this is often supposed to be the case); it is conceivable that a high infantile mortality might be the source of a low number of persons per house, and the unravelling of cause and effect is only possible where we know not only the number of persons per house, but its relation to both the gross and net family of that house.

Let I = Infantile mortality, L = Literacy, S = Scholars, P = Persons per house. Then we have the following results:

Means	Standard Deviations	Correlations
$M_I = 29.30,$	$\sigma_I = 7.608,$	$r_{IL} = -0.1040 \pm 0.1500,$
$M_L = 21.96,$	$\sigma_L = 5.137,$	$r_{IS} = +0.5309 \pm 0.1278,$
$M_S = 5.569,$	$\sigma_S = 2.951,$	$r_{LS} = +0.0093 \pm 0.1508,$
$M_P = 5.643,$	$\sigma_P = 1.428,$	—————

Correlations:

$$r_{PI} = +0.1675 \pm 0.1487, \quad r_{PL} = -0.3421 \pm 0.1437, \quad r_{PS} = +0.0296 \pm 0.1508.$$

* The scholars were taken for 1910-1911, the year of infantile mortality, but this involved the assumption that the foreign scholars were the same in numbers in 1910-11 and 1912-13, probably not a very inaccurate assumption, which in any case affects little more than Cairo, Alexandria, Suez and Ismailia practically. It is the number of Egyptian scholars that is rapidly changing and the scholars dealt with in our ratio are Egyptian only.

TABLE III.

Infantile Mortality. Persons per House and Education.

Town	Infantile Mortality per 100 births	Male Literacy per 100 of population	Scholars per 100 of population	Persons per House
Cairo ...	32.9	28.03	6.12	4.62
Alexandria ...	26.9	30.09	3.41	8.43
Damietta ...	18.1	7.06	1.89	6.96
Port Said ...	21.0	24.13	2.64	4.14
Ismailia ...	16.0	28.05	1.22	4.10
Suez ...	26.9	25.74	0.64	3.85
Benha ...	29.6	20.54	4.87	5.47
Zagazig ...	27.9	25.67	6.47	6.07
Tantah ...	29.6	25.62	9.45	5.18
Mansorah ...	21.4	26.77	6.20	4.60
Chibine El Kom	16.1	18.71	5.10	4.92
Damanhur ...	27.5	19.54	3.55	7.27
Guizeh ...	35.6	19.23	11.31	6.12
Fayoum ...	40.1	15.55	4.78	9.34
Beui Suef ...	37.1	21.15	7.19	5.11
Minia ...	38.2	21.96	8.89	4.96
Assiut ...	33.6	20.92	11.48	6.00
Sohag ...	29.0	22.37	5.58	6.15
Kena ...	37.4	16.76	4.80	5.14
Assuan ...	41.1	21.31	5.78	4.09

It will be clear from these results that there is no significant relation between the literacy of the male population and infantile mortality. There is also no significant relation between the number of persons to a house and the number of scholars, i.e. it does not appear to be the more crowded towns which have the largest percentage of scholars to the population; Alexandria and Damietta, for example, have considerably more than the mean number of persons to the house and relatively few scholars. On the other hand a larger number of literates marks less crowding. Crowding and infantile mortality are slightly related, but considering the probable error, not with definite significance*.

While literacy has no relation to the infantile deathrate, it is noteworthy that there is a significant correlation ($+0.5309 \pm 0.1278$) between the number of scholars and the infantile deathrate, *which is greater where there is more education*. Now this either suggests that many scholars mean large families and large families correspond to increased infantile mortality, which is usual, or that the towns in which there are the classes who educate their children have a higher infantile deathrate. The only means, and those inadequate, of testing the first

* This agrees with the result for overcrowding and infantile mortality in English manufacturing towns, where the correlation is very small and sometimes has one sign and sometimes the other.

assumption are to take the partial coefficient between scholars and deathrate for constant number of persons per house.

We find*:

$$pr_{IS} = +.5336 \pm .1107;$$

similarly

$$pr_{IL} = -.0504 \pm .1543.$$

There is thus a slight increase in the relation of scholars to deathrate when we take a constant number of persons per house, and it is hard to believe that the relation is indirectly due to size of family. The second result shows that literacy has no relation to the infantile deathrate. Towns like Alexandria, Damietta, Port Said, Ismailia, and to a less extent Suez, with a low infantile deathrate have a low education rate, and towns like Cairo, Guizeh, Beni Suef, Minia, and Assiut, with high infantile deathrates have high education rates. The first towns are on the sea or the canal, the second in the Nile Valley; it is conceivable that the latter are the more unhealthy for the infant; it would need special local knowledge to explain why education has been most accepted above Cairo†. There does not, however, seem any relation between ignorance, as measured by literacy, and a heavy infantile mortality, nor on the other hand can we assert that education and European influence have certainly increased criminality.

* The value of pr_{LS} is $+.0207 \pm .1508$, and is therefore not significant.

† It is noteworthy that there is no relation between literacy and number of scholars, i.e. education of children does not appear to follow the power to read and write in their parents, that is to say if we judge by the averages in towns and not by individuals.

HEIGHT AND WEIGHT OF SCHOOL CHILDREN IN GLASGOW

By ETHEL M. ELDERTON, Galton Fellow, University of London.

In 1905-6 an enquiry was made in the Public Schools of the School Board for Glasgow as to the height and weight of all the scholars, the occupation of the parents, the number of rooms occupied etc. By permission of Sir John Struthers, of the Scottish Education Office, these schedules were most kindly placed at the disposal of the Galton Laboratory.

The number of children concerning whom the enquiry was made is over seventy thousand of ages 5 to 18 years. The schools from which these children came were divided into four groups according to the district in which the schools were situated.

Group *A* comprised schools in the poorest districts of the city.

„ *B* „ „ in poor districts of the city.

„ *C* „ „ in districts of a better class.

„ *D* „ „ in districts of a still higher class with which are included four out of five Higher Grade Schools.

The data were originally used by the Galton Laboratory with the object of discovering how far the physique of school children, judged by their height and weight, is affected by the occupation of the father and the employment of the mother. With this end in view the necessary data were entered on cards. Children over 14 were excluded and all children who had not both parents alive were also excluded; this left us with 30,965 girls and 32,811 boys.

The object of the present paper is to ascertain what is the average weight of a child of a given age and a given height.

The first step in this enquiry was to sort the cards and form tables giving the distribution of weight for each height at each age in each school group, and this laborious work was carried out very largely by Miss Augusta Jones; this step necessitated making 72 tables, and she is responsible for 58 of them while the

remaining 14 are due to Miss H. Gertrude Jones; I have to thank most heartily these colleagues for their very efficient help in this matter.

The three factors with which we are concerned are the age, height, and weight of school children. The instructions issued to the teachers in the schools for recording these three facts were that ages were to be given to the nearest year, weights to the nearest pound, and heights to the nearest quarter of an inch*. The method of recording ages is very important. Ages being recorded to the nearest year, this means that children classed as 6 years were from 5·5 years to 6·5 years; and the average age of this group was 6 years; this is not the method most frequently employed for recording ages; "age last birthday" is generally used and if "age last birthday" be given as 6 years then the children of that age are from 6 to 7 and the average age of children in this group is approximately 6·5 years. It will be seen at once that a comparison of weights and heights of two groups of children of 6 years cannot be undertaken until we know which method of recording ages has been adopted. The height and weight of these Glasgow children have been compared by Dr Leslie Mackenzie and Captain Foster† with the height and weight of children as given by the Anthropometric Committee of the British Association and it is pointed out that at each age the average weight of the children is uniformly below the "standard of the Anthropometric Committee," and that generally speaking the same thing applies to height. As a matter of fact this point as to age has not been noticed by these writers and children whose average age is 6 years in Glasgow are compared with children whose average age is 6·5 years, naturally the younger children are shorter and lighter. There is further an important question to be asked: Which standard of the B.A. Anthropometric Committee ought to be selected? To this point I return below.

As I have said the Glasgow children's ages were recorded to the nearest year‡, but the Anthropometric Committee recorded age last birthday, and before these children can be compared the six months extra growth must be allowed for. This is quite easily done by finding the regression of height and weight on age and adding half the regression coefficient to the height and weight of the Glasgow children. We have found the regression for children of 5 to 14 inclusive to be as follows:

		Boys	Girls
Regression of Weight on Age	...	4·564	4·916
„ Height on Age	...	1·807	1·937

* It is not known what record was made when an exact half year, an exact half pound or an exact quarter inch occurred.

† *Report on the Physical Condition of Children attending the Public Schools of the School Board for Glasgow*, by Dr W. Leslie Mackenzie and Captain A. Foster. Wyman and Sons, 1907.

‡ The actual wording of the Glasgow direction to school teachers runs: "In recording age, disregard months and record to nearest year; thus 6 years 7 months record as 7 years, 8 years 3 months record as 8 years." It is not clear how 6 years 6 months would be recorded; we have assumed as no half years are entered in the schedules that an exact calculation was made in the case of each child of doubtful age to ascertain whether it was or was not past the half year.

This means that we must add 2·28 lbs. to the weight of the Glasgow boys and ·90 inches to their height and 2·5 lbs. to the weight of Glasgow girls and ·97 inches to their height before we can compare them with the Anthropometric Committee's standard. The Glasgow children still fall below the "Anthropometric Committee's Average" but not to the appalling extent shown in the diagram at the end of Dr Mackenzie's and Captain Foster's Report. Personally I should hesitate to compare actual height and weight of Glasgow school children with the so-called Anthropometric Committee's standard. The so-called standard is taken from the Final Report of the Anthropometric Committee of the British Association, 1883. In Tables XVI—XIX, the average heights and weights at different ages of males and females of different classes of the population of Great Britain are given. For example in the case of stature we have four classes: Class I, Professional Classes, Town and Country, 10,739 individuals, ages 9 to 60; Class II, Commercial Classes, Towns, 5472 individuals, ages 8 to 60 (5 below 8 are of no service for means); Class III, Labouring Classes, Country, 8727 individuals, ages 3 to 70 (8 below 3 are of no service); Class IV, Artizans, Towns, 126,236 individuals, ages 3 to 60, and 451 babies at birth. All these data are pooled and the column headed "General Population, All Classes, Town and Country," and it is this "General Population" which is so frequently cited by various medical authorities, including Dr Leslie Mackenzie and Captain Foster, as the Anthropometric Committee's "standard." What they understand by such a "standard" it is impossible to say. It does not represent the "General Population" of Great Britain, but the total population measured by the Committee. In this *all* the babies are artizan babies, there are only 8 children from 0 to 2 and these belong to the labouring rural classes, and there is no professional class contribution until after 9 years of age. Then the various age groups are made up from various social classes in proportions which bear no relation whatever to their actual proportions in the kingdom at large. For example, the average height of lads of 18 is determined from 1724 of the professional, 62 of the commercial, 148 of the rural labourer, and 371 of the town artizan classes! It will be quite clear that a "standard" reached in this way means absolutely nothing at all, and yet this is the "standard" which, attached to numerous weighing machines is posted in innumerable public places up and down this country. It does not in the least represent any "General Population" of Great Britain. To be a standard of the general population each class should have been properly weighted, and this cannot be done as in certain classes certain ages are quite inadequately represented, or not represented at all. There is in fact no such thing as an "Anthropometric Committee's standard" for either height or weight. The only thing that is possible is to compare the corresponding social class in that Committee's measurements with the measurements under consideration. In the case of Dr Leslie Mackenzie's and Captain Foster's data, this is undoubtedly the Class IV, "Artizans, Towns." Such a comparison is made in the accompanying diagrams. It will be seen that the Glasgow children as far as height is concerned are the equals if not the superiors of the Anthropometric

Committee's artizan class. In weight they appear to be somewhat less, but here Dr Leslie Mackenzie and Captain Foster have overlooked the fact that the Glasgow children were weighed *without* boots, but the British Association Committee weighed in ordinary indoor clothing, i.e. *with* boots or shoes on. Now girls' boots weigh as much as $1\frac{1}{4}$ to $2\frac{1}{2}$ lbs. and boys' boots $1\frac{3}{4}$ to $3\frac{1}{2}$ lbs.* Hence in comparing children in Glasgow with those six months older, Dr Leslie Mackenzie and Captain Foster have dropped $2\frac{1}{4}$ lbs. in weight, while in comparing children without boots with those with boots they have dropped another $1\frac{1}{4}$ lbs. to possibly $3\frac{1}{2}$ lbs. We should anticipate therefore that their readings would be $3\frac{1}{2}$ to nearly 6 lbs. too small†. There is in our mind very little doubt that the weight of the Glasgow children is at every age equal or superior to the weight of the artizan children measured by the British Association Anthropometric Committee and the statement of Dr Leslie Mackenzie and Captain Foster that "at each age from 5 to 18 the average weight of the [Glasgow] children is uniformly below the standard of the Anthropometrical Committee‡" arises from their having entirely overlooked the conditions as to class, age and manner of weighing which were adopted by that Committee, a knowledge of which was essential to any comparison with the Committee's data. In the diagrams on pp. 292-3 we have given the Glasgow measurements set against those of the artizan class of the Anthropometric Committee, and the reader will see clearly how all the arguments based on differences between the Glasgow and the "Anthropometric standard" fall at once to the ground. There is nothing exceptional in the Glasgow data, they differ of course from data for the children of the professional classes, but this difference is not confined to Glasgow. Apart from this point it is essential that the ages of the two groups of children should be the same and not differ by six months.

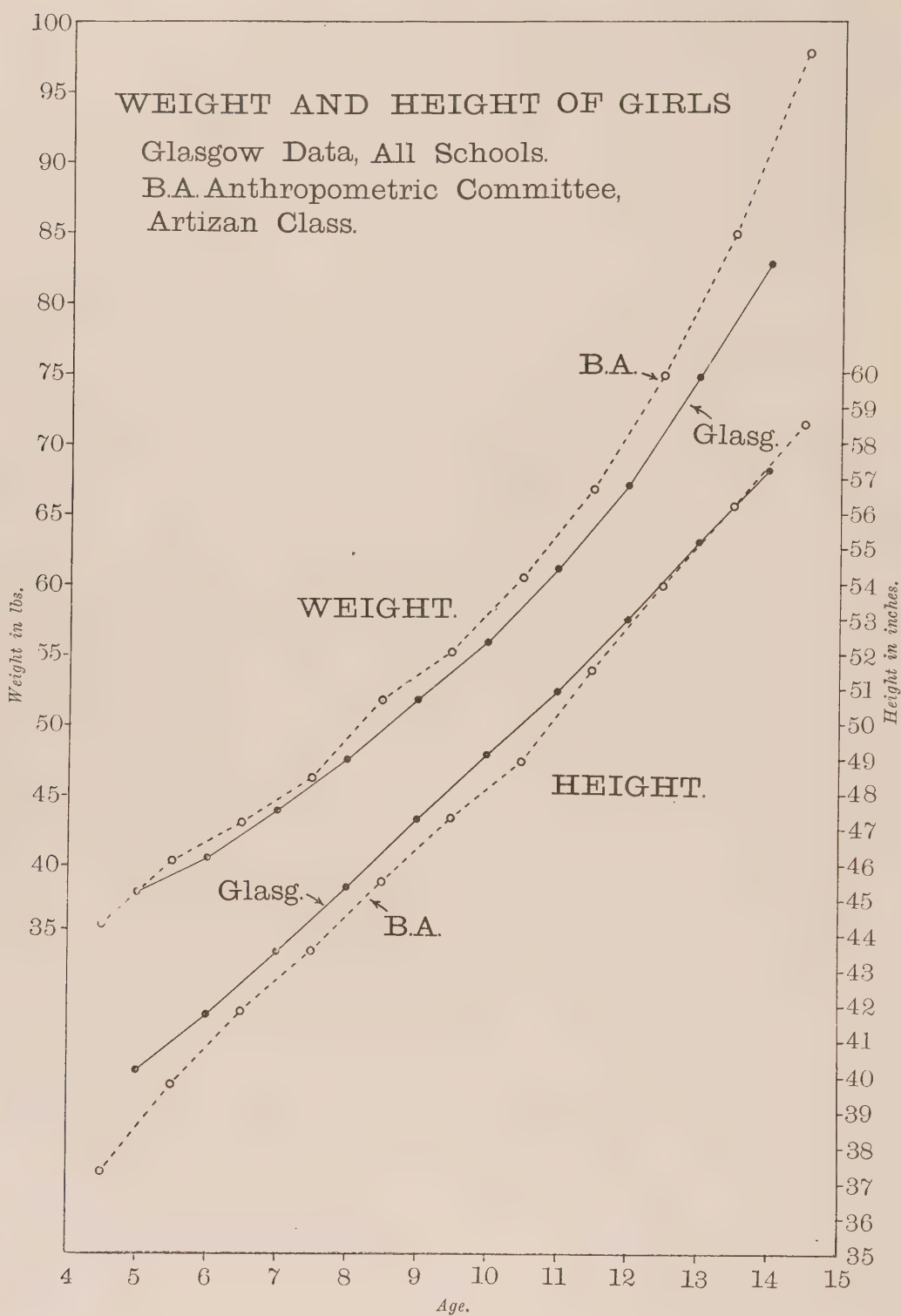
In the data used for this paper, children of 5 were omitted; they are few in number and are not therefore likely to give such reliable results when each age group is used separately. The mean weight for each height in inches was then found and the regression equation calculated. These equations are given in Table I. It will be observed from these equations that, though some irregularities occur, generally speaking weight increases more rapidly for a given height in the better school groups, at the later ages, and for girls more than boys except at ages 6 and 7.

We can see from these equations that the multiple regression surface for weight on height and age is not absolutely planar. It can be shown that it is

* New "tacket" boots for girls of five in Glasgow weight 1 lb. 5 oz. falling to about 1 lb. 3 oz. when the tackets are worn down; for girls of fourteen 2 lbs. 6 oz. falling to about 2 lbs. 2 oz. For boys of five years new tacket boots weigh 1 lb. 14 oz. falling to about 1 lb. 11 oz. when worn down; for boys of fourteen the former weigh 3 lbs. 9 oz. and the latter about 3 lbs. 3 oz. We have to thank Dr Chalmers, M.O.H. for Glasgow, for this information.

† Many public elementary school children have great masses of metal on their boots. Undoubtedly the older children have heavier boots, and we can see from the diagrams that the divergence of the Glasgow children from the Anthropometric Committee's artizan children increases with age.

‡ *Report, Scottish Education Department, 1907, p. iv.*



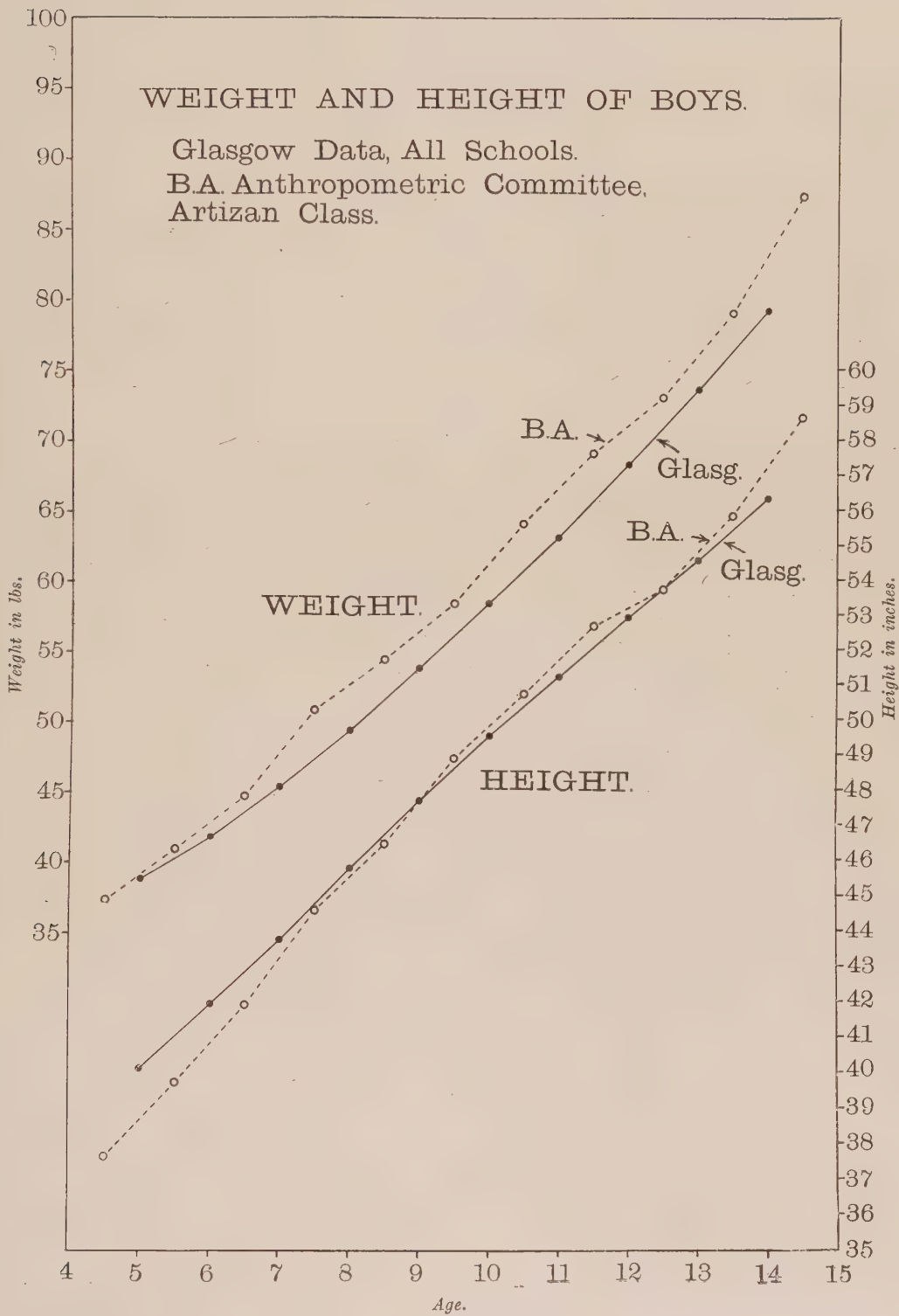


TABLE I.

Glasgow.

Age	Group A, Boys	Group B, Boys	Group C, Boys	Group D, Boys
6	$W = -21.164 + 1.503H$	$W = -22.576 + 1.532H$	$W = -24.489 + 1.591H$	$W = -25.678 + 1.603H$
7	$W = -21.065 + 1.519H$	$W = -26.308 + 1.635H$	$W = -27.390 + 1.667H$	$W = -34.819 + 1.818H$
8	$W = -19.381 + 1.495H$	$W = -28.127 + 1.695H$	$W = -6.640 + 1.227H$	$W = -32.777 + 1.790H$
9	$W = -24.652 + 1.635H$	$W = -32.826 + 1.818H$	$W = -20.671 + 1.562H$	$W = -36.030 + 1.883H$
10	$W = -29.589 + 1.768H$	$W = -35.696 + 1.899H$	$W = -42.942 + 2.055H$	$W = -51.636 + 2.218H$
11	$W = -54.910 + 2.302H$	$W = -39.725 + 2.005H$	$W = -43.628 + 2.088H$	$W = -67.664 + 2.546H$
12	$W = -49.513 + 2.217H$	$W = -62.890 + 2.476H$	$W = -55.386 + 2.337H$	$W = -62.052 + 2.450H$
13	$W = -65.467 + 2.547H$	$W = -63.516 + 2.511H$	$W = -81.342 + 2.854H$	$W = -99.908 + 3.160H$
14	$W = -83.784 + 2.888H$	$W = -76.749 + 2.775H$	$W = -103.661 + 3.251H$	$W = -126.446 + 3.633H$

Age	Group A, Girls	Group B, Girls	Group C, Girls	Group D, Girls
6	$W = -14.561 + 1.329H$	$W = -15.985 + 1.345H$	$W = -23.728 + 1.551H$	$W = -27.563 + 1.624H$
7	$W = -19.762 + 1.465H$	$W = -24.130 + 1.556H$	$W = -29.312 + 1.693H$	$W = -30.226 + 1.694H$
8	$W = -20.721 + 1.503H$	$W = -30.622 + 1.718H$	$W = -29.113 + 1.691H$	$W = -40.156 + 1.927H$
9	$W = -30.133 + 1.730H$	$W = -29.081 + 1.709H$	$W = -38.624 + 1.917H$	$W = -45.066 + 2.045H$
10	$W = -36.478 + 1.878H$	$W = -38.866 + 1.925H$	$W = -46.263 + 2.088H$	$W = -62.015 + 2.397H$
11	$W = -48.707 + 2.153H$	$W = -43.005 + 2.034H$	$W = -51.146 + 2.209H$	$W = -57.754 + 2.339H$
12	$W = -58.277 + 2.360H$	$W = -63.908 + 2.465H$	$W = -77.316 + 2.735H$	$W = -84.298 + 2.859H$
13	$W = -74.156 + 2.694H$	$W = -88.043 + 2.939H$	$W = -83.960 + 2.892H$	$W = -103.594 + 3.229H$
14	$W = -95.464 + 3.084H$	$W = -84.496 + 2.906H$	$W = -106.133 + 3.317H$	$W = -134.197 + 3.804H$

W is weight in lbs., H is height in inches.

The ages are *central* ages, and to obtain the weight corresponding to a given height the child should be taken to the *nearest* whole year.

weight on age and not height on age which is non-linear. The departure from linearity is not great, but Mr H. E. Soper, in order to smooth the material, fitted a parabolic surface to the regression surface of weight on height and age.

Let W be as before the weight in lbs., H = height in inches, and y equal the age of the child measured from 10*. Then

$$W = -\phi_1(y) + \phi_2(y)H$$

is the form of the surface when the relation of W to H for a given age is sensibly linear.

Mr Soper now assumed:

$$\phi_1(y) = a_0 + a_1y + a_2y^2, \quad \phi_2(y) = b_0 + b_1y + b_2y^2$$

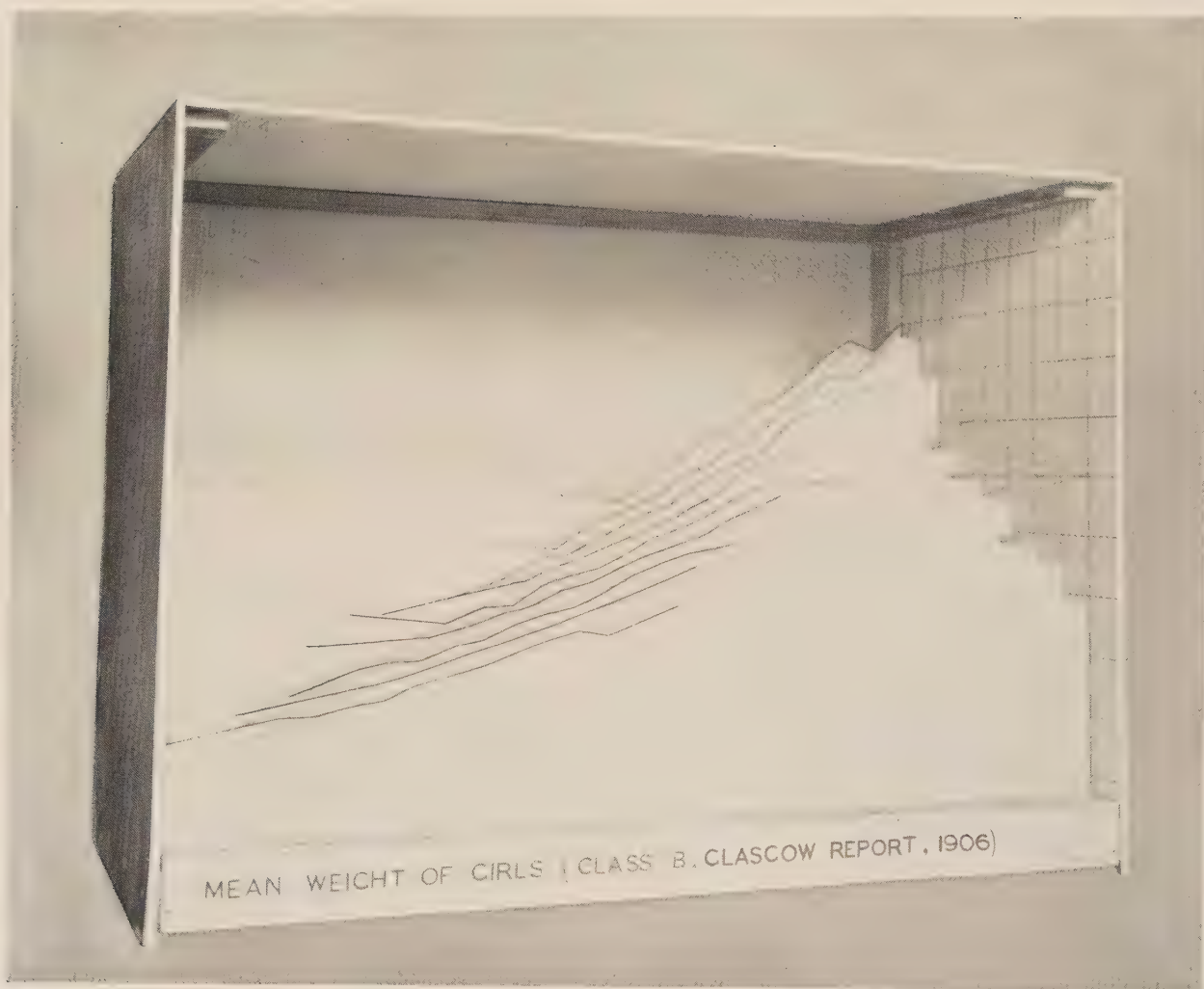
and determined a_0 , a_1 and a_2 , b_0 , b_1 and b_2 , so that:

$$\Sigma \{n(\phi_1(y) - a_0 - a_1y + a_2y^2)\} = \text{minimum},$$

$$\Sigma \{n(\phi_2(y) - b_0 - b_1y - b_2y^2)\} = \text{minimum},$$

where n is the number of individuals in any age group.

* Thus y takes every value from -4 to $+4$ and we have nine equations to deal with.



Model of Regression Surface, giving mean weight of Girls of Class B of Glasgow Schools for a given Height and Age. The mean weight is the vertical coordinate and each section parallel to the front of the model gives the mean weight for the several Heights of Girls of a given Age. See p. 295.

Now $\phi_1(y)$ and $\phi_2(y)$ for given y 's are the values determined in Table I for the constants at each age of the regression lines

$$W = -A + BH,$$

and n is the number of children dealt with at each age. Our type equations are then of the form:

$$\begin{aligned} a_0 + \frac{1}{9}a_1\Sigma(ny) + \frac{1}{9}a_2\Sigma(ny^2) &= \frac{1}{9}\Sigma(A), \\ \frac{1}{9}a_0\Sigma(ny) + \frac{1}{9}a_1\Sigma(ny^2) + \frac{1}{9}a_2\Sigma(ny^3) &= \frac{1}{9}\Sigma(Ay), \\ \frac{1}{9}a_0\Sigma(ny^2) + \frac{1}{9}a_1\Sigma(ny^3) + \frac{1}{9}a_2\Sigma(ny^4) &= \frac{1}{9}\Sigma(Ay^2), \end{aligned}$$

and similar equations for b_0, b_1, b_2 , with B for A .

When these constants had been determined we put $Y = 10 + y$ and obtain the equations given in Table II.

TABLE II.

Glasgow. School Children.

	W =Weight in lbs.	H =Height in inches.	Y =True age.
Boys			
Group A	$W = \{.02181 Y^2 + 2.214 - .2554 Y\} \times H - \{1.13275 Y^2 + 67.542 - 14.7417 Y\},$		
„ B	$W = \{.01533 Y^2 + 1.900 - .1493 Y\} \times H - \{.83314 Y^2 + 53.101 - 9.8662 Y\},$		
„ C	$W = \{.03990 Y^2 + 3.614 - .5796 Y\} \times H - \{2.08397 Y^2 + 139.456 - 31.6570 Y\},$		
„ D	$W = \{.02983 Y^2 + 2.799 - .3636 Y\} \times H - \{1.76624 Y^2 + 111.407 - 24.0174 Y\}.$		
Girls			
Group A	$W = \{.01880 Y^2 + 1.657 - .1644 Y\} \times H - \{.96454 Y^2 + 38.119 - 9.7305 Y\},$		
„ B	$W = \{.02081 Y^2 + 1.907 - .2043 Y\} \times H - \{1.16330 Y^2 + 60.230 - 13.6925 Y\},$		
„ C	$W = \{.02315 Y^2 + 2.222 - .2457 Y\} \times H - \{1.21642 Y^2 + 67.626 - 14.3416 Y\},$		
„ D	$W = \{.02701 Y^2 + 2.385 - .2832 Y\} \times H - \{1.56165 Y^2 + 87.624 - 18.9239 Y\}.$		

A model of the surface for Glasgow Girls of Class B has been made by Mr Soper. Allowing for the points based on few observations at the end of each regression line of weight on height for constant age, the scroll represented by black threads is quite a good fit to the observations represented by card sections. The model is, however, difficult to photograph in a manner which shows effectively the approximation of the thread scroll to the cut card sections. The reader should note that an additional thread is placed between each of the threads which graduate the regression lines for the different ages.

Further eight tables (Tables α — θ) have been constructed in order that the average weight of any boy or girl of a given height and age can be read off at once. See pp. 300—303. It has been stated before that the age groups in Glasgow are from 5.5 to 6.5 years etc. and that 6, 7, 8 etc. are the centres of each age group,

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but since frequently the centres are at 6·5, 7·5 etc. we have constructed Table III which enables anyone to find mean height and weight at any age between 5·5 and 14·5 years. The regression lines are calculated from the original tables in which children of 4·5 to 5·5 years were included. The regression lines omitting the

TABLE III.

Glasgow.

	Mean Height				Mean Weight			
	A	B	C	D	A	B	C	D
Boys: 5·5—6·5	41·3	42·1	42·1	43·0	40·9	42·0	42·5	43·3
6·5—7·5	43·0	44·0	44·0	44·8	44·2	45·6	45·9	46·6
7·5—8·5	45·1	45·9	46·2	46·9	48·0	49·6	50·1	51·2
8·5—9·5	47·0	47·7	48·1	49·0	52·3	53·9	54·4	56·3
9·5—10·5	48·8	49·5	49·9	50·9	56·7	58·4	59·5	61·2
10·5—11·5	50·6	51·1	51·5	52·6	61·6	62·7	63·9	66·3
11·5—12·5	52·3	52·8	53·5	54·2	66·4	67·8	69·1	70·8
12·5—13·5	53·8	54·3	55·0	55·9	71·7	72·9	75·6	76·9
13·5—14·5	55·2	55·5	57·2	57·7	75·6	77·3	82·2	83·2
Regression on Age	1·800 ins.	1·728 ins.	1·847 ins.	1·846 ins.	4·305 lbs.	4·395 lbs.	4·772 lbs.	4·914 lbs.
Girls: 5·5—6·5	41·0	42·0	41·9	42·7	39·9	40·6	41·3	41·8
6·5—7·5	42·9	43·7	43·7	44·8	43·0	43·9	44·7	45·6
7·5—8·5	44·6	45·6	45·6	46·4	46·4	47·7	48·1	49·3
8·5—9·5	46·6	47·4	47·6	48·6	50·5	51·8	52·7	54·3
9·5—10·5	48·5	49·2	49·4	50·4	54·7	55·8	56·9	58·8
10·5—11·5	50·3	51·1	51·2	52·2	59·5	60·8	61·9	64·4
11·5—12·5	52·4	53·0	53·3	54·1	65·3	66·8	68·4	70·5
12·5—13·5	54·4	55·2	55·4	56·5	72·4	74·3	76·1	78·8
13·5—14·5	55·8	57·1	57·0	58·7	76·8	81·3	83·0	89·0
Regression on Age	1·914 ins.	1·859 ins.	1·903 ins.	1·943 ins.	4·551 lbs.	5·083 lbs.	4·944 lbs.	5·489 lbs.

children of 4·5 to 5·5 years were worked out for height on age and weight on age for boys in Group A, and were found to be 1·81 instead of 1·80 for height and 4·39 instead of 4·31 for weight, but such differences are not great enough to matter and the remaining regression coefficients were not calculated with children of five years excluded.

In connection with the tables (1 to 72, pp. 304—339) it should be noted that in transferring the data for boys from the original sheets to cards, 75 of an inch was included in the inch above; for example, 30·75 inches was entered as 31 inches and the centre of the group of 30 inches is 30·125 inches. The data for the girls

were transferred to cards much later and the simpler method was employed, and 30·75 was included in the 30 inch group and the centre of this group is 30·375.

Through the kindness of Dr Priestley, School Medical Officer for Staffordshire, we have been able to obtain the regression of Weight on Height for certain age groups of boys and girls in that county. Staffordshire is a county of very various occupations and contains an agricultural as well as a mining and factory population.

The children measured are "entrants" and "leavers" and a further group of children, namely those from 8 to 9 were measured. The "leavers" include children of 12 to 14 years, "since in general the only 'leavers' at age 12 to 13 are rural, and the only 'leavers' at age 13 to 14 are urban*."

The children were of the age stated, 5 and not yet 6, 8 and not yet 9, on January 1, 1911, but the actual day of weighing may have been any school day from January to December, so that a child entered as 8 may have been only a few days short of 10 when it was actually measured, and therefore the mean age of the group of children of 8 to 9 will be 9 years. "In the case of the group of leavers, 13 to 14, no child can have been more than 14, because on attaining that age the children are entitled to leave school, and generally do leave. With these the mean height and weight in our tables refer to the true mean of the years of the group, viz., 13 and a half†." We shall table to the middle of the group, namely at ages 6, 9, 13 and $13\frac{1}{2}$.

The children were weighed and measured without shoes, but in ordinary indoor clothes. The figures were read to the nearest quarter of an inch and to the nearest quarter of a pound.

Staffordshire Children.

Ages	GIRLS			BOYS		
	Mean Height	Mean Weight	Regression of Weight on Height	Mean Height	Mean Weight	Regression of Weight on Height
6	41·9	39·8	1·705	42·1	41·0	1·741
9	47·7	51·1	2·024	48·1	53·0	2·120
13	56·7	78·0	3·272	55·8	75·3	2·811
$13\frac{1}{2}$	57·1	81·0	3·360	56·3	77·7	3·166

It will be as well to compare these means with those for all Glasgow; so far we have not given them in this paper for all the schools taken together but only for each school group.

* *Staffordshire County Council, Annual Report of the School Medical Officer for the Year 1911.*
J. and C. Mort, Ltd., 39, Greengate Street, Stafford, 1912.

† *Ibid.* p. 25.

Glasgow Children.

Ages	GIRLS		BOYS	
	Mean Height	Mean Weight	Mean Height	Mean Weight
6	41·7	40·5	41·9	41·8
9	47·3	51·3	47·7	53·7
13	55·2	74·8	54·6	73·6

Girls in Staffordshire are taller at ages 6, 9, and 13 than girls in Glasgow, but they are lighter at ages 6 and 9. We might argue from this a lack of physique in Staffordshire girls who are absolutely ·7 lbs. lighter at age 6 than Glasgow children, and relatively to their height even more than this amount. At age 9 the absolute difference is less and at age 13 Staffordshire girls are heavier than Glasgow girls but they are $1\frac{1}{2}$ inches taller, and since the regression of weight on height at age 13 for girls is 3·272 lbs. we should expect Staffordshire girls to be 4·9 lbs. heavier than Glasgow girls, but they are not so much. I should hesitate to say that the physique of Staffordshire girls is inferior to that of Glasgow girls; the difference probably is one of race, but such questions must remain unsolved till we have a far wider range of anthropometric data than is available at present for all the districts of Great Britain. Boys show the same characteristics to a lesser extent; Staffordshire boys are taller at ages 6, 9, and 13, but they are lighter in weight; at age 6 they are ·8 lbs. lighter than Glasgow boys; at age 9 they are ·7 lbs. lighter and at age 13 they are 1·7 lbs. heavier. Again relative to their height Staffordshire boys are lighter than Glasgow boys at the three ages for which a comparison can be made.

Comparing boys and girls in Staffordshire we find that girls of 6 and 9 are shorter and lighter than boys of the same age, but at 13 and $13\frac{1}{2}$ girls are both taller and heavier. At 6 and 9 years the regression of weight on height is practically the same for both sexes, but at 13 and $13\frac{1}{2}$ the regression of weight on height is greater for girls than for boys; girls are heavier proportionally to their height than boys are. For girls of 13 an additional inch in height should mean 3·3 lbs. more weight while for boys the additional pounds expected are only 2·8, while for girls of $13\frac{1}{2}$ we expect 3·4 lbs. increase for every inch of growth and for boys 3·2 lbs. increase. A comparison of the regression coefficients with those given for Glasgow in Table I will show that the coefficient is higher in Staffordshire for children of 6 and boys of 9 than in any of the school groups in Glasgow. The regression coefficient found for girls of 9 and 13 in Staffordshire is practically identical with that found in Group *D* in Glasgow, and boys of 13 in Staffordshire would seem to be most like boys of Group *C* in Glasgow.

In a Drapers' Company Research Memoir * recently published tables are given showing the height and weight of boys and girls of 12 to 13 years who were members of the Worcestershire public elementary schools. These tables are XLIV and LIX and will be found on pp. 100 and 107 of the work cited; we have calculated the mean heights and weights and the regression coefficient of weight on height as we have done for Glasgow and Staffordshire. The mean age of the group of children of 12 to 13 years is 12·5 years, so allowance must be made for the six months age difference in comparing with the Glasgow data. We have already given the mean heights and weights of Glasgow and Staffordshire boys and girls of age 13 so we will calculate what the height and weight of Worcestershire children at age 13 would be. An additional year makes a difference of roughly 1·9 inches and 4·9 lbs. in the height and weight of a girl and of 1·8 inches and 4·6 lbs. in the height and weight of a boy.

	GIRLS			BOYS		
	Mean Height	Mean Weight	Regression of Weight on Height	Mean Height	Mean Weight	Regression of Weight on Height
12·5 } Worcestershire	55·2	72·9	2·829	54·6	72·1	2·800
13 }	56·1	75·3	—	55·5	74·4	—
13 Staffordshire ...	56·7	78·0†	—	55·8	75·3	—
13 Glasgow ...	55·2	74·8	—	54·6	73·6	—

Worcestershire children are taller than Glasgow children but slightly shorter than Staffordshire children. They are also rather heavier than Glasgow children but not relatively to their height. The height of Worcestershire children of 12·5 years is the same as the height of Glasgow children of 13 years, but the weight of girls is 2 lbs. less and of boys is 1½ lbs. less. Worcestershire children are lighter than Staffordshire children, but when allowance is made for the difference in height the Worcestershire children are not much at a disadvantage; girls are a pound lighter and the weight of boys is practically the same.

The differences we have found between the Worcestershire, Staffordshire and Glasgow children may well be due to differences of local race, and not be the results of differential environment or nurture. We should have little hesitation in applying the returns for Glasgow children as an approximate standard—say to the lb. and inch—for all British children of the artizan classes.

* "A Statistical Study of Oral Temperatures in School Children with special reference to Parental Environment and Class Differences," by M. H. Williams, Julia Bell and Karl Pearson. *Studies in National Deterioration*, IX. 1914. Dulau and Co., Ltd., 37, Soho Square, W.

† This weight appears somewhat exaggerated. It may in part be due to local differences in the average ages of 'leavers.'

TABLE α . GLASGOW. BOYS. GROUP A*.*Weights for Height at each Age.*

Actual Age.

	6	7	8	9	10	11	12	13	14
Height.									
33	28.5	—	—	—	—	—	—	—	—
34	30.0	31.0	—	—	—	—	—	—	—
35	31.5	32.5	—	—	—	—	—	—	—
36	32.9	33.9	34.3	—	—	—	—	—	—
37	34.4	35.4	35.8	—	—	—	—	—	—
38	35.8	36.9	37.4	37.3	—	—	—	—	—
39	37.3	38.4	39.0	39.0	38.4	—	—	—	—
40	38.8	39.9	40.5	40.6	40.2	39.3	—	—	—
41	40.2	41.4	42.1	42.3	42.1	41.3	—	—	—
42	41.7	42.9	43.7	44.0	43.9	43.4	42.4	—	—
43	43.2	44.4	45.2	45.7	45.7	45.4	44.7	43.6	42.1
44	44.6	45.9	46.8	47.4	47.6	47.5	47.0	46.2	45.0
45	46.1	47.4	48.4	49.1	49.4	49.5	49.3	48.7	47.9
46	47.6	48.9	49.9	50.7	51.3	51.5	51.6	51.3	50.8
47	49.1	50.4	51.5	52.4	53.1	53.6	53.8	53.9	53.7
48	50.5	51.9	53.1	54.1	54.9	55.6	56.1	56.5	56.6
49	—	53.4	54.6	55.8	56.8	57.7	58.4	59.1	59.6
50	—	54.9	56.2	57.5	58.6	59.7	60.7	61.6	62.5
51	—	—	57.8	59.1	60.5	61.8	63.0	64.2	65.4
52	—	—	59.3	60.8	62.3	63.8	65.3	66.8	68.3
53	—	—	60.9	62.5	64.2	65.8	67.6	69.4	71.2
54	—	—	—	64.2	66.0	67.9	69.9	72.0	74.1
55	—	—	—	65.9	67.8	69.9	72.2	74.5	77.0
56	—	—	—	67.6	69.7	72.0	74.5	77.1	79.9
57	—	—	—	69.2	71.5	74.0	76.7	79.7	82.9
58	—	—	—	—	73.4	76.1	79.0	82.3	85.8
59	—	—	—	—	—	78.1	81.3	84.9	88.7
60	—	—	—	—	—	—	83.6	87.4	91.6
61	—	—	—	—	—	—	—	90.0	94.5
62	—	—	—	—	—	—	—	—	97.4

TABLE β . GLASGOW. BOYS. GROUP B.*Weights for Height at each Age.*

Actual Age.

	6	7	8	9	10	11	12	13	14
Height.									
33	27.5	—	—	—	—	—	—	—	—
34	29.0	—	—	—	—	—	—	—	—
35	30.6	31.4	—	—	—	—	—	—	—
36	32.1	33.0	—	—	—	—	—	—	—
37	33.7	34.6	34.9	—	—	—	—	—	—
38	35.2	36.2	36.6	—	—	—	—	—	—
39	36.8	37.8	38.3	—	—	—	—	—	—
40	38.3	39.4	40.0	40.1	39.8	—	—	—	—
41	39.9	41.0	41.7	41.9	41.8	—	—	—	—
42	41.5	42.6	43.3	43.7	43.7	43.3	—	—	—
43	43.0	44.2	45.0	45.5	45.7	45.5	—	—	—
44	44.6	45.8	46.7	47.3	47.6	47.6	—	—	—
45	46.1	47.4	48.4	49.1	49.5	49.7	49.5	—	—
46	47.7	49.0	50.1	50.9	51.5	51.8	51.8	51.6	—
47	49.2	50.6	51.8	52.7	53.4	53.9	54.2	54.2	54.0
48	50.8	52.2	53.5	54.5	55.4	56.0	56.5	56.7	56.8
49	52.4	53.8	55.2	56.3	57.3	58.1	58.8	59.3	59.6
50	53.9	55.4	56.8	58.1	59.2	60.2	61.1	61.8	62.4
51	—	57.0	58.5	59.9	61.2	62.4	63.4	64.4	65.3
52	—	58.7	60.2	61.7	63.1	64.5	65.7	66.9	68.1
53	—	—	61.9	63.5	65.1	66.6	68.1	69.5	70.9
54	—	—	—	65.3	67.0	68.7	70.4	72.0	73.7
55	—	—	—	67.1	68.9	70.8	72.7	74.6	76.5
56	—	—	—	—	70.9	72.9	75.0	77.1	79.3
57	—	—	—	—	72.8	75.0	77.3	79.7	82.1
58	—	—	—	—	—	77.1	79.6	82.2	85.0
59	—	—	—	—	—	79.3	81.9	84.8	87.8
60	—	—	—	—	—	—	84.3	87.3	90.6
61	—	—	—	—	—	—	—	89.9	93.4
62	—	—	—	—	—	—	—	92.4	96.2
63	—	—	—	—	—	—	—	95.0	99.0

* Throughout weights are given in lbs., heights in inches.

TABLE 7. GLASGOW. BOYS. GROUP C.

Weights for Height at each Age.

Actual Age.

[illegible]

TABLE δ . GLASGOW. BOYS. GROUP *D*.

Weights for Height at each Age.

Actual Age.

[illegible]

TABLE 6. GLASGOW. GIRLS. GROUP A.

Weights for Height at each Age.

Actual Age.

[illegible]

TABLE 2. GLASGOW. GIRLS. GROUP B.

Weights for Height at each Age.

Actual Age.

[illegible]

TABLE η . GLASGOW. GIRLS. GROUP C.*Weight for Height at each Age.*

Actual Age.

Height.	Actual Age.									
	6	7	8	9	10	11	12	13	14	
35	30.0	—	—	—	—	—	—	—	—	
36	31.5	—	—	—	—	—	—	—	—	
37	33.1	33.7	—	—	—	—	—	—	—	
38	34.7	35.3	35.3	—	—	—	—	—	—	
39	36.3	37.0	37.0	—	—	—	—	—	—	
40	37.9	38.6	38.8	38.3	—	—	—	—	—	
41	39.4	40.2	40.5	40.2	—	—	—	—	—	
42	41.0	41.9	42.2	42.1	41.5	—	—	—	—	
43	42.6	43.5	44.0	44.0	43.6	42.7	—	—	—	
44	44.2	45.1	45.7	45.9	45.6	45.0	—	—	—	
45	45.8	46.8	47.4	47.8	47.7	47.3	46.6	—	—	
46	47.3	48.4	49.2	49.6	49.8	49.7	49.2	—	—	
47	48.9	50.1	50.9	51.5	51.9	52.0	51.8	—	—	
48	—	51.7	52.7	53.4	54.0	54.3	54.4	54.3	—	
49	—	53.3	54.4	55.3	56.0	56.6	57.0	57.3	—	
50	—	—	56.1	57.2	58.1	58.9	59.6	60.2	—	
51	—	—	57.9	59.1	60.2	61.3	62.3	63.2	64.0	
52	—	—	—	61.0	62.3	63.6	64.9	66.1	67.3	
53	—	—	—	62.8	64.4	65.9	67.5	69.0	70.6	
54	—	—	—	—	66.4	68.2	70.1	72.0	74.0	
55	—	—	—	—	68.5	70.5	72.7	74.9	77.3	
56	—	—	—	—	70.6	72.9	75.3	77.9	80.6	
57	—	—	—	—	—	75.2	77.9	80.8	83.9	
58	—	—	—	—	—	77.5	80.5	83.7	87.2	
59	—	—	—	—	—	79.8	83.1	86.7	90.6	
60	—	—	—	—	—	—	85.7	89.6	93.9	
61	—	—	—	—	—	—	—	92.6	97.2	
62	—	—	—	—	—	—	—	95.5	100.5	
63	—	—	—	—	—	—	—	98.4	103.8	
64	—	—	—	—	—	—	—	—	107.2	

TABLE θ . GLASGOW. GIRLS. GROUP D.*Weight for Height at each Age.*

Actual Age.

Height.	Actual Age.									
	6	7	8	9	10	11	12	13	14	
36	29.4	—	—	—	—	—	—	—	—	
37	31.1	—	—	—	—	—	—	—	—	
38	32.7	34.0	—	—	—	—	—	—	—	
39	34.4	35.7	—	—	—	—	—	—	—	
40	36.1	37.4	37.8	—	—	—	—	—	—	
41	37.7	39.1	39.6	—	—	—	—	—	—	
42	39.4	40.9	41.5	41.2	—	—	—	—	—	
43	41.0	42.6	43.3	43.3	—	—	—	—	—	
44	42.7	44.3	45.2	45.3	44.7	—	—	—	—	
45	44.4	46.0	47.0	47.3	46.9	—	—	—	—	
46	46.0	47.8	48.9	49.3	49.2	48.4	—	—	—	
47	47.7	49.5	50.7	51.4	51.4	50.9	49.8	—	—	
48	49.3	51.2	52.6	53.4	53.7	53.4	52.7	—	—	
49	—	52.9	54.4	55.4	55.9	56.0	55.6	—	—	
50	—	54.7	56.3	57.4	58.2	58.5	58.4	57.9	—	
51	—	—	58.1	59.5	60.4	61.1	61.3	61.2	60.7	
52	—	—	—	61.5	62.7	63.6	64.2	64.4	64.4	
53	—	—	—	63.5	65.0	66.1	67.1	67.7	68.1	
54	—	—	—	65.5	67.2	68.7	69.9	71.0	71.8	
55	—	—	—	—	69.5	71.2	72.8	74.3	75.5	
56	—	—	—	—	71.7	73.8	75.7	77.5	79.3	
57	—	—	—	—	—	76.3	78.6	80.8	83.0	
58	—	—	—	—	—	78.8	81.4	84.1	86.7	
59	—	—	—	—	—	81.4	84.3	87.3	90.4	
60	—	—	—	—	—	—	87.2	90.6	94.1	
61	—	—	—	—	—	—	90.1	93.9	97.8	
62	—	—	—	—	—	—	—	97.1	101.6	
63	—	—	—	—	—	—	—	100.4	105.3	
64	—	—	—	—	—	—	—	—	109.0	
65	—	—	—	—	—	—	—	—	112.7	

TABLE 1. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 5·5—6·5 years																			Totals	
	23·5—	25·5—	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	—		67·5—
31½—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
32 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
33 "	—	1	—	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
34 "	—	—	4	5	5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
35 "	—	—	2	2	5	8	3	1	—	—	—	—	—	—	—	—	—	—	—	—	21
36 "	—	1	—	2	17	14	10	6	1	1	—	—	—	—	—	—	—	—	—	—	52
37 "	—	—	—	1	7	25	16	15	5	1	1	—	—	—	1	1	—	—	—	—	73
38 "	—	—	—	1	10	20	36	34	15	16	2	—	—	—	—	—	—	—	—	—	134
39 "	—	—	—	—	4	15	32	43	35	20	4	2	—	—	—	—	—	—	—	—	155
40 "	—	—	—	—	1	5	11	54	61	52	27	2	1	—	1	—	—	—	—	1	216
41 "	—	—	—	1	—	1	6	22	56	52	31	16	7	1	1	—	—	—	—	—	194
42 "	—	—	—	—	1	1	2	10	21	54	42	26	7	1	2	—	—	—	—	—	167
43 "	—	—	—	—	—	—	—	5	7	22	22	25	15	7	3	1	1	—	—	—	108
44 "	—	—	—	—	—	—	—	1	4	4	11	7	14	7	2	—	—	—	—	—	50
45 "	—	—	—	—	—	—	1	—	1	2	4	8	10	5	2	4	—	—	—	—	37
46 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	4	2	—	—	—	—	8
47 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
48 "	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	1	—	—	3
Totals	1	3	7	15	52	91	117	192	206	224	145	87	54	22	16	8	2	1	—	1	1244

TABLE 2. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 6·5—7·5 years																				Totals	
	19·5—	21·5—	23·5—	25·5—	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—		59·5—
29½—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
30 "	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
31 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
32 "	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
33 "	—	—	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
34 "	—	—	—	—	—	3	1	2	2	2	1	—	—	—	—	—	—	—	—	—	—	11
35 "	—	—	—	—	—	—	2	6	1	2	—	2	1	—	—	—	—	—	—	—	—	14
36 "	—	—	—	—	—	—	3	2	3	2	—	1	—	—	—	—	—	—	—	—	—	11
37 "	—	—	—	—	—	1	—	10	6	8	3	1	1	—	—	—	—	—	—	—	—	30
38 "	—	—	—	—	—	—	—	11	11	7	14	1	3	—	—	—	—	—	—	—	—	47
39 "	—	—	—	—	—	—	4	8	13	33	32	19	10	—	—	—	—	—	—	—	—	119
40 "	2	—	—	—	—	—	—	4	12	30	37	39	19	3	1	1	—	—	—	—	—	148
41 "	—	—	—	—	—	—	—	1	4	17	32	59	43	13	16	1	1	1	—	1	—	188
42 "	—	—	—	—	—	—	—	1	2	14	25	50	52	36	28	9	—	1	—	—	—	219
43 "	—	—	—	—	—	—	—	—	—	2	5	30	60	44	39	11	2	1	—	—	—	194
44 "	—	—	—	—	—	—	—	—	—	2	2	8	30	38	36	29	7	1	2	—	—	155
45 "	—	—	—	1	—	—	—	—	—	—	1	8	6	15	30	28	17	6	7	—	—	119
46 "	—	—	—	—	—	1	—	—	—	—	—	—	—	3	10	8	14	11	9	2	—	58
47 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	4	4	7	2	1	—	20
48 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	3	—	2	1	1	9
49 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
51 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	2	—	1	1	1	5	14	45	55	119	153	220	225	152	161	93	48	28	23	7	1	1354

TABLE 3. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 7·5—8·5 years																								Totals	
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—		
32½—			1																							1
33 "	1			2																						3
34 "				1		1																				2
35 "							2				1		1													4
36 "				2	3	2	2	1																		10
37 "				2	5	4	3	2	2																	18
38 "				1	6	13	3	2																		25
39 "				2	3	10	10	3	4																	32
40 "				1	1	11	19	16	7	6	3															64
41 "		1		2	4	5	18	23	22	11	7		2													95
42 "					2	4	14	36	53	24	37	4														174
43 "						4	7	39	44	60	43	15	9	—	1											222
44 "							1	1	22	46	52	53	34	18	6	11										244
45 "							1		12	9	30	54	43	33	21	6	2									211
46 "								1	3	14	31	33	34	19	19	8	1			1						164
47 "									1	1	12	18	31	18	22	13	7	1								124
48 "							1			1	2	6	14	14	9	12	8	2	5							74
49 "									1	1	1	1	5	1	8	6	3	2	2							31
50 "									1			1	1	1	5	2	2	2	1						1	15
51 "									1	2	3	1	1	1												9
52 "											1	2	1	1				1				1				7
53 "														1												1
54 "															1											1
55 "																										
56 "											1	1			1	1										4
Totals	1	1	1	13	24	56	80	157	194	202	249	159	150	83	78	47	21	8	8	1	—	1	—	1	—	1535

TABLE 4. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 8·5—9·5 years																											Totals
	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—	77·5—	79·5—	81·5—	
31½—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	1	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
36 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
37 "	—	1	1	—	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
38 "	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	14
39 "	—	—	1	—	1	—	6	2	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
40 "	—	—	1	—	3	5	6	8	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	39
41 "	—	—	1	1	2	4	9	9	5	4	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	49
42 "	—	—	—	—	1	2	17	10	9	4	4	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	99
43 "	—	—	—	—	1	2	11	23	25	13	15	6	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	160
44 "	—	—	—	—	—	2	8	19	33	42	30	18	6	2	—	—	—	—	—	—	—	—	—	—	—	—	—	207
45 "	—	—	—	—	—	—	6	15	23	43	45	33	23	11	6	2	—	—	—	—	—	—	—	—	—	—	—	216
46 "	—	—	—	2	—	1	1	5	10	27	44	49	37	25	10	3	2	—	—	—	—	—	—	—	—	—	—	206
47 "	—	—	—	—	—	1	1	1	6	15	23	40	36	52	20	9	1	—	1	—	—	—	—	—	—	—	—	152
48 "	—	—	—	—	—	—	1	—	—	9	8	16	24	40	26	12	13	2	1	—	—	—	—	—	—	—	—	100
49 "	—	—	—	—	—	—	—	—	—	—	4	6	9	22	19	14	18	4	2	1	1	—	—	—	—	—	—	57
50 "	—	—	—	—	—	—	—	—	—	1	—	2	1	8	12	10	11	7	3	—	1	1	—	—	—	—	—	30
51 "	—	—	—	—	—	—	—	—	—	—	1	—	2	2	3	4	4	6	4	1	2	1	—	—	—	—	—	12
52 "	—	—	—	—	—	—	—	1	1	—	—	—	1	—	1	—	2	3	—	1	1	1	—	—	—	—	—	7
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	1	1	1	—	—	—	—	—	5
54 "	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	1	—	—	—	1	—	—	6
55 "	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
56 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—
60 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1	2	5	3	9	18	67	93	118	162	176	178	145	165	98	56	51	23	13	5	7	4	1	—	1	—	1	1402

TABLE 5. Glasgow. Height and Weight of Boys of Group A.

Height	Weight of Boys of 9·5—10·5 years																				Totals						
	34·5—	35·5—	36·5—	37·5—	38·5—	39·5—	40·5—	41·5—	42·5—	43·5—	44·5—	45·5—	46·5—	47·5—	48·5—	49·5—	50·5—	51·5—	52·5—	53·5—							
34½	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1						
35 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1						
36 "	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2						
37 "	1	—	—	2	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	5						
38 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1						
39 "	—	—	—	1	2	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	6						
40 "	—	1	1	—	2	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	9						
41 "	—	—	2	—	5	2	5	2	1	—	—	—	—	—	—	—	—	—	—	—	17						
42 "	—	—	—	2	3	3	2	5	2	1	—	—	—	—	—	—	—	—	—	—	18						
43 "	—	—	—	—	4	3	4	12	7	1	3	1	—	—	—	—	—	—	—	—	35						
44 "	—	—	—	—	3	4	11	17	14	7	3	4	—	—	—	—	—	—	—	—	63						
45 "	—	—	—	—	—	2	10	24	28	17	21	13	3	2	—	—	—	—	—	—	120						
46 "	—	—	—	1	1	3	7	14	25	19	19	26	7	4	1	—	1	—	—	—	128						
47 "	—	—	—	1	—	—	1	13	24	36	47	45	28	8	8	4	2	1	1	—	219						
48 "	—	—	—	—	—	1	5	9	25	23	58	37	22	10	3	4	3	—	—	—	200						
49 "	—	—	—	—	2	1	—	—	1	14	16	34	38	31	35	15	6	1	—	—	194						
50 "	—	—	—	1	—	—	—	2	2	7	19	19	27	38	16	16	6	4	—	2	159						
51 "	—	—	—	—	—	—	—	1	1	2	1	7	19	19	13	12	12	2	2	1	94						
52 "	—	—	—	—	—	—	—	—	1	—	2	1	4	—	3	4	5	1	3	3	70						
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	21						
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	2	1	—	8						
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1						
56 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	2						
57 "	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2						
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—						
59 "	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2						
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1						
Totals	2	1	3	9	23	19	45	95	114	126	142	205	142	119	121	77	59	38	15	9	10	3	—	1	1	—	1379

TABLE 6. Glasgow. Height and Weight of Boys of Group A.

Height	Weight of Boys of 10·5—11·5 years																				Totals						
	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—		77·5—	79·5—	81·5—	83·5—	85·5—	87·5—
34½—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39 "	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	3
40 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41 "	—	1	—	—	2	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
42 "	—	—	—	1	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
43 "	1	—	1	—	2	4	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
44 "	—	—	—	—	5	6	3	6	—	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
45 "	—	—	2	1	3	10	7	11	9	9	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	55
46 "	—	1	—	1	—	7	9	12	14	18	2	3	3	2	—	—	—	—	—	—	—	—	—	—	—	—	72
47 "	—	—	—	—	2	3	5	13	15	30	23	7	—	3	—	—	1	1	—	—	—	—	—	—	—	—	103
48 "	—	—	—	—	—	3	2	9	21	42	28	25	15	5	3	—	—	—	—	—	—	—	—	—	—	—	153
49 "	—	—	1	1	—	1	2	4	5	32	35	34	33	22	9	8	2	2	—	—	—	—	—	—	—	—	191
50 "	—	—	—	—	—	1	1	3	7	15	23	27	48	28	8	11	6	2	—	—	1	—	—	—	—	—	181
51 "	—	—	—	—	—	—	1	—	1	5	12	18	35	24	21	20	17	8	2	3	—	—	—	—	—	—	167
52 "	—	—	—	—	—	—	1	—	—	2	—	8	14	17	22	16	20	9	4	1	3	—	—	—	—	—	117
53 "	—	—	—	—	—	—	—	—	—	—	2	2	6	11	13	15	14	16	13	3	3	—	1	—	—	—	99
54 "	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	3	5	5	8	10	3	4	—	—	1	—	44
55 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	4	1	3	7	4	1	1	—	1	—	25
56 "	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	—	1	1	—	1	3	1	—	10
57 "	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	1	—	—	4
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	2	4	6	15	41	35	61	73	158	127	125	160	114	82	76	70	48	32	18	17	1	3	5	2	1	1278

TABLE 7. *Glasgow. Height and Weight of Boys of Group A.*

	Weight of Boys of 11·5—12·5 years																							Totals
Height	36·5—	39·5—	42·5—	45·5—	48·5—	51·5—	54·5—	57·5—	60·5—	63·5—	66·5—	69·5—	72·5—	75·5—	78·5—	81·5—	84·5—	87·5—	90·5—	93·5—	96·5—	99·5—	102·5—	
36½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 „	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41 „	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 „	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
43 „	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
44 „	—	—	—	—	2	4	2	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	10
45 „	—	—	1	1	3	4	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
46 „	—	—	—	1	2	12	6	5	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	29
47 „	—	—	1	—	4	7	22	14	7	1	—	1	—	—	—	—	—	—	—	—	—	—	—	57
48 „	—	—	—	1	3	11	20	16	11	8	3	1	—	—	—	1	—	—	—	—	—	—	—	75
49 „	—	—	—	1	—	7	17	27	40	16	14	2	1	—	1	—	—	—	—	—	—	—	—	126
50 „	—	—	—	—	—	1	10	33	41	33	22	12	3	—	1	—	—	—	—	—	—	—	—	156
51 „	—	—	—	—	—	1	13	14	30	41	36	20	7	6	—	1	—	—	—	—	—	—	—	169
52 „	—	—	—	—	—	1	—	6	16	31	38	31	15	7	1	1	2	—	—	—	—	—	—	149
53 „	—	—	—	—	—	—	1	5	8	16	28	31	27	18	4	2	2	—	—	—	—	—	—	142
54 „	—	—	—	—	—	—	1	—	1	8	12	23	29	19	11	4	2	—	—	—	—	—	—	110
55 „	—	—	—	—	—	1	1	1	1	3	7	19	8	13	9	5	—	1	1	—	—	—	—	69
56 „	—	—	—	—	—	—	—	—	—	—	1	1	10	8	6	4	2	—	—	—	—	—	—	32
57 „	—	—	—	—	—	—	—	1	—	—	—	1	1	4	5	3	1	—	2	—	1	—	1	20
58 „	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	2	—	1	—	—	—	—	—	5
59 „	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	—	3
60 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
61 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
63 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
64 „	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	3	1	16	50	98	125	155	160	163	143	91	78	43	26	12	4	3	—	2	—	1	1178

TABLE 8. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 12·5—13·5 years																Totals	
	42·5—	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—		107·5—
42½	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	—	1	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
45 "	1	—	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	6
46 "	—	2	4	5	—	—	—	—	—	—	1	—	—	—	—	—	—	12
47 "	—	2	3	13	6	3	—	—	—	—	—	—	—	—	—	—	—	27
48 "	1	2	3	9	18	5	1	1	1	—	—	—	—	—	—	—	—	41
49 "	1	—	2	18	27	11	6	1	—	—	—	—	—	—	—	—	—	66
50 "	—	1	—	7	27	29	17	10	2	—	—	—	—	—	—	—	—	93
51 "	1	—	1	8	26	34	40	22	6	1	1	—	—	—	—	—	—	140
52 "	—	—	—	3	21	46	51	33	22	6	4	2	—	—	—	—	—	188
53 "	—	—	—	1	7	24	43	57	17	7	2	—	—	1	—	1	—	160
54 "	—	—	—	—	4	14	29	45	37	8	2	1	—	—	—	—	—	140
55 "	—	—	—	—	2	3	10	33	33	16	10	7	1	—	—	—	—	115
56 "	—	—	1	—	—	2	7	11	26	18	15	6	1	2	—	—	—	89
57 "	—	—	—	—	—	—	1	7	7	10	10	8	4	2	—	—	—	49
58 "	—	—	—	—	—	1	—	1	1	3	12	5	2	8	—	—	—	33
59 "	—	—	—	—	—	—	—	1	3	2	2	6	3	1	4	—	—	22
60 "	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	—	—	4
61 "	—	—	—	—	—	—	—	—	—	—	—	—	2	3	1	—	1	7
62 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	2
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	4	8	21	67	139	172	205	222	155	71	59	37	16	17	7	1	1	1202

TABLE 9. *Glasgow. Height and Weight of Boys of Group A.*

Height	Weight of Boys of 13·5—14·5 years																	Totals	
	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	107·5—	—		119·5—
42 ⁵ / ₈ —	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
44 "	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
45 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
46 "	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
47 "	—	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	4
48 "	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
49 "	1	1	3	4	4	2	—	—	—	—	—	—	—	—	—	—	—	—	15
50 "	1	3	1	5	7	8	2	—	—	—	—	—	—	—	—	—	—	—	27
51 "	—	—	1	6	5	10	3	3	—	—	—	—	—	—	—	—	—	—	28
52 "	—	—	—	4	11	22	4	1	1	—	—	—	—	—	—	—	—	—	43
53 "	—	—	1	4	12	16	15	4	7	1	—	—	—	—	—	—	—	—	60
54 "	—	—	—	1	4	11	9	11	6	1	—	1	1	—	—	—	—	—	45
55 "	—	—	—	—	1	3	17	21	7	4	1	—	1	—	—	—	—	—	56
56 "	—	—	—	—	—	—	2	4	10	10	14	6	2	—	1	—	—	—	49
57 "	—	—	1	—	—	3	2	5	7	8	5	4	—	1	—	—	—	—	36
58 "	—	—	—	—	—	—	—	5	6	4	2	2	4	1	—	—	—	—	24
59 "	—	—	—	—	—	—	—	1	1	1	3	1	1	2	2	1	—	—	13
60 "	—	—	—	—	—	—	—	—	—	—	2	—	2	1	2	1	—	—	8
61 "	—	—	—	—	—	—	—	—	—	—	—	2	1	—	1	—	—	—	4
62 "	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	—	—	—	4
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	3	8	11	28	46	77	56	61	45	36	17	15	10	8	5	1	—	1	428

TABLE 10. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 5·5—6·5 years																	Totals
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	
29 ⁵ / ₈ —	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
30 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
31 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
32 "	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	3
33 "	1	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	4
34 "	1	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	5
35 "	—	3	4	4	—	—	—	—	—	—	—	—	—	—	—	—	—	11
36 "	—	—	6	14	3	1	—	—	—	1	—	—	—	—	—	—	—	25
37 "	—	1	3	14	15	6	1	3	—	—	—	—	—	—	—	—	—	43
38 "	—	1	5	15	15	19	9	1	2	—	—	—	—	—	—	—	—	67
39 "	1	—	1	8	29	31	23	11	4	2	—	—	—	—	—	—	—	110
40 "	—	—	1	4	23	39	61	49	9	4	1	—	—	—	—	—	—	191
41 "	—	—	—	6	9	26	46	59	28	14	2	2	—	—	—	—	—	192
42 "	—	—	—	—	2	14	31	71	44	22	22	—	—	—	—	—	—	206
43 "	—	1	—	—	1	—	16	32	33	26	26	11	1	1	—	—	—	148
44 "	—	—	—	—	—	—	1	11	19	15	14	10	5	3	—	—	—	78
45 "	—	—	1	—	—	1	—	2	—	6	18	9	2	4	—	—	—	43
46 "	—	—	—	—	—	—	—	—	1	5	3	7	1	1	1	—	—	19
47 "	—	—	—	—	—	—	—	—	—	2	2	4	—	1	1	1	1	12
48 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	2
49 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	4	7	25	69	97	138	188	239	140	97	88	43	10	10	4	1	1	1161

TABLE 11. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 6·5—7·5 years																		Totals	
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—		63·5—
29½—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
30 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
31 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
32 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
33 "	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
34 "	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
35 "	—	1	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
36 "	—	1	2	3	3	2	1	—	—	—	1	—	—	—	—	—	—	—	—	13
37 "	—	—	2	5	7	6	3	—	—	—	—	—	—	—	—	—	—	—	—	23
38 "	—	1	—	6	8	14	5	4	—	—	—	—	—	—	—	—	—	—	—	38
39 "	—	1	2	3	7	12	9	6	2	5	—	—	—	—	—	—	—	—	—	47
40 "	—	—	—	3	8	27	23	23	14	4	1	—	—	—	—	—	—	—	—	103
41 "	—	—	—	3	2	17	30	44	26	16	6	—	1	1	—	—	—	—	—	146
42 "	—	—	—	—	7	11	26	44	51	32	7	6	2	1	1	—	—	—	—	188
43 "	—	—	—	—	—	2	16	47	63	59	36	16	5	1	1	—	—	—	—	246
44 "	—	—	—	—	—	1	2	17	41	49	55	27	15	5	2	1	—	—	—	215
45 "	—	—	—	—	—	—	—	2	13	35	38	33	14	7	6	1	—	—	—	149
46 "	—	—	—	—	—	—	—	2	7	10	24	22	19	15	12	1	2	—	—	114
47 "	—	—	—	—	—	—	—	—	1	4	5	12	15	13	5	4	—	1	—	60
48 "	—	—	—	—	—	—	—	—	—	—	1	3	4	7	6	2	2	2	—	27
49 "	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	3	2	—	1	9
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
Totals	1	4	10	26	44	93	115	189	218	215	174	119	76	51	33	13	7	3	1	1392

TABLE 12. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 7·5—8·5 years																						Totals
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	
34½—	1	1																					2
35 "			1					1				1											3
36 "							1			1													2
37 "			2	2		1		1		1		1											8
38 "			1		2	5	3	1															12
39 "					5	4	5	5	3			1											23
40 "					2	10	13	15	11	1	1	1											54
41 "						6	14	19	11	11	6	1	3										71
42 "					1		8	27	18	25	8	3	1			1							92
43 "						2	3	27	28	43	26	9	3	1	5								147
44 "						2	1	16	34	47	61	28	17	5	4	2							217
45 "							2	5	11	37	58	57	30	15	9	4							228
46 "							1		5	13	42	38	53	26	29	9	2						218
47 "								1	4	4	13	22	32	36	22	19	5	4					162
48 "									1	2	5	6	10	16	25	23	7	6	3				104
49 "												3	7	6	11	12	6	4	3	1	1		54
50 "				1							1		1		2	4	5	2	4	1			21
51 "														2		2	1	5	3	1			14
52 "											1								1	1		1	4
53 "																					1		1
54 "																							
55 "												2											2
56 "												1			1								2
57 "																							
58 "																							
59 "													1										1
Totals	1	1	4	3	10	30	51	118	126	185	222	174	158	107	108	76	26	21	14	4	2	1	1442

TABLE 13. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 8·5—9·5 years																	Totals
	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	
34½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
36 "	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	2
37 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
39 "	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	3
40 "	—	1	—	4	4	2	3	—	—	—	—	—	—	—	—	—	—	14
41 "	—	1	—	1	5	7	3	1	—	—	—	—	—	—	—	—	—	18
42 "	—	1	1	2	5	11	10	4	1	1	—	1	—	—	—	—	—	37
43 "	—	—	—	1	4	22	22	14	11	6	2	2	—	—	—	—	—	84
44 "	—	—	—	3	6	18	26	31	19	12	6	4	2	—	—	—	—	127
45 "	—	—	—	—	3	16	19	48	43	29	23	13	4	3	—	—	—	201
46 "	—	—	—	—	1	6	10	30	40	35	38	37	14	3	1	—	—	215
47 "	—	—	—	—	3	2	3	17	22	47	39	48	26	16	6	2	2	233
48 "	—	—	—	—	—	—	4	6	18	20	29	54	35	18	12	8	3	207
49 "	—	—	—	—	—	—	—	4	2	8	17	27	29	29	26	10	2	157
50 "	—	—	—	—	—	—	—	—	2	2	4	4	11	14	11	13	6	74
51 "	—	—	—	—	—	—	—	—	1	—	1	5	2	3	10	8	1	35
52 "	—	—	—	—	—	—	—	—	1	—	1	1	—	1	4	3	2	19
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	8
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	4
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2
57 "	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
58 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—
Totals	1	3	3	11	32	85	100	156	160	162	161	198	124	89	73	45	17	1449

TABLE 14. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 9·5—10·5 years																	Totals
	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	
37½—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
38 "	—	—	—	—	1	1	—	—	—	—	—	—	—	—	1	1	—	4
39 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
40 "	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	3
41 "	—	—	1	—	4	1	—	—	1	1	—	—	—	—	—	—	—	8
42 "	—	—	—	2	2	3	5	1	1	—	—	1	—	—	—	1	—	18
43 "	—	—	—	—	5	6	2	3	5	1	1	—	—	—	—	—	—	23
44 "	—	—	—	—	2	4	3	9	7	5	3	1	—	—	—	—	—	34
45 "	—	—	—	—	—	1	4	17	12	13	10	9	2	2	1	—	—	71
46 "	—	—	—	1	2	2	6	12	24	19	26	21	8	1	1	1	—	124
47 "	—	—	—	—	—	—	2	9	16	20	34	39	20	13	1	2	2	158
48 "	—	—	—	—	—	1	—	3	11	18	31	44	48	25	28	4	—	215
49 "	—	—	—	—	—	—	—	3	1	5	16	43	45	40	34	14	6	211
50 "	—	—	—	—	2	—	1	2	2	5	17	24	32	25	21	8	14	160
51 "	—	—	—	—	—	1	—	1	1	4	9	8	13	24	26	17	12	134
52 "	—	—	—	—	—	—	—	—	1	1	1	4	2	13	10	8	9	82
53 "	—	—	—	—	—	—	—	—	—	2	1	—	—	3	2	4	2	45
54 "	—	—	—	—	—	1	—	1	—	—	1	—	—	—	1	—	2	19
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	6
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	4
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
58 "	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
Totals	2	—	1	6	13	19	26	63	80	94	133	188	159	133	132	82	50	1327

TABLE 15. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 10·5—11·5 years																				Totals
	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	
35½—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
37 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
39 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
40 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41 "	1	—	—	1	—	2	—	—	1	—	—	—	—	1	—	—	—	1	—	—	2
42 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	6
43 "	—	—	1	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	5
44 "	—	—	—	—	1	2	3	3	—	—	—	1	1	—	—	—	—	—	—	—	11
45 "	—	—	1	—	3	5	6	6	6	7	2	—	—	—	—	—	—	—	—	—	36
46 "	—	—	—	3	1	6	5	14	11	4	7	3	2	—	—	—	—	—	—	—	56
47 "	—	—	—	—	—	4	6	12	18	28	11	9	6	2	2	2	1	—	—	—	101
48 "	—	—	—	—	1	3	5	7	13	27	27	22	22	6	7	10	5	1	1	—	141
49 "	—	—	—	—	—	—	2	7	13	35	47	30	34	27	10	5	1	1	—	—	212
50 "	—	—	—	—	1	—	1	3	6	15	25	30	39	32	25	12	8	3	—	1	202
51 "	—	—	—	1	—	1	1	1	2	5	8	21	35	27	27	29	15	11	3	3	190
52 "	—	—	—	—	—	1	1	—	1	1	3	7	19	19	19	24	21	9	5	—	132
53 "	—	—	—	—	—	—	—	—	—	—	—	2	8	8	12	11	22	12	11	5	95
54 "	—	—	—	—	—	—	—	—	—	—	1	1	4	4	2	5	13	13	8	11	66
55 "	—	—	—	—	—	—	—	—	—	—	1	1	—	2	1	2	5	6	4	2	39
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	6	2	13
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
58 "	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	3
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	2
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
72 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	—	2	1	6	8	24	31	55	71	124	134	128	171	128	105	88	88	57	31	1319

TABLE 16. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 11·5—12·5 years															Totals
	45·5—	48·5—	51·5—	54·5—	57·5—	60·5—	63·5—	66·5—	69·5—	72·5—	75·5—	78·5—	81·5—	84·5—	87·5—	
38½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	3
43 "	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	3
44 "	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	3
45 "	2	—	8	1	1	1	—	—	—	—	—	—	—	—	—	13
46 "	—	6	7	3	2	2	—	—	—	—	—	—	—	—	—	20
47 "	1	—	9	9	14	6	2	1	—	1	—	—	—	—	—	43
48 "	—	2	10	14	15	18	11	4	—	—	—	—	—	—	—	74
49 "	—	1	7	19	40	29	18	10	2	2	—	—	—	—	—	128
50 "	—	1	2	13	25	35	29	17	9	4	1	1	—	—	—	137
51 "	—	—	2	4	25	36	43	34	24	7	2	1	1	—	1	180
52 "	—	—	—	4	5	20	41	55	36	12	12	2	1	—	—	188
53 "	—	—	—	1	2	5	19	34	61	37	19	5	—	1	—	184
54 "	—	—	—	—	1	2	4	16	22	30	17	7	3	2	—	107
55 "	—	—	—	—	—	1	5	3	9	12	26	13	9	2	—	80
56 "	—	—	—	—	—	—	—	—	4	6	15	13	6	6	3	56
57 "	—	—	—	—	—	—	—	1	—	2	8	7	6	4	5	35
58 "	—	—	—	—	—	—	—	—	—	1	—	—	—	4	4	14
59 "	—	—	—	—	—	—	—	—	1	—	1	—	—	3	1	10
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	5	13	48	70	130	155	172	175	169	113	102	49	26	22	13	1282

TABLE 17. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 12·5—13·5 years																						Totals	
	31·5—	39·5—	43·5—	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	107·5—	111·5—	115·5—	119·5—		
39 ⁵ / ₈ —	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	
40 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	
41 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	
42 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
43 "	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	
44 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	
45 "	1	—	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	
46 "	—	—	—	—	—	3	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	
47 "	—	—	—	1	3	8	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17	
48 "	—	—	—	1	—	4	17	21	4	2	—	—	—	—	—	—	—	—	—	—	—	—	49	
49 "	—	—	—	—	1	9	21	10	5	1	—	—	—	—	—	—	—	—	—	—	—	—	47	
50 "	—	—	—	—	1	12	20	31	22	9	2	1	—	—	—	—	—	—	—	—	—	—	99	
51 "	—	—	—	—	—	2	5	19	35	31	15	6	3	—	—	—	—	—	—	—	—	—	116	
52 "	—	—	—	—	—	—	3	15	32	50	25	14	3	2	—	—	—	—	—	—	—	—	144	
53 "	—	—	—	—	—	2	3	6	15	61	59	30	6	1	1	—	—	—	—	—	—	—	184	
54 "	—	—	—	—	—	1	1	14	17	52	30	15	7	2	—	—	—	—	—	—	—	—	139	
55 "	—	—	—	—	—	1	—	2	3	20	42	43	21	14	3	1	—	1	—	—	—	—	151	
56 "	—	—	—	—	—	—	—	2	1	7	12	27	33	13	10	1	1	1	—	—	—	—	108	
57 "	—	—	—	—	—	1	—	1	2	7	13	17	24	6	5	1	—	—	—	—	—	—	77	
58 "	—	—	—	—	—	—	—	—	1	2	—	5	11	10	15	3	4	2	—	1	—	—	54	
59 "	—	—	—	—	—	—	—	—	—	—	—	1	4	4	5	1	1	1	—	—	—	—	17	
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	1	—	2	2	1	—	—	10	
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	—	2	1	—	—	1	7	
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	
64 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	2	
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	
66 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	
67 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	
Totals	1	—	—	1	6	17	65	113	150	219	223	171	114	78	46	13	8	9	3	2	1	1	2	1243

TABLE 18. *Glasgow. Height and Weight of Boys of Group B.*

Height	Weight of Boys of 13·5—14·5 years																		Totals	
	43·5—	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	107·5—	—		123·5—
42 $\frac{5}{8}$ —	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
43 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
44 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
47 "	—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	—	—	3
48 "	—	—	—	1	1	3	—	—	—	—	—	—	—	—	—	—	—	—	—	5
49 "	—	—	1	1	4	1	3	—	—	—	—	—	—	—	—	—	—	—	—	10
50 "	—	—	—	3	4	8	1	2	—	—	—	—	—	—	—	—	—	—	—	18
51 "	—	—	—	1	4	10	7	5	2	—	—	—	—	—	—	—	—	—	—	29
52 "	—	—	—	—	4	5	13	4	2	—	—	—	—	—	—	—	—	—	—	28
53 "	—	—	—	—	—	6	13	4	10	1	—	1	—	—	—	—	—	—	—	35
54 "	—	—	—	—	4	5	8	17	22	7	—	4	—	—	1	—	—	—	—	68
55 "	—	—	—	—	—	2	5	14	16	12	6	2	—	—	—	—	—	—	—	57
56 "	—	—	—	—	—	—	3	3	6	13	14	5	3	—	—	—	—	—	—	47
57 "	—	—	—	—	—	—	1	—	6	9	12	6	4	1	1	—	—	—	—	40
58 "	—	—	—	—	—	—	—	4	2	—	4	9	5	1	2	—	—	—	—	27
59 "	—	—	—	—	—	—	—	—	—	2	3	2	1	3	3	—	—	—	—	14
60 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
61 "	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	5
62 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	2
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	—	1	8	23	40	54	54	67	44	39	30	15	7	9	1	3	—	1	397

TABLE 19. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 5·5—6·5 years													Totals
	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	52·5—	50·5—	
38½—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
34—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
35—	—	1	—	—	2	—	—	—	—	—	—	—	—	3
36—	4	1	—	1	—	—	—	—	—	—	—	—	—	6
37—	—	7	6	4	1	—	—	—	—	—	—	—	—	18
38—	1	5	12	8	4	4	—	—	—	—	—	—	—	34
39—	1	8	12	23	12	9	1	—	—	—	—	—	—	66
40—	—	2	6	11	14	22	5	2	—	—	—	—	—	62
41—	—	—	1	11	23	21	21	8	5	1	—	—	—	91
42—	—	—	1	5	12	24	23	19	14	—	3	1	—	102
43—	—	—	—	—	5	14	19	10	11	5	1	—	—	65
44—	—	—	—	1	—	2	2	6	8	6	3	—	—	28
45—	—	1	—	—	—	—	—	1	9	5	—	1	1	18
46—	—	—	—	—	—	—	—	—	1	—	2	—	—	3
47—	—	—	—	—	—	—	—	—	—	1	1	1	—	3
Totals	6	25	39	64	73	96	71	46	48	18	10	3	1	500

TABLE 20. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 6·5—7·5 years																Totals	
	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—		61·5—
34½—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
35 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
36 "	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
37 "	—	—	1	—	4	1	—	—	—	1	—	—	—	—	—	—	—	7
38 "	—	—	—	3	2	—	—	—	—	—	—	—	—	—	—	—	—	5
39 "	—	—	2	2	9	10	5	1	1	—	—	—	—	—	—	—	—	30
40 "	—	—	2	2	11	12	9	2	2	—	—	1	—	—	—	—	—	41
41 "	—	—	1	4	13	12	24	15	10	3	2	—	—	—	—	—	—	84
42 "	—	—	—	—	4	18	28	32	17	4	4	—	—	—	—	—	—	107
43 "	—	—	—	—	—	7	15	27	23	24	10	5	2	1	—	—	—	114
44 "	—	—	—	—	—	—	5	14	24	25	27	8	3	2	—	—	—	108
45 "	—	—	—	—	1	1	2	2	10	15	21	9	5	6	—	—	—	72
46 "	—	—	—	—	—	—	1	1	3	13	7	16	3	4	1	—	—	49
47 "	—	—	—	—	—	—	—	1	—	—	3	5	5	3	—	—	—	17
48 "	—	—	—	—	—	—	—	—	—	—	—	—	2	2	—	1	1	6
49 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
50 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
51 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
52 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	2	8	11	44	61	90	95	93	85	74	44	20	18	1	1	2	649

TABLE 21. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 7·5—8·5 years																	Totals
	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	
30 $\frac{1}{8}$ —	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
35 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
36 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	1	1	2	1	1	—	—	—	—	—	—	—	—	—	—	6
39 "	—	1	1	2	4	1	1	1	—	—	—	—	—	—	—	—	—	11
40 "	—	—	1	1	2	2	4	—	—	—	—	—	—	—	—	—	—	10
41 "	—	—	1	—	6	9	6	6	4	—	—	—	—	—	—	—	—	32
42 "	—	—	—	2	3	14	11	5	5	—	1	2	—	—	—	—	—	43
43 "	—	—	—	1	5	4	17	13	18	4	3	1	—	—	—	—	—	66
44 "	—	—	—	—	1	—	5	10	20	24	15	8	3	—	—	—	—	94
45 "	—	—	—	—	—	1	1	10	10	22	27	12	14	3	4	—	—	104
46 "	—	—	—	—	—	—	2	7	12	22	18	16	10	3	—	—	—	90
47 "	—	—	—	—	—	—	—	5	4	12	12	15	14	4	2	—	—	72
48 "	—	—	—	—	—	—	2	1	—	6	5	12	13	5	8	2	2	60
49 "	—	—	—	—	—	—	—	—	1	1	—	3	5	6	3	2	—	21
50 "	—	—	—	—	—	—	1	1	1	—	—	—	2	2	2	1	—	10
51 "	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
52 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	2
53 "	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	1	—	4
54 "	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	2
55 "	—	—	—	—	—	—	—	3	1	—	—	—	—	—	—	—	—	4
56 "	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	2
57 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2
58 "	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
Totals	1	1	5	8	23	37	66	75	94	88	61	72	52	26	17	7	2	642

TABLE 22. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 8·5—9·5 years															Totals
	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	
35 $\frac{5}{8}$ —	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
36 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
38 "	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
39 "	—	—	1	1	—	—	1	—	—	—	—	—	—	—	—	3
40 "	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	2
41 "	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	4
42 "	—	1	2	2	5	4	—	—	1	—	—	—	—	—	—	15
43 "	—	1	4	5	4	9	4	3	1	—	1	—	—	—	—	32
44 "	—	1	2	11	3	12	11	5	4	—	2	—	—	—	—	51
45 "	—	—	2	6	6	17	11	11	5	1	5	1	1	—	—	66
46 "	—	—	1	3	5	5	17	16	18	11	2	4	—	—	—	82
47 "	—	—	—	—	2	5	8	25	30	18	9	9	3	—	—	109
48 "	—	—	1	—	1	1	4	12	15	32	17	8	3	2	1	97
49 "	—	1	1	1	1	—	—	2	11	7	12	13	3	9	1	65
50 "	—	—	—	—	—	—	1	—	—	4	11	11	8	7	2	51
51 "	—	—	1	1	—	—	1	1	—	—	2	4	5	2	1	22
52 "	—	—	—	—	—	—	—	1	—	1	2	2	3	1	—	17
53 "	—	—	—	—	1	2	1	—	—	—	—	—	1	—	—	5
54 "	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
55 "	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	2
56 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	7	17	33	28	56	59	76	85	75	66	52	28	21	10	630

TABLE 23. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 9·5—10·5 years																	Totals
	33·5—	34·5—	35·5—	36·5—	37·5—	38·5—	39·5—	40·5—	41·5—	42·5—	43·5—	44·5—	45·5—	46·5—	47·5—	48·5—	49·5—	
39½—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 "	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
41 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
42 "	—	—	—	—	1	—	—	—	—	2	—	—	—	—	—	—	—	3
43 "	—	—	—	—	—	2	1	1	—	—	—	—	—	—	—	—	—	4
44 "	—	—	—	1	3	5	3	3	—	—	—	—	—	—	1	—	—	17
45 "	—	—	—	—	1	5	4	6	4	4	2	—	—	—	—	—	—	26
46 "	—	—	1	1	2	—	3	9	14	7	7	5	—	—	—	—	—	58
47 "	—	—	—	—	1	—	4	8	7	17	10	11	4	—	1	1	—	65
48 "	—	—	—	—	—	—	6	5	20	19	26	18	11	2	—	1	2	110
49 "	—	—	—	—	—	—	1	2	6	18	25	11	24	4	4	3	—	99
50 "	—	—	—	—	—	—	—	—	3	6	10	18	23	10	8	6	3	88
51 "	—	—	—	—	—	—	—	1	1	2	10	5	13	10	10	4	4	62
52 "	—	—	—	—	—	—	—	—	—	1	—	6	8	7	3	3	7	44
53 "	—	—	—	—	—	—	—	—	—	—	—	—	2	2	2	7	3	20
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	4	11
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	3
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	—	2	1	5	6	19	32	33	66	69	91	68	81	37	28	25	616

TABLE 24. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 10·5—11·5 years																	Totals
	39·5—	40·5—	41·5—	42·5—	43·5—	44·5—	45·5—	46·5—	47·5—	48·5—	49·5—	50·5—	51·5—	52·5—	53·5—	54·5—	55·5—	
35½—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
39 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
40 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	—	—	—	—	1	—	1	2	—	—	—	—	—	—	—	—	—	4
45 "	—	—	1	1	2	—	1	2	2	—	—	—	—	—	—	—	—	9
46 "	1	—	—	1	2	1	3	2	2	3	—	2	—	—	—	—	—	17
47 "	—	—	—	—	1	4	1	5	9	4	8	1	2	1	—	—	—	36
48 "	—	—	—	—	1	2	3	6	16	11	12	8	2	1	—	2	1	65
49 "	—	—	—	—	—	1	4	4	9	13	13	12	1	3	—	—	—	61
50 "	—	—	—	—	—	1	2	5	10	22	17	15	11	8	4	1	—	96
51 "	—	—	—	—	1	—	1	2	6	12	17	10	12	8	6	2	1	80
52 "	—	—	—	—	—	—	—	2	1	6	11	17	6	7	14	4	3	75
53 "	—	—	—	—	—	—	—	—	—	1	3	7	7	9	7	10	5	59
54 "	—	—	—	—	—	—	—	—	1	—	2	2	—	3	6	4	4	30
55 "	—	—	—	—	—	—	—	—	1	—	—	—	—	1	3	3	5	18
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	6
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	6
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	3	3	7	9	11	24	43	46	75	74	67	39	39	44	25	568

TABLE 25. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 11·5—12·5 years																				Totals		
	45·5—	48·5—	51·5—	54·5—	57·5—	60·5—	63·5—	66·5—	69·5—	72·5—	75·5—	78·5—	81·5—	84·5—	87·5—	90·5—	93·5—	96·5—	—	103·5—		—	111·5—
35½—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
44 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
45 "	2	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
46 "	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
47 "	1	1	1	5	3	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	13
48 "	—	—	4	9	7	4	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
49 "	—	—	2	5	5	14	7	4	4	—	—	—	—	—	—	—	—	—	—	—	—	—	41
50 "	—	—	1	7	9	18	12	6	11	2	—	2	—	—	—	—	1	—	—	—	—	—	69
51 "	—	—	2	2	9	14	28	23	7	7	—	1	—	—	—	—	—	—	—	—	—	—	93
52 "	—	—	—	—	3	15	19	18	19	4	6	1	1	—	—	—	—	—	—	—	—	—	86
53 "	—	—	—	—	—	6	7	15	18	17	5	2	2	—	—	—	—	—	—	—	—	—	72
54 "	—	—	—	—	—	1	9	11	17	13	15	6	6	2	1	—	—	—	—	—	—	—	81
55 "	—	—	—	—	—	—	1	4	4	5	8	6	4	3	2	—	—	—	—	—	—	—	37
56 "	—	—	—	—	—	—	1	—	4	3	5	8	10	3	—	2	—	1	—	—	—	—	37
57 "	—	—	—	—	—	—	—	—	4	2	2	3	3	—	2	1	—	—	—	—	—	—	17
58 "	—	—	—	—	—	—	—	—	—	—	—	—	3	2	1	—	—	1	—	—	—	—	7
59 "	—	—	—	—	—	1	—	—	—	1	—	1	1	1	—	—	—	1	—	1	—	—	8
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
66 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	5	4	13	29	36	74	86	83	88	54	42	30	30	11	7	3	1	3	—	1	—	1	601

TABLE 26. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 12·5—13·5 years																		Totals	
	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	—	111·5—	—		123·5—
44½—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
45 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46 "	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
47 "	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
48 "	—	—	3	4	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	9
49 "	—	2	6	3	2	3	2	—	—	—	—	—	—	—	—	—	—	—	—	18
50 "	—	—	4	7	12	7	1	—	—	—	—	—	—	—	—	—	—	—	—	31
51 "	—	1	4	6	24	20	7	1	2	—	—	—	—	—	—	—	—	—	—	65
52 "	—	1	1	9	25	24	15	5	—	—	—	—	—	—	—	—	—	—	—	80
53 "	—	—	—	1	8	16	21	7	4	4	—	—	—	1	—	—	—	—	—	62
54 "	—	—	—	—	3	9	19	18	15	7	2	—	—	1	—	—	—	—	—	74
55 "	—	—	—	—	1	11	16	24	15	9	2	—	—	—	—	—	—	—	—	78
56 "	—	—	—	—	—	2	4	11	6	19	4	3	1	—	—	—	—	—	—	50
57 "	—	—	—	—	—	1	1	12	9	13	6	2	2	—	—	—	—	—	—	46
58 "	—	—	—	—	—	—	—	1	3	4	7	7	1	1	—	—	—	—	—	24
59 "	—	—	—	—	—	—	—	—	—	2	2	3	3	2	1	—	—	—	—	14
60 "	—	—	—	—	—	—	—	—	—	—	1	4	1	1	2	—	—	—	—	9
61 "	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	1	—	—	5
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	1	4
63 "	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	1	—	—	3
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
66 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
67 "	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	2	5	20	32	77	94	88	80	54	58	25	19	8	9	6	—	4	—	1	582

TABLE 27. *Glasgow. Height and Weight of Boys of Group C.*

Height	Weight of Boys of 13·5—14·5 years																		Totals
	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	107·5—	111·5—	115·5—	119·5—	123·5—	
48½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
49 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
50 "	—	—	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	4
51 "	—	2	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	6
52 "	—	2	5	2	6	2	—	—	—	—	—	—	—	—	—	—	—	—	17
53 "	1	2	4	4	6	7	1	1	—	—	—	—	—	—	—	—	—	—	26
54 "	1	—	2	1	6	4	6	4	—	—	—	—	—	—	—	—	—	—	24
55 "	1	—	—	2	5	16	4	7	—	—	—	—	—	—	—	—	—	—	35
56 "	—	1	—	1	5	10	13	4	3	1	2	1	—	—	—	—	—	—	41
57 "	—	—	—	1	3	6	8	7	5	2	1	—	—	—	—	—	—	—	33
58 "	—	—	—	1	1	2	2	4	7	4	3	—	1	—	—	—	—	—	25
59 "	—	—	1	—	—	1	—	1	3	6	—	2	1	—	—	—	—	—	15
60 "	—	—	—	—	—	1	1	1	1	—	2	3	1	1	1	—	—	—	12
61 "	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	1	—	—	4
62 "	—	—	—	—	—	—	—	—	—	—	—	1	1	3	—	1	—	—	6
63 "	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	3
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	1	3
68 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
69 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	4	8	15	14	35	49	35	29	19	15	8	8	7	6	2	3	—	2	259

TABLE 28. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 5·5—6·5 years																	Totals	
	21·5—	—	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—		57·5—
36½—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
37 "	—	—	—	—	1	3	1	2	1	—	—	—	1	—	—	—	—	—	9
38 "	—	—	—	—	—	4	4	3	3	—	1	—	—	—	—	—	—	—	15
39 "	—	—	—	—	—	3	6	4	2	1	—	—	—	—	—	—	—	—	17
40 "	—	—	—	—	—	1	6	16	12	14	4	1	—	—	—	—	—	—	54
41 "	—	—	—	—	1	1	2	6	17	24	12	8	—	—	—	—	—	—	71
42 "	—	—	—	—	—	1	1	8	12	23	31	11	4	—	1	—	—	—	92
43 "	—	—	—	—	—	—	—	1	5	22	20	8	11	3	1	—	—	—	71
44 "	1	—	—	—	1	—	—	—	—	2	12	11	11	2	3	1	—	—	44
45 "	—	—	—	—	—	—	—	—	—	1	2	7	5	3	4	1	1	1	25
46 "	—	—	—	—	—	—	—	—	—	—	—	1	3	2	4	2	—	—	12
47 "	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	—	—	4
48 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	—	1	—	3	14	20	40	52	87	83	47	37	11	14	5	1	1	417

TABLE 29. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 6·5—7·5 years															Totals
	31·5—	32·5—	33·5—	34·5—	35·5—	36·5—	37·5—	38·5—	39·5—	40·5—	41·5—	42·5—	43·5—	44·5—	45·5—	
36½—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	4
37 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	3
39 "	—	—	2	2	2	1	1	—	—	—	—	—	—	—	—	8
40 "	—	2	5	2	3	2	2	1	1	—	—	—	—	—	—	18
41 "	—	1	3	5	7	10	4	3	3	—	—	—	—	—	—	36
42 "	—	—	—	4	10	17	19	10	—	1	—	—	—	—	—	61
43 "	—	—	—	1	8	18	37	18	16	2	1	—	—	—	—	101
44 "	—	—	—	—	—	10	17	32	23	13	5	2	—	—	—	102
45 "	—	—	—	—	—	7	6	11	22	14	9	3	2	—	—	74
46 "	—	—	—	—	—	—	3	8	11	9	8	8	7	1	—	55
47 "	—	—	—	—	—	—	1	—	4	5	5	4	3	1	1	24
48 "	—	—	—	—	—	—	—	—	—	—	—	3	3	3	1	12
49 "	—	—	—	—	—	—	—	—	—	—	1	—	3	1	—	6
50 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	1	4	13	14	32	65	90	84	80	44	29	20	19	6	3	506

TABLE 30. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 7·5—8·5 years																			Totals	
	31·5—	—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—		69·5—
34½—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
35 "	1	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	3
36 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
37 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
38 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39 "	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 "	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
41 "	—	—	—	3	1	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	8
42 "	—	—	2	2	—	12	5	4	1	—	—	—	—	—	—	—	—	—	—	—	26
43 "	—	—	—	—	—	9	16	11	6	3	1	—	—	—	—	—	—	—	—	—	46
44 "	—	—	—	—	2	3	13	15	22	7	6	3	—	—	—	—	—	—	—	—	71
45 "	—	—	—	—	1	4	4	13	32	11	12	7	4	1	—	—	—	—	—	—	89
46 "	—	—	—	—	—	1	1	8	20	29	28	13	5	3	—	—	—	—	—	—	108
47 "	—	—	—	—	—	—	—	2	9	16	20	23	22	5	3	—	—	—	—	—	100
48 "	—	—	—	—	—	—	—	1	2	4	9	17	18	7	2	1	3	—	—	—	64
49 "	—	—	—	—	—	—	—	—	—	1	3	6	9	6	7	4	1	1	—	—	38
50 "	—	—	—	—	—	—	—	—	—	—	—	—	3	3	2	4	1	2	—	1	16
51 "	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	1	—	—	4
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
54 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	2
Totals	1	—	2	5	4	33	42	55	94	72	80	69	63	26	14	9	5	6	—	1	581

TABLE 31. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 8·5—9·5 years																					Totals	
	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—	77·5—	—		93·5—
37½—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
40 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41 "	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
42 "	—	1	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
43 "	—	4	1	4	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
44 "	—	1	2	2	3	5	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
45 "	—	2	3	1	10	10	5	1	3	4	—	—	—	—	—	—	—	—	—	—	—	—	39
46 "	—	—	—	7	13	13	10	7	5	3	1	—	—	—	—	—	—	—	—	—	—	—	59
47 "	—	—	—	—	4	15	24	17	15	8	3	2	—	—	—	—	—	—	—	—	—	—	88
48 "	—	—	—	—	4	6	15	21	19	16	12	7	1	—	—	—	—	—	—	—	—	—	101
49 "	—	—	—	—	—	1	3	11	22	10	12	11	3	3	1	1	—	—	—	—	—	—	78
50 "	—	—	—	—	—	3	2	2	3	11	20	10	9	3	—	—	—	—	—	—	—	—	63
51 "	—	—	—	—	—	—	—	—	1	2	2	5	5	3	5	—	3	—	—	—	—	—	26
52 "	—	—	—	—	—	—	—	—	—	1	1	4	6	4	2	—	—	—	1	—	—	—	19
53 "	—	—	—	—	—	—	—	—	—	—	1	2	1	2	1	1	2	—	—	1	—	—	11
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
58 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	8	8	16	36	54	64	61	71	55	52	41	25	15	9	2	6	1	1	1	—	1	528

TABLE 32. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 9·5—10·5 years																					Totals	
	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—	77·5—	79·5—	81·5—	—		91·5—
36½—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
38 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43 "	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
44 "	1	1	2	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
45 "	1	3	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
46 "	1	3	3	1	6	5	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	23
47 "	—	—	3	11	8	8	8	3	4	3	—	—	—	—	—	—	—	—	—	—	—	—	48
48 "	—	—	1	3	4	12	15	9	3	4	1	2	1	—	—	—	—	—	—	—	—	—	55
49 "	—	—	—	2	5	6	23	17	20	15	5	1	—	—	—	—	—	—	—	—	—	—	94
50 "	—	—	—	1	2	2	13	17	22	26	14	9	3	3	2	1	—	—	—	—	—	—	115
51 "	—	—	—	—	—	1	4	2	9	17	18	13	8	7	3	2	—	—	—	—	—	—	84
52 "	—	—	—	—	—	—	2	—	4	9	9	14	8	6	4	3	—	—	—	—	1	—	60
53 "	—	—	—	—	—	—	—	—	—	3	4	6	2	11	7	2	2	1	2	—	—	—	40
54 "	—	—	—	—	—	—	—	1	—	1	—	—	2	4	2	2	—	—	—	—	—	—	12
55 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	2
56 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	1	—	—	—	3
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	4	8	10	24	27	34	68	50	63	78	52	45	25	31	21	10	2	2	2	1	—	1	558

TABLE 33. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 10·5—11·5 years																				Totals
	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—	77·5—	79·5—	81·5—	83·5—	
45½	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
46 "	—	—	3	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
47 "	—	1	2	2	4	3	4	1	—	1	—	—	—	—	—	—	—	—	—	—	18
48 "	—	—	3	2	4	6	6	7	5	1	—	—	—	—	—	—	—	—	—	—	35
49 "	—	—	1	1	1	10	17	13	12	8	4	—	—	—	—	—	—	—	—	—	67
50 "	—	—	1	—	2	5	11	15	14	11	8	5	4	1	1	1	—	—	—	—	79
51 "	—	—	—	1	—	—	6	10	20	14	16	9	6	5	3	—	—	—	—	—	90
52 "	—	—	—	—	—	1	3	4	10	14	30	17	12	11	4	1	1	1	—	—	109
53 "	—	—	—	—	—	1	2	3	3	11	10	7	14	3	5	5	3	1	1	1	70
54 "	—	—	—	—	—	—	—	—	—	5	5	6	8	4	10	5	3	1	5	3	56
55 "	—	—	—	—	—	—	—	—	—	2	4	3	6	4	4	4	3	3	1	1	32
56 "	—	—	—	—	—	—	—	—	—	—	1	—	1	1	2	2	3	1	—	—	12
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	2	1	1	1	9
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	3
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	4
60 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1	1	11	9	13	26	49	53	64	65	75	49	48	31	31	18	12	12	10	7	594

TABLE 34. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 11·5—12·5 years																	Totals
	42·5—	45·5—	48·5—	51·5—	54·5—	57·5—	60·5—	63·5—	66·5—	69·5—	72·5—	75·5—	78·5—	81·5—	84·5—	87·5—	90·5—	
39½	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
45 "	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
46 "	—	—	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	3
47 "	—	—	—	2	1	2	—	—	—	—	—	—	—	—	—	—	—	5
48 "	—	—	—	1	3	3	1	1	—	—	—	—	—	—	—	—	—	9
49 "	—	—	—	2	2	6	6	1	2	1	—	—	—	—	—	—	—	20
50 "	—	—	—	2	2	13	9	6	7	2	—	—	—	—	—	—	—	41
51 "	—	—	—	—	3	7	9	12	12	6	6	3	1	—	—	—	—	59
52 "	—	—	—	1	1	3	8	18	20	21	6	3	2	—	—	—	—	83
53 "	—	—	—	—	1	2	7	5	27	37	14	8	2	1	—	—	—	104
54 "	—	—	—	—	—	—	4	9	11	23	25	11	4	1	1	—	—	90
55 "	—	—	—	1	—	—	1	3	15	11	10	8	1	1	1	1	—	53
56 "	—	—	—	—	—	—	—	1	5	9	7	10	10	3	—	—	1	46
57 "	—	—	—	—	—	—	—	1	1	3	4	4	2	3	—	2	—	20
58 "	—	—	—	—	—	—	—	—	1	2	2	1	4	4	3	—	2	20
59 "	—	—	—	—	—	—	—	—	—	—	1	—	2	1	2	1	—	8
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	4
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	1	—	1	10	13	39	44	54	83	112	77	50	32	21	13	6	7	571

TABLE 35. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 12·5—13·5 years																		Totals	
	47·5—	51·5—	55·5—	59·5—	63·5—	67·5—	71·5—	75·5—	79·5—	83·5—	87·5—	91·5—	95·5—	99·5—	103·5—	107·5—	111·5—	115·5—		119·5—
44 ⁵ —	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
45 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
47 „	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
48 „	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
49 „	—	—	2	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
50 „	—	1	2	6	4	2	1	—	—	—	—	—	—	—	—	—	—	—	—	16
51 „	—	1	4	6	14	9	1	2	—	—	—	—	—	—	—	—	—	—	—	37
52 „	—	—	1	7	12	17	5	6	—	—	—	—	—	—	—	—	—	—	—	48
53 „	—	—	—	6	12	17	16	16	3	1	—	—	1	—	—	—	—	—	—	72
54 „	—	—	—	1	12	19	22	21	7	2	—	—	—	—	—	—	—	—	—	84
55 „	—	—	—	1	3	11	25	34	14	6	3	3	1	—	—	—	—	—	—	101
56 „	—	—	—	—	1	2	19	24	16	14	2	2	—	—	—	—	—	—	—	80
57 „	—	—	—	—	—	—	9	10	16	20	7	3	1	2	—	—	—	—	—	68
58 „	—	—	—	—	—	1	2	5	8	14	4	4	3	1	—	—	—	—	—	42
59 „	—	—	—	—	—	1	1	1	—	2	4	7	3	—	—	1	1	—	—	21
60 „	—	—	—	—	—	—	—	—	—	3	1	7	1	1	—	1	—	—	—	14
61 „	—	—	—	—	—	—	—	—	1	—	1	—	2	2	—	1	1	1	—	9
62 „	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
63 „	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	1	4
64 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	3	12	31	59	79	101	119	65	62	22	26	12	8	1	3	2	2	2	611

TABLE 36. *Glasgow. Height and Weight of Boys of Group D.*

Height	Weight of Boys of 13·5—14·5 years																						Totals	
	51·5	55·5	59·5	63·5	67·5	71·5	75·5	79·5	83·5	87·5	91·5	95·5	99·5	103·5	107·5	111·5	115·5	119·5	123·5	127·5	131·5	—		147·5
45½	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
50 "	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
51 "	—	—	7	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
52 "	—	—	3	5	5	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	19
53 "	—	—	1	5	4	10	2	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25
54 "	—	—	1	4	5	8	15	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37
55 "	—	—	—	2	3	4	13	10	4	2	—	1	—	—	—	—	—	—	—	—	—	—	—	39
56 "	—	—	—	1	5	12	13	12	11	4	2	—	—	—	—	—	—	—	—	—	—	—	—	60
57 "	—	—	—	—	1	4	12	9	7	7	—	2	—	2	—	—	—	—	—	—	—	—	—	41
58 "	—	—	—	—	—	2	6	10	11	12	4	6	1	1	—	1	—	—	—	—	—	—	—	54
59 "	—	—	—	—	—	—	2	3	3	10	2	3	1	—	—	—	—	—	—	—	—	—	—	24
60 "	—	—	—	—	—	—	1	1	4	5	2	8	6	4	1	1	—	—	—	—	—	—	—	33
61 "	—	—	—	—	1	—	—	—	—	1	5	3	—	1	2	—	—	—	—	—	—	—	—	14
62 "	—	—	—	—	—	—	—	—	—	—	2	1	2	—	1	1	1	—	—	—	—	—	—	8
63 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	—	—	—	1	—	—	—	—	4
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	1	—	1	—	—	5
66 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
70 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	12	22	25	47	64	52	40	41	13	26	14	7	6	5	2	1	2	—	2	—	1	383

TABLE 37. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 5·5—6·5 years																	Totals	
	21·5—	23·5—	25·5—	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—		55·5—
25½—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
26 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
27 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
28 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
29 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
30 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
31 "	—	—	—	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	3
32 "	—	—	2	1	1	1	—	—	—	1	—	—	—	—	—	—	—	—	6
33 "	—	—	—	—	3	1	1	—	1	1	1	—	—	—	—	—	—	—	8
34 "	—	—	—	2	3	4	5	1	—	—	1	—	—	—	—	—	—	—	16
35 "	—	—	—	3	8	12	10	7	1	1	1	—	—	—	1	—	—	—	44
36 "	—	—	—	—	3	12	12	15	5	1	—	—	—	—	1	—	—	—	49
37 "	1	—	1	—	3	9	30	13	11	10	2	1	—	1	—	—	—	—	82
38 "	—	—	—	1	1	7	28	34	29	17	4	3	—	—	—	—	—	—	124
39 "	—	—	—	—	—	4	13	43	59	23	19	6	—	2	—	1	—	—	170
40 "	—	—	—	—	1	—	7	24	42	55	46	14	6	2	—	—	—	—	197
41 "	—	—	—	—	—	—	3	8	20	45	57	19	5	5	2	1	—	—	165
42 "	—	1	—	—	—	—	—	1	9	23	51	29	17	11	2	1	1	—	146
43 "	—	—	—	—	—	—	—	—	3	5	15	21	10	8	8	2	—	—	72
44 "	—	—	—	—	—	—	—	—	1	3	7	6	7	10	4	—	—	—	38
45 "	—	—	—	—	—	—	1	—	—	1	1	1	1	2	3	—	1	—	11
46 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	—	1	6
47 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	3
Totals	1	1	4	8	25	50	110	146	183	186	205	100	47	41	24	8	2	2	1143

TABLE 38. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 6·5—7·5 years																		Totals	
	25·5—	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—		61·5—
30½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
31 „	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
32 „	—	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	4
33 „	—	—	—	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	4
34 „	—	—	1	2	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	6
35 „	—	—	1	2	7	—	4	2	—	1	1	—	—	—	—	—	—	—	—	18
36 „	—	1	—	7	1	5	5	1	2	—	—	1	—	—	—	—	—	—	—	23
37 „	—	—	1	2	10	14	6	5	—	—	—	—	—	—	—	—	—	—	—	38
38 „	—	—	—	2	15	17	11	13	7	2	—	1	1	—	—	—	—	—	—	69
39 „	—	—	—	4	16	21	34	18	17	5	3	—	1	—	—	—	—	—	—	119
40 „	—	—	—	2	3	21	37	44	32	22	1	2	—	—	—	—	—	—	—	164
41 „	—	—	1	—	3	11	33	49	63	28	17	7	4	1	—	1	—	—	—	218
42 „	—	—	—	—	2	2	11	35	70	46	22	22	2	2	1	—	—	—	—	215
43 „	—	—	—	1	—	2	8	16	49	49	36	35	10	3	1	—	—	—	1	211
44 „	—	—	—	—	—	—	2	5	21	28	40	37	20	8	1	2	1	1	—	166
45 „	—	—	—	—	—	—	—	—	4	6	18	24	18	13	7	1	—	—	—	91
46 „	—	—	—	—	—	—	—	—	—	2	3	12	8	2	5	2	—	—	—	34
47 „	—	—	—	—	—	—	—	—	—	2	2	1	3	—	3	1	3	1	1	17
48 „	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	2	1	—	—	6
Totals	1	3	5	25	58	95	153	189	265	191	143	113	67	30	19	9	5	2	2	1405

TABLE 39. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 7·5—8·5 years																			Totals
	25·5	27·5	29·5	31·5	33·5	35·5	37·5	39·5	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	
29 $\frac{1}{8}$	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
30 "	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	2
31 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
32 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
33 "	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
34 "	—	—	—	1	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	4
35 "	—	—	—	1	1	2	1	—	1	—	2	—	—	—	—	—	—	—	—	8
36 "	—	—	1	2	2	4	—	—	—	—	—	—	—	—	—	—	—	—	—	9
37 "	—	—	—	—	1	1	5	1	1	1	—	—	—	—	—	—	—	—	—	10
38 "	—	—	—	1	5	4	11	6	5	3	—	1	—	—	—	—	—	—	—	36
39 "	—	—	—	—	8	5	13	8	4	3	—	—	1	—	—	—	—	—	—	42
40 "	—	—	—	—	1	10	15	15	19	11	2	4	3	—	—	—	—	—	—	80
41 "	—	—	—	—	1	3	13	31	41	15	13	2	1	—	1	—	—	—	—	121
42 "	—	—	—	1	2	6	8	28	46	41	27	14	2	3	—	—	—	—	—	178
43 "	—	—	—	—	—	1	3	14	39	54	37	42	17	6	—	2	—	—	—	215
44 "	—	—	—	—	1	—	2	3	13	42	53	49	29	14	8	5	2	1	—	222
45 "	—	—	—	—	1	—	—	1	11	20	31	39	30	16	14	6	1	—	—	170
46 "	—	—	—	—	—	1	—	1	—	6	18	32	31	19	22	13	4	1	—	148
47 "	—	—	—	—	—	—	—	—	—	1	4	10	12	10	20	9	10	—	—	76
48 "	—	—	—	—	—	—	—	—	1	1	1	3	5	8	3	8	7	2	2	42
49 "	—	—	—	—	—	—	—	—	—	1	—	—	1	3	3	2	5	2	1	20
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1
51 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
53 "	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	3
Totals	2	—	1	7	25	39	71	108	183	199	189	197	132	80	71	46	29	5	5	1393

TABLE 40. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 8·5—9·5 years																			Totals
	29·5	31·5	33·5	35·5	37·5	39·5	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	63·5	65·5	
32 $\frac{1}{8}$	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
33 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
34 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	—	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
36 "	—	—	2	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	4
37 "	—	1	1	—	—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	5
38 "	—	—	1	3	6	3	—	—	1	—	—	—	—	—	—	—	—	—	—	14
39 "	—	—	2	2	5	1	1	—	1	—	1	—	1	—	—	—	—	—	—	14
40 "	—	—	—	1	5	3	11	3	1	2	—	—	—	—	—	—	—	—	—	26
41 "	—	—	—	3	3	6	11	12	4	6	—	1	—	—	—	—	—	—	—	46
42 "	—	—	—	—	3	11	17	13	18	5	3	1	1	—	—	—	—	—	—	72
43 "	—	—	—	2	4	7	14	19	17	29	20	6	3	—	—	—	—	—	—	121
44 "	—	—	—	—	—	4	17	22	34	47	20	7	5	3	—	—	1	—	—	160
45 "	—	—	—	—	—	2	7	12	33	51	46	33	10	10	3	—	2	—	—	209
46 "	—	—	—	—	—	—	4	5	15	42	40	30	32	28	10	2	3	—	—	212
47 "	—	—	—	—	—	—	1	2	7	17	26	32	23	30	12	6	6	1	—	163
48 "	—	—	—	—	—	—	—	2	2	11	6	15	24	38	15	12	4	3	1	133
49 "	—	—	—	—	—	—	—	—	—	7	3	11	9	16	9	7	2	1	—	65
50 "	—	—	—	—	—	—	—	—	1	1	1	—	1	3	10	9	1	2	3	37
51 "	—	—	—	—	—	—	—	1	—	—	—	—	1	1	—	2	4	3	4	17
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	4	1	—	7
53 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	2
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	3	7	12	27	38	85	90	135	214	170	129	112	122	68	40	28	16	10	1314

TABLE 41. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 9·5—10·5 years																	Totals
	29·5	31·5	33·5	35·5	37·5	39·5	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	
34½	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
35 "	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	2
36 "	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	3
37 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	2
39 "	—	—	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	4
40 "	—	—	2	1	—	1	1	—	1	—	—	—	—	—	—	—	—	6
41 "	—	—	—	1	6	8	4	3	1	—	—	—	—	—	—	—	—	23
42 "	—	—	1	1	3	5	10	7	7	1	1	3	—	—	—	—	—	39
43 "	—	—	1	—	3	6	6	8	8	4	3	—	—	1	—	—	—	40
44 "	—	—	1	—	4	4	13	15	22	12	6	4	5	1	1	—	—	88
45 "	—	—	—	—	2	6	6	22	30	22	24	10	10	—	—	—	—	132
46 "	—	—	—	—	—	4	3	10	25	26	30	30	12	3	4	—	1	148
47 "	—	—	—	—	—	—	1	3	23	26	40	38	26	16	13	5	2	193
48 "	—	—	—	—	—	—	1	3	11	9	20	30	45	32	21	12	6	193
49 "	—	—	—	—	—	—	—	—	1	5	14	13	34	25	27	24	11	161
50 "	—	—	—	—	—	—	—	—	—	1	6	6	13	25	21	13	10	112
51 "	—	—	—	—	—	—	—	—	1	—	—	—	6	3	11	11	10	58
52 "	—	—	—	—	—	—	—	1	—	2	—	1	1	3	1	5	7	35
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	25
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	7
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	3
56 "	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2	2
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	3
58 "	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	2
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	1	—	5	6	22	36	45	74	131	107	147	135	153	112	100	74	48	1284

TABLE 42. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 10·5—11·5 years																	Totals
	39·5	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	63·5	65·5	67·5	69·5	71·5	
35½	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
37 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—
39 "	—	1	2	—	1	—	—	—	—	1	—	—	—	—	—	—	—	5
40 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
41 "	1	1	1	2	—	2	1	—	—	—	—	—	—	—	—	—	—	8
42 "	2	3	1	2	1	—	2	—	1	—	—	—	—	—	—	—	—	12
43 "	—	1	5	5	1	2	3	—	1	—	—	—	—	—	—	—	—	18
44 "	—	2	4	2	7	5	5	1	2	1	—	—	—	—	—	—	—	30
45 "	1	—	5	6	12	8	10	8	3	—	2	—	—	—	—	—	—	55
46 "	—	1	1	4	15	17	13	14	6	7	5	3	1	—	—	—	—	87
47 "	—	—	—	2	7	16	18	18	26	12	7	9	4	1	1	—	—	121
48 "	—	—	1	—	4	13	19	26	56	29	15	7	9	—	—	1	—	180
49 "	—	—	2	—	2	6	14	16	24	34	23	18	16	8	—	3	3	170
50 "	—	—	—	—	—	4	8	19	28	20	34	15	6	10	2	—	1	153
51 "	—	—	—	—	—	—	1	4	5	11	25	22	27	18	8	6	2	144
52 "	—	—	—	1	1	—	1	1	—	3	4	8	12	16	6	15	6	86
53 "	—	—	—	—	—	—	—	—	1	2	4	10	9	6	7	9	4	59
54 "	—	—	—	—	—	—	—	—	—	3	2	3	5	2	5	5	4	36
55 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	12
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	6
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	4	10	22	24	52	69	92	96	143	127	106	108	98	63	29	51	34	1187

TABLE 43. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 11·5—12·5 years																	Totals
	39·5	42·5	45·5	48·5	51·5	54·5	57·5	60·5	63·5	66·5	69·5	72·5	75·5	78·5	81·5	84·5	87·5	
34 ⁷ / ₈	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
40 "	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
43 "	—	1	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	5
44 "	—	—	3	2	6	2	—	—	—	—	—	—	—	—	—	—	—	13
45 "	—	—	3	6	5	4	2	—	—	—	—	—	—	—	—	—	—	20
46 "	—	1	3	6	16	2	1	2	—	—	—	—	—	—	—	1	—	32
47 "	—	1	3	12	10	18	16	3	2	—	—	—	—	—	—	—	—	65
48 "	—	—	—	6	10	23	19	11	11	2	1	—	—	—	—	—	—	83
49 "	—	—	—	3	12	29	29	26	23	8	4	5	—	—	—	—	—	139
50 "	—	—	2	3	6	20	29	43	35	17	3	4	—	—	—	—	—	162
51 "	—	—	—	—	3	13	24	37	40	18	15	2	6	1	1	—	1	161
52 "	—	—	1	1	—	8	5	21	37	25	26	20	7	1	1	—	—	153
53 "	—	—	—	—	—	1	4	12	21	30	32	12	7	1	1	—	—	133
54 "	—	—	1	—	—	1	—	3	14	14	15	13	14	9	5	—	1	92
55 "	—	—	—	—	—	—	—	2	7	13	14	10	5	5	5	1	3	65
56 "	—	—	—	—	—	—	—	—	1	4	6	10	3	2	2	5	2	41
57 "	—	—	—	—	—	—	—	—	3	1	1	5	3	2	2	3	3	27
58 "	—	—	—	—	—	—	—	1	—	—	—	—	—	2	1	1	—	9
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3
67 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	6	20	41	70	121	129	159	186	125	115	77	64	32	19	13	10	1214

TABLE 44. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 12·5—13·5 years																	Totals
	41·5	45·5	49·5	53·5	57·5	61·5	65·5	69·5	73·5	77·5	81·5	85·5	89·5	93·5	97·5	101·5	105·5	
41 ⁷ / ₈	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
42 "	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
45 "	—	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	4
46 "	—	—	1	5	2	1	—	—	—	—	—	—	—	—	—	—	—	9
47 "	2	3	6	9	4	4	2	—	—	—	—	—	—	—	—	—	—	30
48 "	—	—	4	6	10	7	3	—	—	—	—	—	—	—	—	—	—	30
49 "	—	2	3	12	10	26	6	2	—	—	1	—	1	—	—	—	—	63
50 "	—	—	2	5	21	26	12	7	1	—	—	—	—	—	—	1	—	75
51 "	—	—	—	3	12	26	24	15	6	3	—	1	1	—	—	—	—	91
52 "	—	—	—	2	12	24	20	32	11	8	4	1	—	—	—	—	—	114
53 "	—	—	—	—	4	11	26	29	22	13	2	3	—	—	—	—	—	110
54 "	—	—	—	—	4	6	18	27	29	14	8	4	1	—	—	—	—	111
55 "	—	—	—	—	—	3	12	14	18	29	11	9	4	3	2	—	—	105
56 "	—	—	—	1	—	—	3	8	18	18	13	13	10	1	1	—	—	86
57 "	—	—	—	—	—	1	—	4	9	15	7	12	4	1	2	—	—	55
58 "	—	—	—	—	—	—	—	1	1	5	3	6	8	3	1	1	2	34
59 "	—	—	—	—	—	—	—	—	2	2	1	2	6	—	1	1	2	17
60 "	—	—	—	—	—	—	—	—	—	1	2	2	4	2	2	—	1	14
61 "	—	—	—	—	—	—	—	—	1	—	—	1	1	2	—	2	1	8
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	3	—	—	—	4
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2
64 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
66 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
67 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	5	9	20	45	79	136	126	139	120	108	53	54	44	12	9	3	6	976

TABLE 45. *Glasgow. Height and Weight of Girls of Group A.*

Height	Weight of Girls of 13·5—14·5 years																		Totals	
	41·5—	—	49·5—	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—		113·5—
46½—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
47 "	1	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
48 "	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
49 "	—	—	—	2	3	3	2	—	—	—	—	—	—	—	—	—	—	—	—	10
50 "	—	—	2	2	3	3	—	3	—	—	—	—	—	—	—	—	—	—	—	13
51 "	—	—	—	2	6	3	2	4	1	—	—	—	—	—	—	—	—	—	—	18
52 "	—	—	—	2	—	5	9	10	4	1	2	—	—	—	—	—	—	—	—	33
53 "	—	—	—	2	2	3	2	6	4	4	3	—	1	—	—	—	—	—	—	27
54 "	—	—	—	—	1	5	1	10	8	5	5	1	—	1	—	—	—	—	—	37
55 "	—	—	—	—	—	—	1	9	11	7	5	3	1	—	—	—	—	—	—	37
56 "	—	—	—	—	—	2	1	7	2	3	4	6	6	—	—	—	1	—	—	32
57 "	—	—	—	—	—	—	—	—	3	5	4	3	7	2	—	1	—	—	—	25
58 "	—	—	—	—	—	—	—	1	—	1	2	3	7	4	1	1	—	—	—	20
59 "	—	—	—	—	—	1	—	—	—	1	4	3	4	3	—	1	1	2	2	22
60 "	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	3
61 "	—	—	—	—	—	—	—	—	—	—	—	1	2	1	1	—	—	—	—	5
62 "	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	2
Totals	1	—	3	13	15	25	20	50	34	28	30	22	28	11	2	3	2	2	2	291

TABLE 46. *Glasgow. Height and Weight of Girls of Group B.*

Height	Weight of Girls of 5·5—6·5 years																Totals	
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—		59·5—
21 ⁷ / ₈ —	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
30 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
31 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
32 "	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
33 "	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
34 "	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
35 "	—	2	4	3	5	—	—	—	—	—	—	—	—	—	—	—	—	14
36 "	1	2	6	10	9	—	—	—	—	—	—	—	—	—	—	—	—	28
37 "	—	3	9	14	15	6	1	1	—	—	—	—	—	—	—	—	—	49
38 "	—	1	9	26	24	10	5	3	—	—	—	—	—	—	—	—	—	78
39 "	1	2	3	18	38	38	33	14	4	—	1	—	—	—	—	—	—	152
40 "	—	1	3	14	31	53	61	30	13	2	3	—	—	—	—	—	—	211
41 "	—	—	—	6	18	48	39	45	22	6	3	1	—	—	—	—	—	188
42 "	—	—	—	1	4	9	31	53	34	14	5	—	—	—	—	—	—	151
43 "	—	—	1	—	2	6	12	37	36	22	21	4	—	1	—	—	—	142
44 "	—	—	—	—	1	—	4	14	17	13	12	8	—	—	1	—	—	70
45 "	—	—	—	—	1	1	4	2	5	2	7	2	—	—	—	—	—	24
46 "	—	—	—	—	—	—	2	1	—	1	2	2	2	—	—	—	—	10
47 "	—	—	—	—	—	—	1	1	2	1	—	1	1	1	—	—	—	8
48 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	3
54 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	4	17	37	93	148	171	193	201	133	61	55	18	3	2	3	1	1	1141

TABLE 47. *Glasgow. Height and Weight of Girls of Group B.*

	Weight of Girls of 6·5—7·5 years																			
Height	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	Totals
32½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
33 "	—	2	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
34 "	—	4	2	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	9
35 "	—	3	—	7	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	11
36 "	—	1	—	5	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
37 "	—	—	3	5	3	4	2	1	—	—	—	—	—	—	—	—	—	—	—	18
38 "	—	1	3	7	10	10	4	3	1	—	—	—	—	—	—	—	—	—	—	39
39 "	—	2	—	8	17	25	15	7	3	—	—	—	—	—	—	—	—	—	—	77
40 "	—	—	1	3	20	36	28	20	8	2	1	1	1	—	—	—	—	—	—	121
41 "	—	1	—	4	10	28	41	39	21	12	5	3	2	—	—	—	—	—	—	166
42 "	—	—	—	—	5	15	37	60	47	21	12	2	2	—	—	—	—	—	—	201
43 "	—	—	—	—	1	2	22	52	57	45	23	7	3	—	1	—	—	—	—	213
44 "	—	—	—	—	—	3	6	30	56	46	36	20	8	3	2	—	—	—	—	210
45 "	—	—	—	—	—	—	1	9	14	30	31	23	15	4	2	—	—	—	—	129
46 "	—	—	—	—	—	—	1	2	5	9	18	18	13	3	5	2	—	—	—	76
47 "	—	—	—	—	—	—	—	—	3	1	6	9	7	7	1	2	1	1	—	38
48 "	—	—	—	—	—	—	—	—	—	1	1	—	3	2	4	1	—	1	—	13
49 "	—	—	—	—	—	—	—	—	—	2	—	—	—	1	1	—	—	—	1	5
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	3
Totals	1	14	9	43	71	124	157	224	215	170	133	83	54	21	17	5	1	3	1	1346

TABLE 48. *Glasgow. Height and Weight of Girls of Group B.*

		Weight of Girls of 7·5—8·5 years																					
Height		27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	—	69·5—	Totals
33½—		—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
34 "		—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
35 "		—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
36 "		—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "		—	1	1	3	1	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	8
38 "		—	—	—	3	2	3	1	2	1	1	—	—	—	—	—	—	—	—	—	—	—	13
39 "		—	—	—	2	3	11	6	7	4	1	—	—	—	—	—	—	—	—	—	—	—	34
40 "		1	—	—	1	5	8	14	13	2	—	1	—	—	—	—	—	—	—	—	—	—	45
41 "		—	—	—	1	5	13	19	23	12	7	3	2	2	—	—	—	—	—	—	—	—	87
42 "		—	—	1	2	3	6	22	35	30	16	9	2	1	—	—	—	—	—	—	—	—	127
43 "		—	—	—	—	3	6	8	39	31	36	30	15	5	2	2	—	—	—	—	—	—	177
44 "		—	—	—	—	1	2	3	24	51	31	49	28	11	6	—	1	1	—	—	—	—	208
45 "		—	—	—	1	—	1	3	6	22	48	50	49	24	16	7	1	—	1	—	—	—	229
46 "		—	—	—	—	—	—	1	6	7	23	38	26	26	27	9	2	1	—	—	—	—	166
47 "		—	—	—	—	—	—	1	—	1	6	12	25	31	24	16	10	2	4	—	—	—	132
48 "		—	—	—	—	—	—	—	—	—	—	5	9	12	14	11	9	4	3	2	—	—	69
49 "		—	—	—	—	—	—	—	—	—	—	1	2	1	6	12	4	4	3	—	—	—	33
50 "		—	—	—	—	—	—	—	—	—	—	—	1	1	1	4	3	5	—	—	1	—	16
51 "		—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	—	—	—	4
52 "		—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
53 "		—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals		1	1	4	15	25	51	78	135	162	170	198	159	115	96	61	30	19	12	3	—	1	1356

TABLE 49. Glasgow. Height and Weight of Girls of Group B.

Height	Weight of Girls of 8·5—9·5 years																	Totals
	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	
29½	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
33 "	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
34 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
36 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	—	2	2	1	1	1	—	—	—	—	—	—	—	—	—	7
39 "	—	—	—	—	2	3	4	—	1	—	—	—	—	—	—	—	—	10
40 "	—	—	—	—	5	6	7	1	2	—	—	1	—	—	—	—	—	22
41 "	—	—	—	1	1	8	14	5	5	1	1	1	—	—	—	—	—	37
42 "	—	—	—	—	3	7	14	19	16	8	—	—	—	1	—	—	—	68
43 "	—	—	—	—	—	3	10	14	15	11	8	1	—	1	1	—	—	64
44 "	1	—	—	—	1	9	11	22	28	37	21	10	8	4	—	1	—	153
45 "	—	—	—	—	1	2	7	10	25	44	33	25	12	10	5	—	1	177
46 "	—	—	—	—	—	1	2	10	19	34	50	36	23	22	3	6	1	207
47 "	—	—	—	—	—	1	1	7	7	15	27	36	53	23	14	6	6	199
48 "	—	—	—	—	—	2	2	5	11	11	19	36	24	33	14	9	7	176
49 "	—	—	—	—	—	1	—	—	—	2	8	11	14	27	24	8	10	115
50 "	—	—	—	—	—	—	—	—	—	—	2	8	14	10	17	8	2	63
51 "	—	—	—	—	—	—	—	—	—	—	—	—	1	2	7	7	6	35
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	1	12
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	3
55 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	2	1	1	3	15	42	74	92	123	163	161	142	156	132	98	58	44	1360

TABLE 50. Glasgow. Height and Weight of Girls of Group B.

Height	Weight of Girls of 9·5—10·5 years																	Totals
	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	
36½	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
38 "	1	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	3
39 "	—	—	—	—	2	1	—	—	1	—	—	—	—	—	—	—	—	5
40 "	—	—	—	—	2	1	—	—	—	—	—	—	—	1	—	—	—	4
41 "	—	1	—	1	1	1	3	2	1	—	—	—	—	—	—	—	—	10
42 "	—	—	1	2	7	5	4	1	1	—	—	—	—	—	—	—	—	21
43 "	—	—	2	2	4	5	9	8	7	2	—	2	—	—	—	—	—	41
44 "	—	—	—	4	3	13	15	12	9	3	2	—	1	—	—	—	—	62
45 "	—	—	—	1	2	3	17	30	23	22	8	7	2	1	1	—	—	117
46 "	—	—	—	—	2	2	17	22	28	22	20	13	8	4	2	—	2	142
47 "	—	1	—	1	1	3	3	13	39	41	39	32	9	6	5	1	—	194
48 "	—	—	—	—	—	1	6	9	11	28	30	46	31	14	4	7	1	190
49 "	—	—	—	—	—	—	1	3	3	11	15	44	41	29	17	6	8	185
50 "	—	—	—	—	—	—	—	1	3	4	9	18	26	30	21	10	6	136
51 "	—	—	—	—	—	—	1	2	1	1	3	9	15	17	19	14	7	110
52 "	—	—	—	—	—	—	—	—	—	—	—	3	2	8	9	9	7	60
53 "	—	—	—	—	—	—	—	—	—	—	—	—	2	1	2	7	1	24
54 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	2	8
55 "	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2	8
56 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	6
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	—	4
Totals	2	3	4	11	24	35	76	103	127	136	126	176	138	116	81	55	35	1333

TABLE 51. *Glasgow. Height and Weight of Girls of Group B.*

Height	Weight of Girls of 10·5—11·5 years																				Totals
	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	75·5—	
35½	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
38 "	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
39 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 "	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
41 "	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
42 "	—	—	—	1	1	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	6
43 "	—	—	1	—	1	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
44 "	—	1	—	—	4	6	4	5	2	—	—	—	—	—	—	—	—	—	—	—	22
45 "	—	—	1	2	5	5	6	7	7	2	1	1	—	—	—	—	—	—	—	—	37
46 "	—	—	2	—	4	9	9	10	12	11	5	4	3	—	—	—	—	—	—	—	69
47 "	—	—	—	1	2	13	12	16	19	13	13	8	6	2	1	1	—	—	—	—	107
48 "	—	—	—	—	—	7	14	14	18	40	29	15	10	8	—	1	1	—	—	1	158
49 "	—	—	1	—	1	1	5	11	18	29	33	21	24	10	8	6	1	1	—	—	171
50 "	—	—	—	—	1	2	—	5	11	18	35	32	39	17	3	4	5	2	—	1	176
51 "	—	—	—	—	—	—	—	1	4	16	17	20	24	15	21	7	8	4	1	2	144
52 "	—	—	—	—	—	—	—	—	—	2	8	12	25	25	9	16	12	4	3	4	122
53 "	—	—	—	—	—	—	—	1	—	1	3	3	5	13	10	13	11	11	5	2	89
54 "	—	—	—	—	—	—	—	—	—	—	1	1	1	1	8	9	8	7	3	3	55
55 "	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	5	6	3	5	33
56 "	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	2	—	1	1	2	17
57 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	1	1	1	2	8
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
62 "	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	2	4	7	5	21	51	52	73	94	133	146	118	138	96	61	65	53	36	21	18	1244

TABLE 52. *Glasgow. Height and Weight of Girls of Group B.*

Height	Weight of Girls of 11·5—12·5 years																			Totals
	39·5—	42·5—	45·5—	48·5—	51·5—	54·5—	57·5—	60·5—	63·5—	66·5—	69·5—	72·5—	75·5—	78·5—	81·5—	84·5—	87·5—	90·5—	93·5—	
39½	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
43 "	—	1	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
44 "	—	1	3	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	8
45 "	—	—	3	6	4	2	—	1	—	—	—	—	—	—	—	—	—	—	—	16
46 "	—	1	—	3	8	7	4	—	1	—	—	—	—	—	—	—	—	—	—	24
47 "	1	1	1	8	11	12	6	4	1	2	—	—	—	—	—	—	—	—	—	47
48 "	—	—	2	4	17	17	18	9	5	1	—	3	1	—	—	—	—	—	—	77
49 "	1	—	—	1	5	31	20	20	19	5	4	—	—	—	—	—	—	—	—	106
50 "	—	—	—	—	4	13	39	35	31	20	5	1	2	1	—	—	—	—	—	151
51 "	—	—	—	1	1	14	30	37	33	22	24	4	2	2	—	—	—	—	—	170
52 "	—	—	—	—	1	5	13	30	43	28	29	12	12	5	2	2	—	—	—	182
53 "	—	—	—	—	—	1	3	14	24	21	26	20	12	4	6	2	—	—	—	133
54 "	—	—	—	—	—	—	4	7	8	19	15	22	16	13	7	—	—	—	—	111
55 "	—	—	—	1	1	—	2	2	4	8	20	12	15	12	4	2	3	1	—	88
56 "	—	—	—	—	—	—	—	2	2	1	5	8	4	13	7	5	2	2	2	55
57 "	—	—	—	—	—	—	—	—	—	1	2	4	5	5	6	1	—	6	2	32
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	3	3	—	2	3	1	12
59 "	—	—	—	—	—	—	1	—	—	—	—	—	2	1	2	1	4	—	4	19
60 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	1	1	1	5
61 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1	—	3
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
69 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	3	4	14	29	54	103	140	162	171	129	131	86	74	59	36	13	12	12	11	1252

TABLE 53. *Glasgow. Height and Weight of Girls of Group B.*

Height	Weight of Girls of 12·5—13·5 years																						Totals
	41·5—	45·5—	49·5—	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—	113·5—	117·5—	121·5—	125·5—	
42½	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
45 "	—	1	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
46 "	—	1	3	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
47 "	—	2	4	5	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	16
48 "	—	—	3	5	11	4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
49 "	—	—	2	7	13	13	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	41
50 "	—	—	—	4	19	16	9	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	54
51 "	—	—	1	5	21	31	20	17	7	—	—	—	—	—	—	—	—	—	—	—	—	—	102
52 "	—	—	—	2	9	32	31	29	11	14	1	1	—	—	—	—	—	—	—	—	—	—	130
53 "	—	—	—	2	5	23	33	31	21	12	6	3	1	—	—	—	—	—	—	—	—	—	137
54 "	—	—	1	—	1	14	17	40	35	22	12	8	4	1	—	—	—	—	—	—	—	—	155
55 "	—	—	—	1	2	5	14	30	43	24	16	10	4	1	—	—	1	—	—	—	—	—	151
56 "	—	—	—	—	—	1	3	10	15	22	27	8	8	4	—	1	1	—	—	—	—	—	100
57 "	—	—	—	1	—	2	1	9	14	13	17	11	17	6	5	1	—	1	—	—	—	—	98
58 "	—	—	—	—	—	—	—	3	2	10	7	13	14	2	4	4	—	—	—	1	—	1	61
59 "	—	—	—	—	—	—	1	1	5	6	4	6	10	4	3	3	1	—	—	—	—	—	44
60 "	—	—	—	—	—	—	—	—	—	1	1	2	3	4	4	2	3	2	—	—	—	—	22
61 "	—	—	—	—	1	—	—	—	—	—	1	1	—	1	—	1	—	—	2	1	—	—	8
62 "	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	1	—	—	—	—	—	3
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
66 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
67 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
68 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
69 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	4	18	34	87	144	133	180	154	125	93	63	63	23	16	12	7	3	3	2	—	1	1166

TABLE 54. *Glasgow. Height and Weight of Girls of Group B.*

Height	Weight of Girls of 13·5—14·5 years																		Totals	
	45·5—	49·5—	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—	—		121·5—
45½—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
46 „	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
47 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48 „	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	2
49 „	—	—	1	—	2	—	—	2	—	—	—	—	—	—	—	—	—	—	—	5
50 „	—	—	1	1	4	—	2	—	—	—	—	—	—	—	—	—	—	—	—	8
51 „	—	—	2	2	6	1	2	2	1	—	—	—	—	—	—	—	—	—	—	16
52 „	—	—	—	2	2	6	6	5	1	—	—	—	—	—	—	—	—	—	—	22
53 „	—	—	1	1	3	8	11	8	2	4	—	1	—	—	—	—	—	—	—	39
54 „	—	—	1	—	1	5	11	9	6	2	5	1	2	1	—	—	—	—	—	44
55 „	—	—	—	1	—	4	4	11	9	5	4	1	1	—	—	—	—	—	—	40
56 „	—	—	1	—	—	1	2	13	11	11	9	5	—	2	—	—	1	—	—	56
57 „	—	1	—	1	1	—	4	6	6	12	9	5	2	7	1	—	1	—	—	56
58 „	—	—	—	1	—	—	1	2	5	9	9	7	5	5	1	—	2	—	—	47
59 „	—	—	—	—	—	—	1	—	—	1	7	5	4	1	2	1	1	—	1	25
60 „	—	—	—	—	—	—	—	—	1	1	2	3	4	1	2	1	1	—	1	17
61 „	—	—	—	—	1	—	—	—	1	—	2	1	3	3	2	2	—	—	—	15
62 „	—	—	—	—	—	—	—	1	—	—	—	1	—	1	1	1	2	—	—	7
63 „	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	1	7	11	21	25	44	59	44	45	47	31	21	21	10	5	8	—	2	403

TABLE 55. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 5·5—6·5 years															Totals	
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	—		63·5—
31½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
32 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
33 "	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
34 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
35 "	1	—	—	3	1	—	—	—	—	—	—	—	—	—	—	—	5
36 "	—	—	2	2	1	1	—	1	—	—	—	—	—	—	—	—	7
37 "	—	1	4	6	6	4	1	—	—	—	—	—	—	—	—	—	22
38 "	—	—	5	4	8	14	7	4	—	—	—	1	—	—	—	—	43
39 "	—	—	2	7	12	18	5	8	2	1	1	—	—	—	—	—	56
40 "	—	—	1	1	7	18	19	13	7	3	—	—	—	—	—	—	69
41 "	—	—	—	—	7	13	21	26	10	2	—	1	—	—	—	—	80
42 "	—	—	—	—	1	1	11	15	15	6	5	2	—	—	—	—	56
43 "	—	—	—	—	—	1	6	11	10	9	9	1	2	—	—	—	49
44 "	—	—	—	—	1	1	1	—	9	5	7	4	2	—	—	—	30
45 "	—	—	—	—	—	—	—	1	—	1	3	1	2	2	—	—	10
46 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
47 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	1	14	24	44	71	71	79	53	27	26	10	6	2	—	1	431

TABLE 56. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 6·5—7·5 years															Totals	
	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—		59·5—
33½—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
34 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
35 "	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
36 "	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	3
37 "	—	—	3	—	3	1	—	—	—	—	—	—	—	—	—	—	7
38 "	1	2	1	1	6	4	3	1	—	—	—	—	—	—	—	—	19
39 "	—	2	1	7	12	7	3	—	1	1	—	—	—	—	—	—	34
40 "	—	—	2	4	9	12	11	6	4	1	—	—	1	—	—	—	50
41 "	—	1	1	7	10	18	38	18	5	7	—	—	—	—	—	—	105
42 "	1	—	—	—	3	27	21	20	20	5	3	2	—	—	—	—	102
43 "	—	—	1	3	2	10	15	29	19	12	8	3	—	—	—	1	103
44 "	—	—	—	—	—	2	9	18	14	17	7	7	4	1	—	—	79
45 "	—	—	—	—	—	—	2	5	9	13	16	12	3	—	—	1	61
46 "	—	—	—	—	—	—	—	1	1	2	5	7	5	4	2	—	27
47 "	—	—	—	—	—	—	—	—	—	1	3	4	3	2	—	1	14
48 "	—	—	—	—	—	—	—	1	—	—	—	2	1	1	3	1	9
49 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	2	7	10	23	45	81	103	100	73	59	42	37	17	8	6	4	617

332 *Height and Weight of School Children in Glasgow*

TABLE 57. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 7·5—8·5 years																	Totals
	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	
36½	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	3
37 "	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
38 "	1	—	—	1	1	—	—	—	—	—	1	—	—	—	—	—	—	4
39 "	—	1	3	1	2	2	—	1	2	—	—	—	—	—	—	—	—	12
40 "	—	2	—	7	2	4	3	2	2	1	—	—	—	—	—	—	—	21
41 "	—	—	1	8	8	8	2	2	6	2	—	2	—	—	—	—	—	39
42 "	—	—	—	5	15	13	13	8	8	3	1	—	—	—	—	—	—	66
43 "	—	—	1	3	8	12	31	16	19	3	3	1	—	—	—	—	—	97
44 "	—	—	—	—	—	19	26	30	28	13	7	2	1	1	—	—	—	127
45 "	—	—	—	—	2	4	13	17	37	18	16	8	2	1	—	—	—	118
46 "	—	—	—	—	—	—	1	9	17	17	14	13	5	2	2	—	—	80
47 "	—	—	—	—	—	2	—	1	7	8	15	17	9	4	1	1	—	66
48 "	—	—	—	—	—	—	—	—	4	1	4	6	10	3	3	3	2	36
49 "	—	—	—	—	—	—	—	1	—	—	1	1	3	3	2	1	1	13
50 "	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	1	—	3
51 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
52 "	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	2
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
54 "	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	1	6	5	25	40	65	90	87	129	66	62	50	31	14	9	6	4	691

TABLE 58. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 8·5—9·5 years																	Totals
	29·5—	—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	
32½	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
38 "	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
39 "	—	—	—	3	2	1	1	—	—	—	—	—	—	—	—	—	—	7
40 "	—	—	—	1	1	1	2	—	1	—	—	—	—	—	—	—	—	6
41 "	1	—	—	—	2	3	1	2	—	—	—	—	—	—	—	—	—	9
42 "	—	—	—	—	1	2	4	3	1	1	2	—	—	—	—	—	—	14
43 "	—	—	—	—	1	8	8	9	4	6	—	1	—	—	—	—	—	37
44 "	—	—	—	1	—	4	6	9	16	9	6	3	1	1	—	—	—	56
45 "	—	—	—	—	2	2	9	11	20	20	17	9	4	1	—	—	—	95
46 "	—	—	—	—	—	1	6	6	21	31	20	12	10	3	—	—	1	111
47 "	—	—	—	—	—	—	—	5	7	21	17	20	14	9	2	6	—	102
48 "	—	—	—	—	—	—	—	—	5	6	14	10	17	16	10	6	—	89
49 "	—	—	—	—	—	—	1	—	2	2	3	10	12	9	6	7	2	59
50 "	—	—	—	—	—	—	—	—	1	1	1	—	2	5	6	5	3	27
51 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2	2	5	16
52 "	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	4
53 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	2
Totals	1	—	2	2	11	23	38	46	78	97	80	66	61	46	26	27	10	639

TABLE 59. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 9·5—10·5 years																	Totals
	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	
40½	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41 "	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	3
42 "	1	—	1	3	1	1	—	—	—	—	—	—	—	—	—	—	—	7
43 "	—	—	2	2	3	1	1	1	—	—	—	—	—	—	—	—	—	10
44 "	—	—	2	6	5	12	5	3	2	1	—	—	—	—	—	—	—	36
45 "	1	—	1	1	8	6	8	9	6	1	—	—	—	—	—	—	—	41
46 "	—	—	1	2	3	7	10	10	12	2	2	—	1	1	—	1	—	52
47 "	—	1	—	—	1	5	13	23	11	17	9	8	3	2	—	—	—	93
48 "	—	1	—	—	—	3	4	10	14	12	25	8	7	—	2	2	—	88
49 "	—	—	—	—	—	—	3	5	11	15	13	16	14	4	7	1	—	89
50 "	—	—	—	—	—	—	—	1	6	6	14	15	15	4	1	3	1	68
51 "	—	—	—	—	—	—	—	—	1	2	4	6	16	6	4	3	2	47
52 "	—	—	—	—	—	—	—	—	1	—	3	3	1	6	3	4	—	27
53 "	—	—	—	—	—	—	—	—	—	1	—	—	1	2	1	1	2	10
54 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3	3
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	4
56 "	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	3
57 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
59 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	2	2	10	14	21	36	45	62	64	58	70	57	60	25	18	15	6	585

TABLE 60. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 10·5—11·5 years																	Totals
	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	
35½	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
39 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
40 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
42 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	—	—	1	2	—	—	1	—	—	—	—	—	—	—	—	—	—	4
44 "	—	—	4	2	2	1	—	—	—	—	1	—	—	—	—	—	—	10
45 "	1	—	1	2	4	1	—	1	—	1	—	—	1	—	—	—	—	12
46 "	—	—	1	—	2	4	7	6	2	3	—	1	1	1	—	—	—	28
47 "	—	—	—	1	5	9	6	12	2	7	4	1	—	—	—	—	—	47
48 "	—	—	—	2	9	7	7	16	12	11	9	—	2	2	—	—	—	77
49 "	—	—	—	1	1	2	6	17	8	23	11	8	6	—	—	1	—	84
50 "	—	—	—	—	2	1	5	8	9	14	13	14	5	11	2	2	1	89
51 "	—	—	—	—	—	1	—	2	6	13	20	9	16	10	5	2	1	87
52 "	—	—	—	—	—	—	—	2	7	10	14	7	3	5	5	2	1	60
53 "	—	—	—	—	—	—	—	2	—	4	4	7	4	4	4	3	4	46
54 "	—	—	—	—	—	—	—	—	1	1	—	2	3	4	3	2	—	17
55 "	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	2	2	9
56 "	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	1	—	7
57 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	2
58 "	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	2	4
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
60 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
68 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	2	—	3	8	10	27	30	30	60	44	83	75	54	44	34	21	18	589

TABLE 61. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 11·5—12·5 years																	Totals
	45·5	48·5	51·5	54·5	57·5	60·5	63·5	66·5	69·5	72·5	75·5	78·5	81·5	84·5	87·5	90·5	93·5	
42½	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
45 "	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
46 "	—	1	3	2	—	—	1	—	—	—	—	—	—	—	—	—	—	7
47 "	—	3	3	6	4	1	1	—	—	—	—	—	—	—	—	—	—	18
48 "	—	2	4	8	8	5	4	1	—	—	—	—	—	—	—	—	—	32
49 "	1	—	2	13	15	10	5	3	3	—	—	—	—	—	—	—	—	52
50 "	—	1	1	5	18	17	15	7	5	1	1	—	—	—	—	—	—	71
51 "	—	—	—	3	11	20	16	11	6	3	—	1	—	—	—	—	—	71
52 "	—	—	—	1	7	21	14	20	16	5	3	5	2	1	—	1	—	96
53 "	—	—	—	—	2	7	15	16	10	10	9	2	5	2	1	—	—	79
54 "	—	—	—	—	—	2	2	5	10	11	14	6	6	4	—	2	—	64
55 "	—	—	—	—	—	—	1	3	5	10	9	8	5	1	2	—	—	44
56 "	—	—	—	—	—	—	—	—	6	6	4	3	4	1	2	2	—	29
57 "	—	—	—	—	—	—	—	1	—	—	1	3	1	—	2	—	—	9
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	3	2	1	3	9
59 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	3
60 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
62 "	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	2
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
64 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	3	9	14	39	66	83	74	68	62	46	41	23	23	12	10	7	3	595

TABLE 62. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 12·5—13·5 years																	Totals
	41·5	45·5	49·5	53·5	57·5	61·5	65·5	69·5	73·5	77·5	81·5	85·5	89·5	93·5	97·5	101·5	105·5	
38½	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
40 "	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
41 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46 "	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
47 "	—	—	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	4
48 "	—	1	4	1	3	2	—	—	2	—	1	—	—	—	—	—	—	14
49 "	—	—	1	2	1	6	1	2	2	2	—	—	—	—	—	—	—	17
50 "	—	—	—	6	9	10	2	4	2	—	1	—	—	—	—	—	—	34
51 "	—	—	—	1	4	11	10	14	3	1	1	—	—	—	—	—	—	45
52 "	—	—	—	—	7	9	20	7	11	1	—	1	—	—	—	—	—	56
53 "	—	—	—	—	1	9	14	17	14	4	3	3	2	1	—	—	—	63
54 "	—	—	—	—	—	7	6	24	18	13	3	2	—	1	—	—	1	75
55 "	—	—	—	—	1	1	2	9	19	11	5	6	1	1	1	—	—	57
56 "	—	—	—	—	—	1	1	2	7	5	16	9	4	1	—	—	—	46
57 "	—	—	—	—	1	—	—	2	5	10	10	12	6	3	1	1	—	52
58 "	—	—	—	—	—	—	—	—	4	5	7	8	1	2	2	—	—	29
59 "	—	—	—	—	1	—	—	—	1	2	5	1	5	1	3	1	—	20
60 "	—	—	—	—	—	—	—	—	3	1	—	1	1	3	1	1	2	16
61 "	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	—	4
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	2
Totals	1	2	5	10	29	58	57	81	84	55	48	46	25	15	9	7	4	544

TABLE 63. *Glasgow. Height and Weight of Girls of Group C.*

Height	Weight of Girls of 13·5—14·5 years															Totals	
	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—		113·5—
47½—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
48 "	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
49 "	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
50 "	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
51 "	—	1	1	2	2	—	—	—	—	—	—	—	—	—	—	—	6
52 "	—	—	1	2	2	3	1	2	1	—	1	—	—	—	—	—	13
53 "	—	1	2	3	1	—	1	—	—	—	—	—	—	—	—	—	8
54 "	—	—	1	2	3	7	2	4	1	—	1	—	—	—	—	—	21
55 "	—	—	1	—	4	6	3	6	2	—	—	1	—	—	—	—	23
56 "	—	—	—	1	1	—	6	5	4	2	—	2	2	—	—	—	23
57 "	—	—	—	—	—	3	—	5	2	3	1	1	1	—	1	—	17
58 "	—	—	—	—	—	—	—	3	5	6	2	1	1	—	—	—	18
59 "	—	—	—	—	—	1	—	3	2	2	2	2	2	—	—	3	17
60 "	—	—	—	—	—	—	1	—	—	1	2	2	—	1	—	—	7
61 "	—	—	—	—	—	—	—	—	—	1	—	2	1	1	—	—	5
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	3	5	6	11	13	20	14	28	17	15	10	11	7	2	1	3	166

TABLE 64. *Glasgow Height and Weight of Girls of Group D.*

Height	Weight of Girls of 5·5—6·5 years															Totals	
	27·5—	29·5—	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—		57·5—
34½—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
35 "	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36 "	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	4
37 "	—	1	1	3	4	4	—	1	—	—	—	—	—	—	—	—	14
38 "	2	—	1	3	3	4	—	—	—	—	—	—	—	—	—	—	13
39 "	1	—	2	3	9	12	5	3	—	1	—	—	—	—	—	—	36
40 "	—	—	—	2	9	13	13	13	3	1	—	—	—	—	—	—	54
41 "	—	—	—	—	8	12	21	26	9	4	1	—	—	—	—	—	81
42 "	—	—	—	1	3	3	20	23	16	11	2	1	—	—	—	—	80
43 "	—	—	—	—	1	1	6	6	19	11	4	1	—	—	—	—	49
44 "	—	—	—	—	—	1	4	4	9	5	4	1	1	—	—	—	29
45 "	—	—	—	—	—	—	—	—	3	4	6	3	—	2	—	—	18
46 "	—	—	—	—	—	—	—	—	—	—	1	—	3	1	—	1	6
47 "	—	—	—	—	—	—	—	—	—	1	—	—	2	—	—	—	3
Totals	3	3	7	12	38	50	69	76	59	38	18	6	6	3	—	1	389

TABLE 65. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 6·5—7·5 years															Totals
	31·5—	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	
37½—	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	3
38 "	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	3
39 "	—	—	2	4	2	3	2	—	—	—	—	—	—	—	—	13
40 "	—	1	—	4	4	6	1	—	—	—	—	—	—	—	—	16
41 "	—	2	5	7	8	7	9	3	1	—	—	—	—	—	—	42
42 "	—	—	5	4	20	20	7	13	3	1	—	—	—	—	—	74
43 "	—	—	—	1	7	23	20	21	15	5	1	—	—	—	—	93
44 "	—	—	—	1	3	11	15	18	19	6	1	2	—	—	—	76
45 "	—	—	—	—	—	5	8	14	19	11	5	2	2	—	—	66
46 "	—	—	—	—	1	1	2	6	7	7	5	2	1	1	—	40
47 "	—	—	—	—	—	—	1	1	4	5	5	6	1	2	1	26
48 "	—	—	—	—	—	—	—	1	—	2	—	2	1	—	—	6
49 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
50 "	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2
Totals	2	4	12	23	46	76	65	77	68	37	19	19	6	3	3	460

TABLE 66. *Glasgow. Height and Weight of Girls of Group D*

Height	Weight of Girls of 7·5—8·5 years																	Totals
	33·5—	35·5—	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	
36½—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39 "	—	—	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	4
40 "	1	—	1	1	2	—	—	1	—	—	—	—	—	—	—	—	—	6
41 "	—	3	2	2	6	4	3	2	—	—	—	—	—	—	—	—	—	22
42 "	—	—	4	2	12	12	1	5	—	1	—	—	—	—	—	—	—	37
43 "	—	—	—	2	10	9	7	2	3	—	1	—	—	—	—	—	—	34
44 "	—	—	—	2	11	24	16	19	10	5	1	1	—	—	—	—	—	89
45 "	—	—	—	—	5	15	10	19	15	11	6	2	1	—	—	—	1	85
46 "	—	—	—	—	1	3	9	15	22	17	7	2	2	—	—	—	—	78
47 "	—	—	—	—	—	2	4	7	16	12	6	11	7	3	1	1	—	70
48 "	—	—	—	—	—	1	—	—	3	7	5	9	3	4	4	—	—	36
49 "	—	—	—	—	—	—	—	—	—	—	3	3	4	2	—	1	—	13
50 "	—	—	—	—	—	—	—	—	—	—	2	—	1	2	1	1	—	7
51 "	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	2
55 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1	3	9	11	48	70	50	70	69	53	32	28	18	11	7	3	1	486

TABLE 67. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 8·5—9·5 years																	Totals
	37·5—	39·5—	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	
40 ⁷ / ₈ —	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	2
41 "—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	2
42 "—	1	2	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	7
43 "—	—	1	4	6	5	2	3	—	—	1	—	—	—	—	—	—	—	22
44 "—	—	—	5	6	4	9	4	6	2	—	—	—	—	—	—	—	—	36
45 "—	—	—	—	1	5	5	17	12	13	3	2	—	—	—	—	1	—	59
46 "—	—	—	—	—	1	3	13	18	19	6	6	3	1	—	—	—	—	70
47 "—	—	—	1	1	2	9	16	16	11	10	7	—	2	1	1	—	—	77
48 "—	—	—	—	—	—	1	4	8	12	13	17	8	3	5	2	—	—	73
49 "—	—	—	—	—	—	—	—	1	10	7	15	12	12	2	4	4	1	69
50 "—	—	—	—	—	—	—	—	—	3	7	6	10	5	4	6	4	1	49
51 "—	—	—	—	—	—	—	—	—	2	2	2	1	2	4	1	1	—	15
52 "—	—	—	—	—	—	—	—	—	—	—	1	—	3	1	4	1	1	12
53 "—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	2	—	6
54 "—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	4
55 "—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	2	3	13	22	20	55	63	79	51	59	43	23	21	20	15	5	3	504

TABLE 68. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 9·5—10·5 years																	Totals
	41·5—	43·5—	45·5—	47·5—	49·5—	51·5—	53·5—	55·5—	57·5—	59·5—	61·5—	63·5—	65·5—	67·5—	69·5—	71·5—	73·5—	
41 ⁷ / ₈ —	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
42 "—	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	3
43 "—	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	4
44 "—	—	1	1	3	2	1	1	—	—	—	—	—	—	—	—	—	—	9
45 "—	—	5	2	2	1	3	1	—	—	2	—	—	—	—	—	—	—	16
46 "—	—	1	5	10	11	3	8	3	2	—	1	—	—	—	—	—	—	44
47 "—	—	—	4	4	6	16	17	8	8	1	1	3	—	—	—	—	—	68
48 "—	—	—	4	4	6	12	9	19	15	4	3	2	—	—	—	—	—	78
49 "—	—	—	—	—	4	7	13	21	13	17	14	6	4	1	2	—	—	102
50 "—	—	—	—	—	—	6	7	11	14	15	14	14	7	3	—	1	—	92
51 "—	—	—	—	—	—	—	1	—	7	16	6	13	13	3	3	1	—	64
52 "—	—	—	—	—	—	—	—	2	4	7	2	8	6	6	2	2	1	42
53 "—	—	—	—	—	—	—	1	—	—	3	2	3	2	2	1	3	—	18
54 "—	—	—	—	—	—	—	1	—	—	—	1	—	2	—	3	2	1	12
55 "—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	—	6
56 "—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	3	9	17	24	30	49	58	63	61	57	52	42	37	17	17	8	7	560

338 *Height and Weight of School Children in Glasgow*TABLE 69. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 10·5—11·5 years																				Totals
	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	63·5	65·5	67·5	69·5	71·5	73·5	75·5	77·5	79·5	81·5	
42½	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
43 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
44 "	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
45 "	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
46 "	—	3	2	2	1	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	11
47 "	—	2	2	5	7	3	3	5	1	—	—	—	—	—	—	—	—	—	—	—	28
48 "	—	1	1	4	5	10	4	7	3	1	1	1	—	—	—	—	—	—	—	—	39
49 "	—	—	—	1	9	14	16	14	5	5	3	2	1	1	1	—	—	—	—	—	72
50 "	—	1	—	—	4	7	8	14	10	15	7	1	1	3	1	—	1	—	1	—	74
51 "	—	—	—	—	1	6	6	4	12	14	10	11	4	5	—	2	2	—	—	—	77
52 "	—	—	—	—	—	1	4	13	8	8	8	10	3	2	1	—	1	2	—	1	62
53 "	—	—	—	—	—	—	—	8	3	7	3	12	9	5	6	1	—	1	—	—	55
54 "	—	—	—	—	—	1	—	—	3	1	3	7	5	2	2	4	3	—	2	—	34
55 "	—	—	—	—	—	—	—	—	—	—	3	—	1	6	1	3	2	1	1	—	18
56 "	—	—	—	—	—	2	1	—	—	—	—	1	—	1	2	2	1	—	—	—	14
57 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2
58 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	4
64 "	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	2	9	10	15	24	44	43	44	54	47	43	34	35	30	14	18	13	4	5	498

TABLE 70. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 11·5—12·5 years																	Totals
	48·5	51·5	54·5	57·5	60·5	63·5	66·5	69·5	72·5	75·5	78·5	81·5	84·5	87·5	90·5	93·5	96·5	
46½	1	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	4
47 "	—	4	2	7	1	1	—	—	—	—	—	—	—	—	—	—	—	15
48 "	1	3	4	6	—	3	—	—	—	—	—	—	—	—	—	—	—	18
49 "	2	3	7	5	11	5	2	—	1	—	—	—	—	—	—	—	—	36
50 "	—	—	7	17	12	10	4	1	1	2	—	1	—	—	—	—	—	55
51 "	—	2	8	5	14	18	17	9	3	1	1	—	—	—	—	—	—	78
52 "	—	—	—	2	8	13	17	8	9	3	—	1	—	1	—	—	—	63
53 "	—	1	—	—	9	13	15	18	9	7	5	—	1	—	—	—	—	78
54 "	—	1	—	—	5	10	12	21	17	15	6	4	2	1	1	—	—	95
55 "	—	—	—	—	1	1	8	4	6	7	5	10	6	3	2	—	—	53
56 "	—	—	1	—	—	2	—	1	13	5	3	13	5	2	1	—	—	47
57 "	—	—	—	—	—	—	1	2	1	6	6	4	3	1	1	—	1	27
58 "	—	—	—	—	—	—	—	1	1	—	4	—	1	—	—	2	1	11
59 "	—	—	—	—	—	—	—	1	—	—	—	1	—	1	2	1	—	7
60 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	5
61 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
62 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
63 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	4	15	30	42	61	77	76	65	62	47	26	38	17	11	7	3	5	594

TABLE 71. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 12·5—13·5 years																		Totals
	45·5—	49·5—	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—	113·5—	
48½—	—	—	—	1	4	1	—	—	—	—	—	—	—	—	—	—	—	—	6
49 "	—	1	3	9	1	1	1	—	—	—	—	—	—	—	—	—	—	—	16
50 "	1	—	3	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	9
51 "	—	—	1	8	9	7	4	—	2	2	—	—	—	—	—	—	—	—	33
52 "	—	1	—	6	2	13	8	4	1	1	—	1	—	—	—	—	—	—	37
53 "	—	—	—	2	10	14	13	8	2	1	1	—	—	—	—	—	—	—	51
54 "	—	—	—	—	6	4	19	15	13	6	1	3	—	—	—	—	—	—	67
55 "	—	—	—	1	1	3	11	22	9	10	4	4	—	—	—	1	—	—	66
56 "	—	—	—	—	—	1	6	18	11	14	9	8	1	1	1	—	—	1	71
57 "	—	—	—	—	—	—	3	3	12	20	12	16	6	2	1	—	—	1	76
58 "	—	—	—	—	—	—	—	4	7	4	11	11	2	5	1	—	1	—	46
59 "	—	—	—	—	—	—	1	2	3	4	7	5	5	2	2	1	—	—	32
60 "	—	—	—	—	—	—	—	—	1	—	2	3	3	1	1	1	2	—	14
61 "	—	—	—	—	—	—	—	—	—	—	—	3	2	3	1	1	—	—	10
62 "	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	2
63 "	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
Totals	1	2	7	29	35	45	66	76	61	62	47	55	20	16	7	4	3	2	538

TABLE 72. *Glasgow. Height and Weight of Girls of Group D.*

Height	Weight of Girls of 13·5—14·5 years																						Totals
	49·5—	53·5—	57·5—	61·5—	65·5—	69·5—	73·5—	77·5—	81·5—	85·5—	89·5—	93·5—	97·5—	101·5—	105·5—	109·5—	113·5—	117·5—	121·5—	125·5—	—	134·5—	
46½—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
47 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
48 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
49 "	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
50 "	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
51 "	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
52 "	—	—	1	3	1	4	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	10
53 "	—	—	—	—	—	3	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
54 "	—	—	—	—	3	6	10	6	4	2	1	—	—	—	—	—	—	—	—	—	—	—	32
55 "	—	—	—	1	1	1	14	7	4	3	—	2	—	1	1	—	—	—	—	—	—	—	35
56 "	—	—	—	—	2	2	7	10	11	6	3	3	1	—	—	—	—	—	—	—	—	—	45
57 "	—	—	—	—	—	1	4	3	8	15	12	5	3	—	—	—	—	—	—	—	—	—	51
58 "	—	—	—	—	—	1	5	5	10	11	1	5	4	2	3	—	—	—	—	—	—	—	47
59 "	—	—	—	—	—	1	1	5	5	10	8	10	4	2	1	2	3	—	—	—	—	—	52
60 "	—	—	—	—	—	—	—	4	3	6	3	8	6	3	3	1	1	—	1	—	—	—	39
61 "	—	—	—	—	—	1	—	—	—	1	4	2	2	3	2	2	3	—	—	—	—	—	20
62 "	—	—	—	—	—	—	—	—	—	1	3	—	—	2	—	2	1	1	—	—	—	1	11
63 "	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	3	—	1	1	—	—	7
64 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
65 "	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	2
Totals	1	1	3	6	7	19	43	33	41	47	50	24	30	20	11	13	10	5	2	2	—	1	369

NUMERICAL ILLUSTRATIONS OF THE VARIATE DIFFERENCE CORRELATION METHOD.

BY BEATRICE M. CAVE AND KARL PEARSON, F.R.S.

IN 1904 Miss F. E. Cave in a memoir on the correlation of barometric heights, published in the *R. S. Proc.* Vol. LXXIV. pp. 407 *et seq.*, endeavoured to get rid of seasonal change by correlating first differences of daily readings at two stations. A similar method was used by Mr R. H. Hooker in a paper published some time later in the *Journal of the Royal Statistical Society*, Vol. LXVIII. pp. 396 *et seq.*, 1905. This method was generalised by "Student" in the last number of *Biometrika* (Vol. x. pp. 179, 180). He showed that if there were two variates x and y , such that

$$\begin{aligned}x &= \phi(t) + X, \\ \bar{y} &= f(t) + Y,\end{aligned}$$

where X and Y are the parts of x and y independent of the time t , then the spurious correlation arising from x and y being both functions of the time could be got rid of by correlating the differences of x and y , and that ultimately, when m is sufficiently large :

$$r_{\Delta^m x \Delta^m y} = r_{\Delta^{m+1} x \Delta^{m+1} y} = \text{etc.} = r_{XY},$$

so that the correlation of x and y , free from the spurious time (or it might be position) correlation, i.e. r_{XY} , could be found by correlating the successive differences of x and y . When the correlations of the differences remain steady for several successive values, then we may reasonably suppose that we have reached the correlation r_{XY}^* .

This method is still further developed by Dr Anderson of Petrograd, who in a valuable memoir published in this *Journal* has provided the probable errors of the successive difference correlations of a system of variables :

$$\begin{aligned}X_1, X_2, \dots, X_n, \\ Y_1, Y_2, \dots, Y_n,\end{aligned}$$

* Having been in communication with "Student," while he was writing his paper, I know that the interpretation put by Dr Anderson (*Biometrika*, Vol. x. p. 279) on "Student's" words (*Ibid.* p. 180) is incorrect. "Student" had in mind, if he did not clearly express it, the ultimate steadiness of $r_{\Delta^m x \Delta^m y}$ for a succession of values of m . K. P.

where the correlations of random pairs of values of the variates, or the product sums,

$$\begin{aligned} S\{(X_p - \bar{X})(X_q - \bar{X})\}, \\ S\{(Y_p - \bar{Y})(Y_q - \bar{Y})\}, \end{aligned}$$

are both zero.

Dr Anderson has further provided us with the values of the standard deviations of the successive differences, i.e.

$$\sigma_{\Delta^m X} \text{ and } \sigma_{\Delta^m Y},$$

which represent the ultimate values of $\sigma_{\Delta^m x}$ and $\sigma_{\Delta^m y}$, when we have carried m so far that the time effect has been eliminated.

The new method appears to be one of very great importance, and like many new methods it has been developed in a co-operative manner, which is a good reason for not entitling it by the name of any single contributor. We prefer to term it the *Variate Difference Correlation Method*.

With the exception of a few illustrations given by "Student," no numerical work on the correlation of the higher differences has yet been attempted. It is clear that much numerical work will have to be undertaken before we can feel complete confidence in our knowledge of the range and of the limitations of the new method. We have yet to ascertain how far in different types of material a real stability of difference correlations is ultimately reached, and how far various assumptions made in the course of the fundamental demonstration apply in dealing practically with actual statistical data. One of the most important assumptions made if there be n values of the variates is that arising from the reduction in the number of values as we take the means which occur in successive differences, and a like assumption is made in the case of standard deviations. Thus for example:

$$\frac{1}{n} \sum_1^n (X_s) = \bar{X},$$

but $\frac{1}{n-1} \sum_1^{n-1} (X_s - X_{s+1}) = (X_1 - X_n) / (n-1)$, and will not be sensibly zero, although it is assumed to be, unless n be very large. Similar remarks apply to the sums used in the standard deviations, i.e. we assume in the proof $\frac{1}{n-1} \sum_1^{n-1} (X_s^2) = \frac{1}{n-1} \sum_1^{n-1} (X_{s+1}^2)$.

Ultimately with the m th differences we come in the proof to relations of the type

$$\frac{1}{n-m} \sum_1^{n-m} (X_s) = \frac{1}{n} \sum_1^n (X_s)$$

and

$$\frac{1}{n-m} \sum_1^{n-m} (X_s^2) = \frac{1}{n} \sum_1^n (X_s^2).$$

Now such relations will undoubtedly be very approximately true, if the X 's are random variates uncorrelated to each other, and provided m is small compared with n . These conditions seem amply satisfied when we proceed to fourth or sixth differences in barometric pressures, taken, say, over ten or twelve years; the addition of four or five daily pressures will hardly affect sensibly either the mean or the standard deviation. But such extensive data, while not only involving a great deal of labour in the difference work* are not those which, perhaps, most frequently demand the attention of the statistician, whether he be economist, sociologist or a student of scientific agriculture. In such cases it not infrequently happens that the available data only provide a range of 20 to, perhaps, at most 50 years; and we need to discover whether there is a true relationship between our variates, apart from a continuous change in both due to the time factor. At present accurate statistics of annual trade or revenue, or satisfactory annual demographic data hardly extend at most beyond a period of 50 years. Very often—under even approximately like methods of record—we shall hardly have more than twenty years' trustworthy returns. Not only has the method of record been changed, but the conditions of transit and trade may have been immensely modified and in a manner which we could not suppose to be even approximately represented by a continuous function of the time.

The object of the present paper is to illustrate the theory of the variate difference correlation method in its present stage of development on a *short* series of economic data, in order to test what approximation there is in such short series to stability, and further how nearly Dr Anderson's values for the successive standard deviations apply to such cases. We have selected as our data ten economic indices of Italian prosperity for the years 1885 to 1912, together with a "Synthetic Index," formed by taking the arithmetic mean of the ten economic indices referred to. These eleven indices are given by Professor Georgio Mortara in an interesting memoir: "Sintomi statistici delle condizioni economiche d' Italia" which was published in the *Giornale degli Economisti e Rivista di Statistica*, for February, 1914, and form Tabella I, of that memoir, which we here reproduce in part as Table I. The indices in each case are obtained by dividing the returns for any year by the means of the returns for the years 1901—05, inclusive, and multiplying as usual by 100.

The indices are for returns of (i) Gross Receipts of Railways, (ii) Shipping, loaded and unloaded at the ports, (iii) Effective Revenue of the State, (iv) International Commerce, Value of Imports and Exports, (v) Number of postal Letters and private Telegrams, (vi) Amount of Stamp Duties, (vii) Savings Banks' Returns, (viii) Importation of Coal, (ix) Gross returns of consumption of Tobacco, (x) Returns of Coffee imported. Professor Mortara has drawn attention to the very high correlations of these individual indices with each other, and of each of them with the "Synthetic Index." The latter correlation is, however, to a certain

* A discussion of the correlations of the higher differences in barometric pressures will we hope be shortly issued.

extent spurious. For if $I_1, I_2, \dots, I_{10}$ be the individual indices and I_s the synthetic index, then $I_s = \frac{1}{10}(I_1 + I_2 + \dots + I_{10})$ and any individual index I_q , if there be no correlation between such individual indices, would give

$$\frac{1}{n} S(I_s - \bar{I}_s)(I_q - \bar{I}_q) = \frac{1}{10n} S(I_q - \bar{I}_q)^2 = \frac{1}{10} \sigma^2_{I_q},$$

$$\sigma^2_{I_s} = \frac{1}{n} S(I_s - \bar{I}_s)^2 = \frac{1}{100} (\sigma^2_{I_1} + \sigma^2_{I_2} + \dots + \sigma^2_{I_{10}}),$$

and accordingly

$$r_{I_s I_q} = \frac{\frac{1}{10} \sigma^2_{I_q}}{\sigma_{I_s} \times \sigma_{I_q}} = \frac{\sigma_{I_q}}{\sqrt{\sigma^2_{I_1} + \sigma^2_{I_2} + \dots + \sigma^2_{I_{10}}}},$$

TABLE I.

Professor Mortara's Table of Index Values of Italy.

Numerical Index (Mean 1901—1905) = 100.

Year	Railways	Shipping	Revenue	International Commerce	Post and Telegraphs	Stamp Duties	Savings Banks	Coal	Tobacco	Coffee	Synthetic Index or Arithmetic Mean Index
1885	61	63	78	72	38	82	47	53	82	98	67.4
1886	62	63	79	74	40	87	53	52	86	94	69.0
1887	68	73	82	78	42	94	55	64	88	91	73.5
1888	70	71	83	62	43	98	57	69	86	81	72.0
1889	71	77	85	70	45	99	58	71	86	78	74.0
1890	72	78	86	66	46	97	59	78	87	81	75.0
1891	72	72	85	60	48	96	60	70	88	80	73.1
1892	71	75	86	64	51	96	64	69	89	80	74.5
1893	70	70	85	64	55	95	65	66	90	73	73.3
1894	72	72	86	63	57	93	65	84	89	71	75.2
1895	73	76	89	66	60	92	68	77	88	70	75.9
1896	75	77	90	67	64	94	70	73	88	73	77.1
1897	78	80	90	68	68	96	73	76	87	75	79.1
1898	81	84	91	78	73	96	75	79	89	78	82.4
1899	86	88	93	88	75	96	79	87	91	82	86.5
1900	89	89	94	91	78	96	83	88	92	82	88.2
1901	90	91	96	92	85	96	87	86	95	92	91.0
1902	96	99	98	95	93	97	92	96	97	94	95.7
1903	101	103	99	99	103	99	99	99	99	102	100.3
1904	105	102	101	103	111	101	107	105	102	103	104.0
1905	108	105	105	111	108	107	115	114	106	109	108.8
1906	119	123	108	132	107	115	127	137	109	118	119.5
1907	126	125	108	144	114	120	144	148	115	125	126.9
1908	134	129	113	139	122	120	155	150	124	132	131.8
1909	139	140	121	149	132	123	168	165	131	140	140.8
1910	146	146	129	159	140	130	180	166	138	147	148.1
1911	155	156	135	167	149	136	187	171	144	153	155.3
1912	165	169	138	179	158	141	192	179	151	160	163.2
Mean	94.8	116.3	97.6	96.4	82.3	103.3	95.9	99.0	100.6	98.6	96.5

which would have a substantial, but spurious value amounting to .316 if $\sigma_{I_1} = \sigma_{I_2} = \dots \sigma_{I_{10}}$. If there be high correlation between the individual indices, of course, the correlation of each individual index with the arithmetic mean or synthetic index will also be high. Thus our Table IV shows it to range from .952 in the case of Coffee to .998 in the case of Railways. Possibly a third of this correlation may in some cases have a spurious origin. But the individual indices are very highly correlated together; only two such correlations are below .9, and the lower of these two is as high as .885. We are accordingly left with fifty correlations ranging from .885 to .997 between the individual indices, and if we accept these as true measures, then it is clear that any one of these ten indices might be used as a reasonable index of Italian prosperity; it would for practical purposes be idle to calculate them all or to take their arithmetic mean.

But the high correlations found lay themselves open from our present standpoint to some suspicion of being solely due to the fact that during the 28 years under consideration Italy has progressively increased in population and accordingly the consumption of innumerable goods and the means of interchange have all grown together with the time. In other words the correlations we give under the heading of "quantities" in each separate section of Table IV are very high solely because the individual indices are variates increasing one and all as continuous functions of the time.

The material therefore seems especially suited to the application of the variate difference correlation method. For example, the correlation between the indices for tobacco and savings is .984; are we to interpret this to signify, that, if there are large savings this means that much will be spent on tobacco? Or, is this high correlation simply in whole or part spurious, merely indicating that both savings and consumption of tobacco increased markedly with the time? Actually the correlation of first differences drops from .984 to .766, that of second differences is negative if insensible, while from there onwards it steadily increases *negatively*, till with the sixth differences we reach $-.431$, which seems to indicate that, when time has been eliminated, expenditure on tobacco in any year means less money saved. Again the coffee and tobacco indices appear very highly correlated, .955, but by the third difference correlation we have reached about a third of the relationship, .319, which is scarcely altered in the sixth difference correlation, .326; we may assume therefore that there is probably a moderate "organic" relationship between the expenditures on coffee and tobacco, but the association is nothing like as close as would be suggested by the correlation of the raw indices.

The work has been done in the following manner: The successive differences of the indices up to the sixth were found. The means and standard deviations of these differences were calculated, and the correlations were then worked out in the product-moment manner. This involved the very laborious work of determining 385 coefficients, and then to these coefficients were added the

probable errors as found by Dr Anderson's formulae. These probable errors are of course those of the correlations of $\Delta^m X$ and $\Delta^m Y$, and will not be the correct values for the probable errors of the correlations of $\Delta^m x$ and $\Delta^m y$, until $\Delta^m x = \Delta^m X$ and $\Delta^m y = \Delta^m Y$, i.e. until m is sufficiently great for t to have been eliminated. Further their accuracy depends on the vanishing of the means of the differences or on the equalities of the sums like

$$\frac{1}{n-1} \sum_1^{n-1} S(X_s), \quad \frac{1}{n-1} \sum_2^n S(X_s), \text{ etc.}$$

which, while true on the average, will only be approximately true in the actual instance if n be large. We give in Table II, the Mean Values of Index Differences.

TABLE II.

Mean Values of Indices and their Differences.

	Railways	Shipping	Revenue	International Commerce	Post and Telegraphs	Stamp Duties	Savings Banks	Coal	Tobacco	Coffee	Synthetical or Arithmetic Mean Index
Quantity ...	94.8	116.3	97.6	96.4	82.3	103.3	95.9	99.0	100.6	98.6	96.5
1st difference	-3.85	-3.93	-2.22	-3.96	-4.44	-2.19	-5.37	-4.67	-2.56	-2.30	-3.55
2nd "	-.35	-.50	-.08	-.38	-.27	.00	+.04	-.35	-.12	-.42	-.24
3rd "	+.16	+.28	+.20	-.08	.00	+.12	-.08	+.40	-.12	.00	+.09
4th "	-.33	-.88	-.13	-1.17	.00	-.28	+.04	-.79	-.16	-.42	-.40
5th "	+.57	+2.26	+.26	+2.52	+.30	.00	-.52	+1.87	+.22	+.87	+.83
6th "	-.82	-2.00	-.55	-6.41	+.18	-.73	-.05	+2.00	-.95	-1.68	-1.10

It will be seen from this table that the means of the differences are far from zero even when we have reached a difference for which we may suppose the time to have been eliminated. This arises from the smallness of the series dealt with and shows us that we ought not to anticipate more than a rough accordance with theory, or only an approximate steadiness, for sums like

$$\frac{1}{n-m} \sum_1^{n-m} S(X_s)$$

may grow less and less steady as m increases.

Similar considerations apply to the standard deviations of the differences. These will not at first obey Dr Anderson's formulae because they are the values for $\sigma_{\Delta^m x}$ and $\sigma_{\Delta^m y}$, and when we have taken m sufficiently high for

$$\sigma_{\Delta^m x} \text{ and } \sigma_{\Delta^m y} \text{ to be theoretically equal to } \sigma_{\Delta^m X} \text{ and } \sigma_{\Delta^m Y},$$

and accordingly the correlation should have begun to be steady, then some failure to obey Dr Anderson's formulae will arise, because the means of the differences are not truly zero and equalities of the type

$$\frac{1}{n-1} \sum_1^{n-1} (X_s^2) = \frac{1}{n-1} \sum_1^{n-1} (X_{s+1}^2), \text{ etc.}$$

will not be satisfied when n is relatively small.

Dr Anderson gives the value of $\sigma^2_{\Delta^m X}$ in terms of σ^2_X , but we do not of course know σ^2_X , which will be very different from σ^2_x , and can only practically be found from the value of $\sigma^2_{\Delta^m x}$ itself, after that value has become equal to $\sigma^2_{\Delta^m X}$, i.e. after steadiness has set in. In order therefore to test the formulae we have formed the ratio of

$$\sigma^2_{\Delta^m x} / \sigma^2_{\Delta^{m-1} x} \text{ from } m=1 \text{ to } 6.$$

This equals $4 - \frac{2}{m}$ in Dr Anderson's formulae for $\sigma^2_{\Delta^m X} / \sigma^2_{\Delta^{m-1} X}$ and therefore we have a good measure of the approach of $\Delta^m x$ to $\Delta^m X$, or of the growth of steadiness as apart from the correlations. The following Table III gives the values of the ratios of the squares of the standard deviations, theoretical values, actual values and the mean value for each of the differences of the ten individual indices.

TABLE III.

Values of $\sigma^2_{\Delta^m x} / \sigma^2_{\Delta^{m-1} x}$ and their approach to $4 - \frac{2}{m}$.

m	Theoretical Series	Synthetic Index	Rail	Shipping	Revenue	International Commerce	Post	Stamp Duties	Savings	Coal	Tobacco	Coffee	Mean of 10 Index Difference Standard Deviation Ratios
1	2	·012	·012	·031	·019	·038	·009	·040	·010	·035	·022	·036	·025
2	3	·705	·708	1·834	·763	1·720	·799	·585	·350	2·074	·352	·843	1·003
3	3·333	3·107	2·816	3·093	2·124	3·032	1·959	1·660	2·214	3·075	2·213	3·307	2·549
4	3·5	3·167	3·128	3·174	2·747	3·213	2·597	2·008	3·106	3·379	3·025	3·619	3·000
5	3·6	3·143	3·449	3·189	3·020	3·104	3·010	2·328	3·275	3·580	3·117	3·701	3·177
6	3·667	3·149	3·711	3·195	3·164	2·881	3·208	2·499	3·455	3·682	3·101	3·791	3·269

It will be clear that until we reach the ratio of the square standard deviations of the third and second differences, there is no general approach to steadiness. After $m=3$, however, for $m=4, 5$ and 6 , the ratio of the values for the mean of the series of individual indices to the theoretical value is ·86, ·88 and ·89, respectively.

Thus, there is increasing approach to agreement in the observed and theoretical values, but the approach is slow, and we believe that there is greater steadiness than is really indicated by this test. The source of this apparent unsteadiness lies we

think in the relative largeness of m compared with n (i.e. at a maximum 6 as compared with 28), rather than in our not having taken sufficiently high differences.

We now turn to the correlations. These are given in Table IV, the actual values of the standard deviations of the quantities and their differences being recorded along the diagonal cells, while the other cells contain the correlations of each pair of variates and of their successive differences.

We will now consider these correlations in detail.

(a) *Synthetic Index* (Arithmetic Mean) with Individual Indices.

We see at once that the *Synthetic Index* is highly associated with *Shipping* ($> .85$), with *Importation of Coal* ($? > .75$), and *International Commerce* ($? < .68$), and fairly highly with *Revenue* (c. $.55$). On the other hand the sixth difference correlations with *Post* (c. $.15$), with *Stamp Duties* (c. $.24$) and with *Savings* ($> .23$) are all such (i) that they might well have arisen from the spurious element in the *Synthetic Index* correlations, and are all less than their Andersonian (steady value) probable errors. Almost the same may be said of the *Railway Index*; it is not beyond suspicion of being spurious, and is scarcely significant having regard to its probable error. The *Consumption of Coffee* is also not very closely associated with the *Synthetic Index*; it is only about twice its probable error ($.427 \pm .205$), and a good deal of its value may be spurious. Further in the case of both *Coffee* and *Railways*, the correlations are still falling between $.04$ and $.05$ for each difference. The last individual index remaining is that for *Consumption of Tobacco* and although the correlation of sixth differences is not really significant it is negative ($-.247 \pm .235$), and is exhibiting a steady negative rise.

Stripped therefore of the common time factor the *Synthetic Index* will be seen to be no very appropriate measure of trade, business activity, and spare money for savings and luxuries. With *Post*, *Stamp Duties* and *Savings*, it has probably only a spurious relationship, expenditure on railways has little influence, that on luxuries is very slightly significant, or indeed in the case of tobacco negative. It is, however, closely related to variations in external trade, i.e. imports (including coal), to exports and shipping and to effective revenue. It appears to us that a suitable general index of prosperity, which will distinguish between a continuous growth in all factors with the time, and favourable and unfavourable fluctuations from this growth, can only be obtained, when there has been far more ample study of the associations of individual indices among themselves, and of these indices after they have been freed from the time factor, i.e. of associations between high difference correlations. From this standpoint the study of General Index theories is at present in its infancy.

(b) *Railway Index*. This index is very noteworthy in the nature of its associations after removal of the time factor. We have reached a steady correlation (c. $.62$) with *Shipping*, but beyond this no values of first class importance appear. The relation of *Railways* and *Revenue* after falling practically

TABLE IV. *Italian Indices. Correlations and Probable Errors and Standard Deviations.*

Synthetic Index	Synthetic Index	Rail	Shipping	Revenue	International Commerce	Post	Stamp Duties	Savings	Coal	Tobacco	Coffee
Quantities	28.46	.998 ± .005	.996 ± .005	.989 ± .005	.988 ± .005	.972 ± .007	.965 ± .009	.997 ± .005	.989 ± .005	.985 ± .005	.952 ± .012
1st Diff.	3.14	.874 ± .037	.839 ± .047	.669 ± .057	.789 ± .060	.401 ± .133	.651 ± .091	.777 ± .063	.660 ± .089	.732 ± .073	.759 ± .067
2nd "	2.64	.539 ± .130	.888 ± .039	.370 ± .158	.804 ± .065	-.065 ± .182	.383 ± .156	.164 ± .178	.709 ± .091	-.064 ± .182	.504 ± .135
3rd "	4.65	.505 ± .151	.895 ± .040	.494 ± .151	.806 ± .071	.038 ± .202	.357 ± .175	.139 ± .198	.713 ± .099	-.057 ± .202	.526 ± .146
4th "	8.28	.451 ± .175	.887 ± .047	.546 ± .154	.787 ± .084	.108 ± .217	.328 ± .196	.160 ± .214	.709 ± .109	-.060 ± .219	.509 ± .163
5th "	14.68	.392 ± .199	.874 ± .056	.554 ± .163	.744 ± .105	.146 ± .231	.275 ± .219	.194 ± .227	.721 ± .113	-.134 ± .232	.468 ± .184
6th "	26.05	.353 ± .219	.857 ± .066	.547 ± .177	.680 ± .135	.153 ± .245	.242 ± .236	.232 ± .237	.755 ± .108	-.247 ± .235	.427 ± .205
Quantities	.998 ± .005	29.80	.997 ± .005	.989 ± .005	.982 ± .005	.979 ± .005	.961 ± .010	.996 ± .005	.989 ± .005	.979 ± .005	.938 ± .015
1st Diff.	.874 ± .037	3.23	.769 ± .065	.546 ± .111	.561 ± .109	.393 ± .134	.646 ± .092	.640 ± .093	.590 ± .103	.660 ± .089	.604 ± .100
2nd "	.539 ± .130	2.72	.620 ± .112	.074 ± .182	.226 ± .173	-.140 ± .181	.258 ± .170	-.198 ± .175	.529 ± .132	-.204 ± .175	.044 ± .182
3rd "	.505 ± .151	4.56	.621 ± .124	.241 ± .190	.207 ± .194	.153 ± .198	.159 ± .197	-.291 ± .185	.489 ± .154	.164 ± .197	.018 ± .202
4th "	.451 ± .175	8.06	.616 ± .136	.329 ± .196	.158 ± .214	.184 ± .212	.017 ± .220	.341 ± .194	.442 ± .177	.157 ± .214	.053 ± .219
5th "	.392 ± .199	14.97	.609 ± .148	.381 ± .201	.077 ± .235	-.209 ± .226	.119 ± .233	.386 ± .200	.402 ± .197	.198 ± .227	.139 ± .232
6th "	.352 ± .219	28.84	.621 ± .154	.422 ± .206	-.007 ± .250	-.214 ± .239	-.261 ± .233	.431 ± .204	.383 ± .214	-.243 ± .236	-.204 ± .240
Quantities	.996 ± .005	.997 ± .005	29.14	.990 ± .005	.981 ± .005	.970 ± .008	.967 ± .008	.990 ± .005	.986 ± .005	.978 ± .006	.937 ± .016
1st Diff.	.839 ± .047	.769 ± .065	5.17	.602 ± .101	.705 ± .079	.196 ± .152	.563 ± .108	.402 ± .132	.632 ± .095	.419 ± .130	.532 ± .113
2nd "	.888 ± .039	.620 ± .112	7.00	.514 ± .134	.667 ± .101	-.182 ± .177	.295 ± .167	-.085 ± .181	.604 ± .116	.173 ± .177	.388 ± .155
3rd "	.895 ± .040	.621 ± .124	12.31	.673 ± .111	.682 ± .108	-.127 ± .199	.249 ± .190	-.074 ± .201	.594 ± .131	.122 ± .199	.397 ± .170
4th "	.887 ± .047	.616 ± .136	21.94	.733 ± .102	.661 ± .124	-.079 ± .214	.160 ± .214	-.066 ± .219	.578 ± .146	.123 ± .216	.360 ± .191
5th "	.874 ± .056	.609 ± .148	39.18	.756 ± .101	.614 ± .147	-.054 ± .236	.087 ± .235	-.049 ± .236	.571 ± .156	.189 ± .228	.298 ± .215
6th "	.857 ± .066	.621 ± .154	70.03	.766 ± .103	.542 ± .177	-.059 ± .249	-.009 ± .250	-.034 ± .250	.579 ± .166	-.271 ± .229	.232 ± .237
Quantities	.989 ± .005	.989 ± .005	.990 ± .005	16.28	.962 ± .010	.971 ± .007	.961 ± .010	.988 ± .005	.974 ± .007	.983 ± .005	.918 ± .020
1st Diff.	.669 ± .087	.546 ± .111	.602 ± .101	2.27	.369 ± .136	.368 ± .137	.471 ± .123	.541 ± .112	.279 ± .146	.665 ± .088	.491 ± .120
2nd "	.370 ± .158	.074 ± .182	.514 ± .134	1.98	.229 ± .173	-.282 ± .168	.060 ± .182	-.144 ± .179	.155 ± .178	.102 ± .181	.267 ± .170
3rd "	.494 ± .151	.241 ± .190	.673 ± .111	2.88	.344 ± .178	-.381 ± .173	.002 ± .192	-.247 ± .190	.222 ± .192	.210 ± .193	.390 ± .172
4th "	.546 ± .154	.329 ± .196	.733 ± .102	4.78	.357 ± .192	-.381 ± .188	-.084 ± .218	-.211 ± .210	.240 ± .207	.243 ± .207	.428 ± .179
5th "	.554 ± .163	.381 ± .201	.756 ± .101	8.31	.308 ± .214	-.374 ± .202	.168 ± .230	-.181 ± .229	.249 ± .222	.194 ± .227	.419 ± .194
6th "	.547 ± .177	.422 ± .206	.766 ± .103	14.78	.214 ± .239	-.388 ± .213	-.255 ± .234	-.154 ± .244	.270 ± .232	.115 ± .247	.400 ± .210
Quantities	.988 ± .005	.982 ± .005	.981 ± .005	.962 ± .010	35.83	.949 ± .013	.941 ± .015	.979 ± .005	.970 ± .008	.967 ± .008	.969 ± .008
1st Diff.	.789 ± .060	.561 ± .109	.705 ± .079	.369 ± .136	7.02	.136 ± .155	.481 ± .121	.518 ± .116	.449 ± .126	.433 ± .128	.589 ± .103
2nd "	.804 ± .065	.226 ± .173	.667 ± .101	.229 ± .173	9.20	-.027 ± .182	.410 ± .152	.265 ± .170	.300 ± .166	.070 ± .182	.342 ± .161
3rd "	.806 ± .071	.207 ± .194	.682 ± .108	.344 ± .178	16.02	.182 ± .196	.442 ± .163	.266 ± .188	.273 ± .187	.117 ± .200	.340 ± .179
4th "	.787 ± .084	.158 ± .214	.661 ± .124	.357 ± .192	28.72	.317 ± .197	.461 ± .173	.249 ± .206	.233 ± .208	.151 ± .215	.304 ± .199
5th "	.744 ± .105	.077 ± .235	.614 ± .147	.308 ± .214	50.61	.405 ± .197	.452 ± .187	.251 ± .222	.193 ± .228	.117 ± .233	.233 ± .224
6th "	.680 ± .135	-.007 ± .250	.542 ± .177	.214 ± .239	85.89	.475 ± .194	.463 ± .196	.275 ± .231	.171 ± .243	.015 ± .250	.142 ± .245

to zero, now stands at something greater than .42 and might rise higher, but the relation to *International Commerce* as a whole is zero, which suggests that the goods imported and exported are not in the bulk carried by rail. Further although the final value of the *Railway* and *Post* correlation is scarcely sensible ($-.214 \pm .239$), it has been continuously negative from the second difference, and thus suggests that increased expenditure on the post means lessened profit for the railways. This might be interpreted in two ways: (i) that business conducted by post or telegram lessens rail intercommunication by person, or (ii) that in the case of state-railways, there is not an increased profit to the railways from carrying larger mails. But still more remarkable are the negative correlations of *Stamp Duties*, *Savings*, *Tobacco* and *Coffee* with *Railways*; none of them are very large, and all but savings, perhaps, of the order of their probable errors. But taken as a whole they suggest that when the Italian spends little money in going about, then he saves more, or spends more on such luxuries as tobacco and coffee. Lastly we have the *Coal Index*. It might be supposed that a year with great coal importation would signify great railway activity, and this is the judgment which would be made from the raw correlations of these variates. But the actual facts are exhibited in a correlation still falling at the sixth difference and hardly significant having regard to its probable error. The inferences formed must be: (i) that imported coal is used largely at the ports of disembarkation or travels inland by other than railway transit, (ii) that the imported coal is largely used on the railways themselves and that its cost is a heavy tax on their resources.

(c) *Shipping Index*. As we might anticipate this is highly correlated with (i) *Railways* (c. .62), (ii) *Revenue* (c. .75) and less highly but very significantly with (iii) *International Commerce* (c. 54) and (iv) *Coal* (c. .58), but it appears to have no relation whatever with *Post*, *Stamp-Duties* and *Savings*, and when we come to luxuries, their importation is clearly not a factor of shipping prosperity. Neither in the case of *Tobacco* nor of *Coffee* are the correlations really significant; with the former we have an increasing negative correlation and with the latter a decreasing positive one already below its probable error. Thus we see that neither directly by bulk of importation nor indirectly by *immediate* increase of consumption, does a rise of shipping mark significant rises in the use of luxuries such as *Tobacco* and *Coffee*. It would be of interest to ascertain whether increased consumption of luxuries does not rather *follow* than accompany favourable trade fluctuations.

(d) *Revenue Index*. This index as we might expect is fairly highly correlated with *Shipping* (c. .75). It has relatively small relation to *Railways* ($.422 \pm .206$) at least at the sixth difference and a somewhat similar value (c. $.42 \pm .20$) for *Coffee*. Thus the suggestions arise that revenue is but little produced by the railways and that coffee is not a very large factor of the custom dues. It is astonishing to find, however, that *Post*, *Stamps* and *Savings* have *negative* correlations with *Revenue* of $-.388 \pm .213$, $-.255 \pm .234$ and $-.154 \pm .244$ respectively, which, if scarcely significant, have been in each case for several

differences persistent in sign. Even the correlation with *Tobacco* is small, falling and insignificant ($< .115 \pm .247$), and that with *Coal* which might be supposed to be high as marking good trade times is hardly significant although apparently rising ($? > .270 \pm .232$). Lastly the correlation of *Revenue* with *International Commerce* is again small, falling, and insignificant ($< .214 \pm .239$). Thus *Revenue* or the "entrato effettivo dello stato" seems to provide an index which has little valuable relation to any other characteristic of prosperity beyond shipping.

(e) *International Commerce*. Here we find no single final individual index correlation greater than .54, which is that for *Shipping*. The next most important correlations are with *Post* ($> .47$) and *Stamp Duty* (c. .46). With *Railways* the correlation is zero, and with *Revenue* also falling and insignificant. With *Savings*, *Coal*, *Tobacco* and *Coffee* the correlations are all insignificant; in fact in the last three cases not only are the values less than their probable errors, but they are still falling. It is thus clear that in Italy the total of Exports and Imports is no measure of all-round prosperity, they do not immediately increase either savings or the consumption of luxuries.

(f) *Post and Telegrams*. Here we have the lowest series of correlations we have yet reached. *Post* values have no significant relation to fluctuations in *Railway* (c. $-.20 \pm .24$), to *Shipping* ($-.059 \pm .249$), *Stamp Duties* ($-.027 \pm .250$), *Coal* ($-.050 \pm .250$), *Tobacco* ($+.108 \pm .247$) or *Coffee* ($-.133 \pm .246$) Indices. It is significantly correlated only with *International Commerce* ($> +.47 \pm .19$) and, perhaps, significantly with *Savings* ($+.336 \pm .222$) but negatively with *Revenue* (c. $-.38 \pm .21$). In short the number of letters and telegrams in Italy is hardly a mark of any other favourable fluctuation in prosperity, beyond *International Commerce*.

(g) *Stamp Duties*. This Index is correlated positively and significantly with *International Commerce* (c. $+.46 \pm .20$) and positively, and doubtfully with *Savings* (c. $+.35 \pm .22$). It is correlated insignificantly and negatively with *Railways* ($-.261 \pm .233$), *Shipping* ($-.009 \pm .250$), *Revenue* ($-.255 \pm .234$), *Post* ($-.027 \pm .250$), and *Tobacco* ($-.129 \pm .246$); it is correlated positively and insignificantly with *Coal* ($+.052 \pm .250$) and *Coffee* ($+.222 \pm .238$). Thus again freed from continuous time changes, fluctuations in the *Stamp Duty Index* are of small value as a measure of contemporaneous general prosperity.

(h) *Savings Bank Index*. There are practically only two correlations of any importance with *Savings* and these are both negative, namely those with *Railways* ($-.431 \pm .204$) and with *Tobacco* ($-.431 \pm .204$). Hence it would appear that when the Italian people is in a saving mood, it spares on transit by rail and on the consumption of tobacco, and when it expends on these luxuries, then it does not save. *Savings* have small and possibly not significant correlations with *Post* ($> +.33 \pm .22$) and *Stamp Duties* ($< +.353 \pm .219$), and insignificant and positive correlations with *International Commerce* ($> +.27 \pm .23$), *Coal* ($> +.19 \pm .24$)

and *Coffee* ($c. + \cdot 05 \pm \cdot 25$); they have insignificant *negative* correlations with *Shipping* ($< - \cdot 03 \pm \cdot 25$) and *Revenue* ($< - \cdot 15 \pm \cdot 24$).

Savings are thus—apart from continual time change—no very satisfactory measure of general prosperity, and a fluctuating increase is usually accompanied by a reduction of luxuries.

(i) *Coal Index*. The importation of coal has little relation to any factor of prosperity besides *Shipping* ($c. + \cdot 58 \pm \cdot 17$). With *Railways* the correlation is not quite double the probable error and the value, even at the sixth difference, appears still falling. The correlation with *Revenue* only just exceeds the probable error ($+ \cdot 270 \pm \cdot 232$). With *International Commerce* ($+ \cdot 171 \pm \cdot 243$), *Stamp Duties* ($+ \cdot 052 \pm \cdot 250$), *Savings* ($+ \cdot 196 \pm \cdot 241$) and *Coffee* ($+ \cdot 152 \pm \cdot 245$) the correlations are less than their probable errors, small and in some cases still falling. With the *Postal Index*, the correlation is negative, insignificant and falling. Alone in the case of the *Tobacco Index* does the correlation appear to be nearly as significant as in that of *Shipping*, but it is *negative* and increasing* ($- \cdot 514 \pm \cdot 184$), while in the case of *Shipping* it was steady. It is singular to find that *Coal*, the increased import of which should mark increased industrial activity, is, beyond the naturally influenced *Shipping*, alone effectively associated with the consumption of *Tobacco*.

(j) *Tobacco Index*. This is of considerable interest as marking the association of indices of trade prosperity with the consumption of a luxury. With four exceptions *Tobacco* is negatively correlated, although often insignificantly, with the other indices. *Revenue* ($+ \cdot 115 \pm \cdot 247$), *International Commerce* ($+ \cdot 015 \pm \cdot 250$), and *Post* ($+ \cdot 108 \pm \cdot 247$) are all positive, insignificant, and in the first two cases still falling. The correlation with *Coffee* is positive and might, perhaps, be significant ($+ \cdot 326 \pm \cdot 224$), but it appears to be still falling. With *Coal* and *Savings* there are probably significant negative correlations ($- \cdot 514 \pm \cdot 184$, and $- \cdot 431 \pm \cdot 204$ respectively); with *Railways* ($- \cdot 243 \pm \cdot 236$), *Shipping* ($- \cdot 271 \pm \cdot 229$) and *Stamp Duties* ($- \cdot 129 \pm \cdot 246$) there are insignificant negative correlations, but they tend to confirm each other in sign. Thus we see that the consumption of tobacco can hardly be considered as a measure of general prosperity; it appears to be greatest when trade conditions are unfavourable, and in particular when savings are least and manufacturing conditions as measured by the importation of coal are slack. The result suggests the pipe of the unemployed at the street corner, rather than the increased expenditure of the fully occupied artisan.

(k) *Coffee Index*. This is another luxury and the results are very similar. There appears a significant correlation with *Revenue* ($+ \cdot 400 \pm \cdot 210$), which might easily be explained, and there is a falling but possibly significant correlation with *Tobacco* ($+ \cdot 326 \pm \cdot 224$). With all other indices the relationships are

* It is, perhaps, hard to believe that so much smuggling could be carried on in colliers, that it would seriously affect the profits of the tobacco monopoly!

insignificant. *Railways* ($-.204 \pm .240$), *Shipping* ($+.232 \pm .237$), *International Commerce* ($+.142 \pm .245$), *Post* ($-.133 \pm .246$), *Stamp Duties* ($+.222 \pm .238$), *Savings* ($+.046 \pm .250$) and *Coal* ($+.152 \pm .245$). Apart therefore from the general increase of consumption with the time, during which time the general prosperity of the nation has increased, it would not appear that the consumption of a luxury has any organic relationship to prosperity. We do not find that a favourable trade fluctuation is associated with increased consumption of luxuries. In fact the suggestion arises that in the case of tobacco the consumption may be greater in a period of depression.

Conclusions. While we lay no special stress on any of the results suggested by the difference correlations above studied—far more intimate economic knowledge of Italian affairs and methods of measurement would be requisite—we yet venture to insist on one or two general considerations.

The very superficial statements, so frequently met with, that such and such variates, both changing rapidly with the time, are essentially causative will doubtless cease to have any scientific currency, directly the method of variate differences is fully appreciated. We shall no longer assert that the fall of the phthisis death-rate can be off-hand causatively associated with the contemporaneous rise in the number of persons dying in institutions, or that the increased expenditure on luxuries is necessarily a measure of increased national prosperity.

If we turn as in the present paper to the actual correlations of the indices themselves, we find in every case an arid and scarcely undulating waste of high correlation. No one can obtain any nourishment whatever from the statement that the *Tobacco Index* is correlated with the *Revenue Index* to the amount of .983 and with the *Savings Bank Index* to the extent of .984! The organic relationship between these variates is wholly obscured by the continuous increase of all three of them with the time. But when we proceed to sixth differences and see that the consumption of tobacco has little, if any, relation to revenue, and is associated substantially but *negatively* with savings, we seem to touch realities, and realities of some worth. Again what can we learn, if we are told that the *Shipping Index* is correlated to the extent of .99 with both the *Revenue* and the *Savings Bank* Indices? We might imagine, that increase of shipping was not only the primary cause of increase in Italian revenue, but also the essential origin of any increase in the Italian peasant's and artisan's savings! An appeal to the variate difference method shows how fallacious such imaginings would be! An examination of the sixth difference correlations shows that while prosperity of the revenue is closely associated with trade as measured by shipping (.77), the correlation is not nearly perfect; on the other hand there appears to be no significant organic correlation at all ($-.154 \pm .244$) between the prosperity of the revenue and the savings of the Italian populace. As we have noted a knowledge of local conditions and methods

of reckoning quantities might enable us to put other and, perhaps, more luminous interpretations on our results. But there can be small doubt that to proceed from the actual correlation of such indices to the correlations of their higher differences gives the feeling of clearing away the sand of the desert, and reaching all the ordered arrangements of an excavated town below; the slight undulations of the waste above are really fallacious, and enable us to appreciate nothing of the actual topography of the city.

The method is at present in its infancy, but it gives hope of greater results than almost any recent development of statistics, for there has been no source more fruitful of fallacious statistical argument than the common influence of the time factor. One sees at once how the method may be applied to growth problems in man and in lower forms of life with a view to measuring common extraneous influences, to a whole variety of economic and medical problems obscured by the influences of the national growth factor, and to a great range of questions in social affairs where contemporaneous change of the community in innumerable factors has been interpreted as a causative nexus, or society assumed to be at least an organic whole; the flowers in a meadow would undoubtedly exhibit highly correlated development, but it is not a measure of mutual causation, and the development of various social factors has to be freed from the time effect, before we can really appreciate their organic relationships.

In the present paper we have dealt only with very sparse "populations" (only 28 values of the variates), but this has enabled us to consider not only a very large number of correlations, but to see the practical influence of terminal conditions on our theory. This may we think be summed up in the statement that the Andersonian formulæ for the standard deviations will hardly in many practical cases be more than very roughly approximated before the size of the population becomes too small to make the deductions reliable. Further in most cases our difference correlations have hardly even with the sixth differences reached a steady state. Possibly they have done so in the cases of *Rail and Shipping*, *Shipping and Post*, *Shipping and Coal*, *Revenue and Post*, *International Commerce and Stamp Duties*, *International Commerce and Savings*, *Savings and Coffee*, and in one or other additional pair. But in the great bulk of instances there is still a more or less steady rising or falling appreciable in the difference correlations, and all we can really say is that the final value, the true r_{XY} , will be somewhat greater or less than a given number. From an examination of the actual numerical working of the correlations, it appears to us that the terminal values are in the case of these short series of very great importance. It is further clear that the theory as given by "Student" depends upon certain equalities which are not fulfilled in practice in short series. We await with much interest the complete publication of Dr Anderson's work, and hope to find a fuller discussion of the allowance to be made in short series for the influence of the terminal state of

affairs* on the steadiness of the series and on the approach to the standard-deviation formulae. But apart from these lesser points, our present numerical investigation has convinced us of the very great value of the new method of Variate Difference Correlations.

* For example if we measure X from its mean,

$$\sigma^2_{\Delta X} = \frac{1}{n-1} \sum_1^{n-1} (X_s - X_{s+1})^2 - (\overline{\Delta X})^2 = \frac{1}{n-1} \left(\sum_1^n (X_s^2) - X_n^2 + \sum_1^n (X_s^2) - X_1^2 \right) - (\overline{\Delta X})^2,$$

since $\sum_1^{n-1} (X_s X_{s+1})$ is by hypothesis zero, $= 2\sigma^2_{X_s} + \frac{2}{n-1} \{ \sigma^2_{X_s} - \frac{1}{2} (X_1^2 + X_n^2) \} - (\overline{\Delta X})^2$. The first term $2\sigma^2_{X_s}$ gives Dr Anderson's value of $\sigma^2_{\Delta X}$. Now $\overline{\Delta X}$ equals $\frac{1}{n-1} \sum_1^{n-1} (X_s - X_{s+1}) = \frac{1}{n-1} (X_1 - X_n)$. Thus the remainder is $\frac{2}{n-1} \left[\sigma^2_{X_s} - \frac{1}{2} \left\{ X_1^2 + X_n^2 + \frac{1}{n-1} (X_1 - X_n)^2 \right\} \right]$. Now the average value on many trials of $\frac{1}{2} (X_1^2 + X_n^2)$ will be $\sigma^2_{X_s}$ and of $(X_1 - X_n)^2$, $2\sigma^2_X$, so that the full value may be $\frac{2}{(n-1)^2} \sigma^2_X$ and small for n large; but for n small as above such a relation as $\sigma^2_{\Delta X} = 2\sigma^2_X$ and the similar but more complex relations of the standard deviation formulae for the higher differences need not hold for any individual case, and thus the steadiness of the difference correlation series, and the approach to the Andersonian formulae are very far from attained.

AN EXAMINATION OF SOME RECENT STUDIES OF THE INHERITANCE FACTOR IN INSANITY

By DAVID HERON, D.Sc.*

IN the last few years a number of studies of the inheritance factor in insanity have been published in America, Germany and England. The value of investigation of such a topic cannot be overestimated. We are quite certain that the prevalence of insanity is not falling; many of us indeed believe that the statistics suffice to demonstrate that it is substantially increasing, and that we can attribute this increase not in the first place to the intenser strain of modern life, but to the greater power of modern treatment to check or temporarily cure attack, and thus allow wider possibility of reproduction to members of affected stocks. Indeed the problem seems closely associated with an essential difficulty of modern civilisation, the greater protection of physically and mentally degenerate stocks unaccompanied by any adequate limitation of their thereby increased power of procreation; the inheritance factor thus tends to aid the relatively greater survival of the socially unfit. The studies we have referred to would be of great importance from this aspect of eugenics if (i) the data were collected without conscious or unconscious bias, and (ii) the inferences drawn from them followed logically from the data thus collected.

Unfortunately it is not only in the interpretation of statistics that adequate training is required. It is equally important that in the actual collection of them we should proceed, not only free from the bias which arises from the hurried acceptance of dogmatic theories of heredity, but what is often still more needful, free from the bias which is almost certain to waylay our progress, if we have not initially considered with trained insight the fallacies which may result from our method of recording or even tabulating our material. The day of the amateur in science is gone; no one now pays any attention to men who propound elaborate atomic theories or stellar hypotheses, without having had preliminary training in physical or astronomical science. There are still, however, some who appear willing to accept the statement of statistical data or the inferences drawn from those

* This paper formed the second portion of a lecture given at the Galton Laboratory on March 3, 1914.

data by men who have clearly had no adequate training in statistical science. The craniologist, the anthropologist, even the biological student of heredity and evolution are recognising that a statistical training is needful for the true interpretation of many of the facts in their special fields of research. The physiologist still appears to believe that he can deal with the average effects of diverse dietaries or the pathologist with the "mass-phenomena" of the hereditary factor in insanity without any training in statistical method. A physicist might just as logically assume that without mathematical training he could give an adequate mathematical account of a physical phenomenon, or a cosmic theorist suppose that he was effectively furnished for astronomical research by the perusal of a popular primer on the stars! The statistical calculus cannot be mastered by any easier road than the differential calculus, or, to put a more apt illustration, statistical training is as needful a preliminary to the handling of statistics, as time spent in a physiological laboratory to the effective handling of tissues. In twenty years it will be unnecessary to insist on these points, they will be universally recognised in the courts of science; but at present it is not only necessary to reiterate unpleasant truths, but to emphasise their validity by illustrations which bring home forcibly to scientist and layman alike the danger of amateur statistical handling. To state that a man is in error is not sufficient, if he continues time after time to repeat his assertions, apparently under the belief that incessant repetition will convince the world of the value of his theories.

In the case of the inheritance factor in insanity we are not dealing with any purely academic question of science. We are up against one of the most difficult problems of modern life, where true advice is of urgent importance to the nation as well as to the individual. It is not only the medical man but the layman who seeks guidance in the question of the marriage of members of insane stocks, and a laboratory like the Galton Laboratory knows how often advice on such points is sought. It is disheartening when help is rendered to the seeker to be faced with the criticism: "But Professor ——— says I may marry if I take a wife of sound stock," or "Dr ——— recommends marriage, although my father was insane, because I am over twenty-five and still sane myself." When teaching of this kind, arising solely from false interpretation of defective data, is spread widecast in a dozen different papers or journals, it is not sufficient to issue a brief statement of its futility. It is needful to give it the *coup de grâce* by a more lengthy criticism of its fallacies and their illustration in a form more likely to impress the imagination. The attempt is made in this paper to deal with only one of the authors, who have contributed fallacious eugenic rules to those seeking knowledge on the influence of the hereditary factor in insanity.

In a long series of papers Dr F. W. Mott, Pathologist to the London County Asylums, has stated that when the children of insane parents become insane, they do so at a much earlier age than did their parents, and on the basis of this assertion he has drawn some very sweeping conclusions for practical conduct. Thus in the

British Medical Journal of May 11, 1912 (p. 1060), he states that "this signal tendency of insane offspring to suffer with a more intense form of the disease and at an early age, as shown in the above figures and tables, is of great importance for the following reasons: first, it is one of Nature's methods of ending or mending a degenerate stock; secondly, it is of importance to the physician, for he can say that there is a diminishing risk of the child of an insane parent becoming insane after he has passed 25, a matter of great importance in the question of marriage; thirdly, it is of importance in connection with the subject of social surgery of the insane, for when the first attack of insanity occurs in the parent the children for the most part have all been born....Sterilization would therefore be applicable to relatively few parents admitted to asylums."

Put briefly, Dr Mott's views are that in "Antedating" or "Anticipation," in this alleged tendency of the offspring to become insane at any earlier age than their parents, we have Nature's method of purifying degenerate stocks, that the children of insane parents who are still normal at the age of 25 may safely marry*, and that it is useless to take any special measures to limit the reproduction of the insane since nearly all their children are born before the onset of insanity.

These conclusions, if proved to be correct, would be of the utmost importance to the Eugenist. If the Law of Antedating or Anticipation really acts in the way Dr Mott has suggested, then it would seem to be unnecessary to take any special Eugenic action in the case of the insane and indeed the "Law" has already been used in support of this view. Thus in a leading article in the *British Medical Journal*†, which deals with Dr Mott's work, it is stated that "This intensification of mental disease in the young—this 'anticipation' as it is called, which is one of Nature's methods of ending or mending a degenerate stock, is specially important in connection with sterilization, as the figures given by Dr Mott show that when the first attack of insanity occurs in the parent the children have for the most part all been born. Sterilization, therefore, would be applicable in relatively few cases."

It is at least obvious that when views such as these are taken of the "Law of Anticipation," it merits the most careful examination. Let us consider, then, first of all, Dr Mott's presentation of the case for anticipation. For some years past Dr Mott has been engaged in the collection of cases in which two or more members of a family are or have been resident in London County Asylums, and has noted wherever possible the age of onset of the insanity. Information was thus obtained regarding 217 pairs of father and offspring, and 291 pairs of mother and offspring and the results are summed up in the following table.

Thus in comparing the age at onset of insanity in father and offspring, we find that among the fathers only 1·4% became insane before the age of 20, while among the offspring the percentage was 26·2. These figures are also shown graphically in

* See for instance *Problems in Eugenics*, p. 426.

† May 11, 1912, p. 1089.

Figs. 1 and 2*. Here the horizontal scale represents the age of onset in 5-year groups—the vertical scale the percentages of cases occurring in each age group.

TABLE I.

Percentages of Cases whose First Attack of Insanity occurred within Various Age-periods.

Age-periods	Father	Offspring	Mother	Offspring
	Per cent.	Per cent.	Per cent.	Per cent.
Under 20 years	1·4	26·2	0·6	27·8
20—24 years...	0·4	18·0	3·4	15·7
25—29 "	1·4	18·0	4·4	18·2
30—34 "	9·6	13·0	7·8	13·4
35—39 "	11·5	7·3	9·2	10·0
40—44 "	9·2	6·4	10·3	5·8
45—49 "	14·3	6·0	12·0	3·7
50—54 "	17·5	0·9	12·3	2·4
55—59 "	13·8	3·7	14·0	1·7
60—64 "	10·1	—	11·6	1·3
65—69 "	5·0	—	8·8	—
70—74 "	4·6	0·4	3·1	—
75—79 "	0·4	—	1·3	—
80 "	0·4	—	0·6	—

I have been obliged to follow Dr Mott in treating the "under 20" group as a 5-year group as otherwise my diagrams would bear no resemblance to his, but this procedure is far from satisfactory when such a large proportion of the cases in this group are congenital cases in which the age of onset should be taken at 0 years. The tables and diagrams show that among the parents more than half the cases occur after the age of 50, while among the offspring, more than half occur before 30, and this is taken to prove that there is Anticipation or Antedating in Insanity.

This will perhaps be made more evident if the percentages of those who became insane before the age of 25 are given in each case. Among the fathers, 2% and among the mothers, 4% became insane before the age of 25. Among the offspring, on the other hand, the percentage is 44. Another way of looking at the matter is to take the average age of onset of insanity in each case. Dr Mott gives a Table showing these averages but unfortunately has omitted the congenital cases so that the extent of anticipation is considerably under-estimated, and the form in which the data are given does not permit of an accurate calculation of the actual averages. From the information given it appears, however, that the average age at onset of insanity among the parents is about 50 years, among the offspring about 26 years, showing an anticipation or antedating of some 24 years.

* I am very grateful to Miss H. Gertrude Jones, the Hon. Secretary of the Galton Laboratory, for the diagrams which illustrate this lecture.

Figs. 1 and 2. Diagrams to illustrate the Distribution of Age at Onset of Insanity in Parent and Offspring. (Mott.)

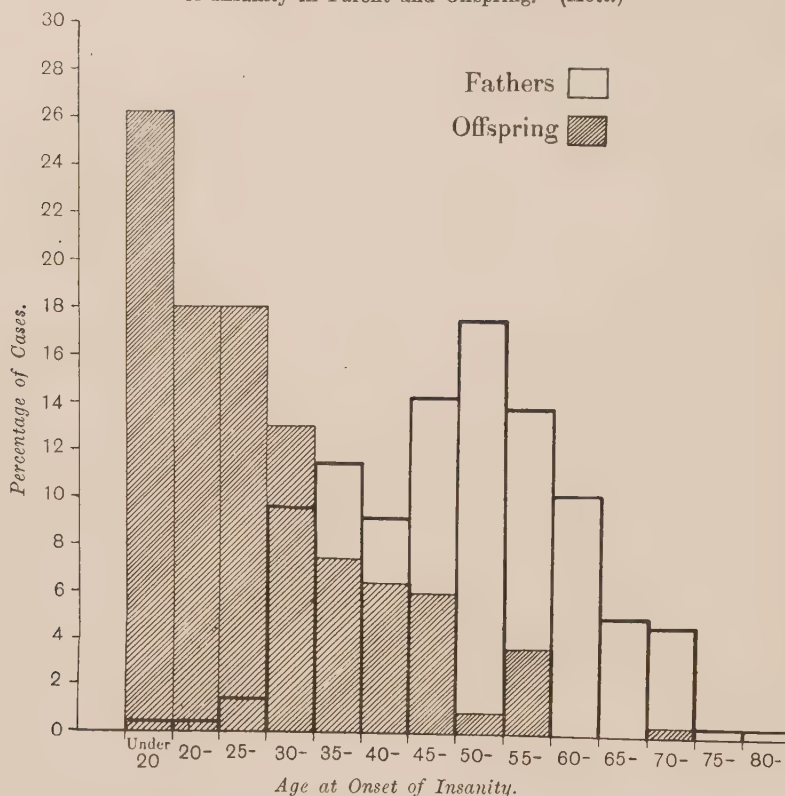


Fig. 1.

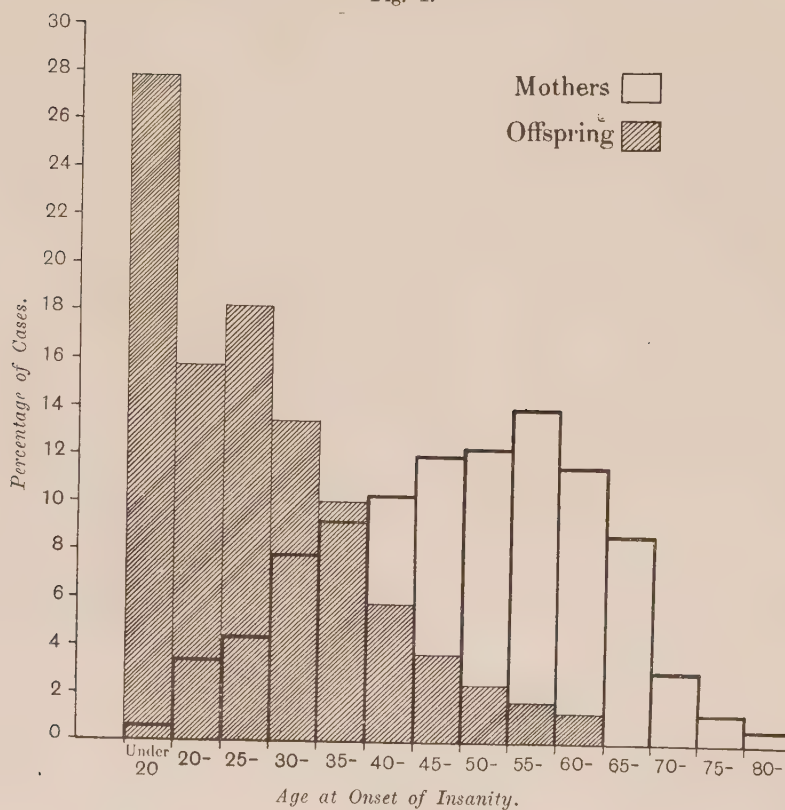


Fig. 2.

Now these conclusions, if satisfactorily demonstrated, would obviously be of the highest importance, but they were immediately challenged by Professor Karl Pearson in a letter which appeared in *Nature* of November 21, 1912 (p. 334). Professor Pearson's letter is as follows:

On an Apparent Fallacy in the Statistical Treatment of "Antedating" in the Inheritance of Pathological Conditions.

The problem of the antedating of family diseases is one of very great interest, and is likely to be more studied in the near future than ever it has been in the past. The idea of antedating, i.e. the appearance of an hereditary disease at an earlier age in the offspring than in the parent, has been referred to by Darwin and has no doubt been considered by others before him. Quite recently, studying the subject on insanity, Dr F. W. Mott speaks of antedating or anticipation as "Nature's method of eliminating unsound elements in a stock" ("Problems in Eugenics," papers communicated to the First International Eugenics Congress, 1912, p. 426).

I am unable to follow Dr Mott's proof of the case for antedating in insanity. It *appears* to me to depend upon a statistical fallacy, but this apparent fallacy may not be real, and I should like more light on the matter. This is peculiarly desirable, because I understand further evidence in favour of antedating is soon forthcoming for other diseases, and will follow much the same lines of reasoning. Let us consider the whole of one generation of affected persons at any time in the community, and let n_s represent the number who develop the disease at age s , then the generation is represented by

$$n_0, n_1, n_2, \dots n_s, \dots n_{100}, \text{ say.}$$

Possibly some of these groups will not appear at all, but that is of little importance for our present purpose.

Let us make the assumptions (1) that there is no antedating at all; (2) that there is no inheritance of age of onset; thus each individual reproduces the population of the affected reduced in the ratio of p to 1. Then the family of any affected person, whatever the age at which he developed the disease, would represent on the average the distribution

$$pn_0, pn_1, pn_2, \dots pn_s, \dots pn_{100}.$$

The sum of such families would give precisely the age distribution at onset of the preceding generation.

Now let us suppose that for any reason certain of the groups of the first generation do not produce offspring at all, or only in reduced numbers. Say that q_s only of the n_s are able to reproduce their kind; then of the older generation, *limited to parents*, the distribution will be

$$q_0 n_0 + q_1 n_1 + q_2 n_2 + \dots + q_s n_s + \dots + q_{100} n_{100},$$

but the younger generation will be

$$p(q_0 n_0 + q_1 n_1 + q_2 n_2 + \dots + q_s n_s + \dots + q_{100} n_{100})(n_0 + n_1 + \dots + n_s + \dots + n_{100}),$$

i.e. the relative proportions will remain absolutely the same.

The average age at onset and the frequency distribution of the older generation, that of the *parents*, will be entirely different from that of the offspring and will depend wholly on what values we give to the q 's. If frequency curves be formed of the two generations they will differ substantially from each other. This difference is not a result or a demonstration of any physiological principle of antedating but is solely due to the fact that those who develop the disease at different ages are not equally likely to marry and become parents.

A quite striking instance of the fallacy, if it be such, would be to consider the antedating of "violent deaths." Fully a quarter of such deaths in males, nearly a half in females, occur before the age of twenty years. Consider now the parents and offspring who die from violent deaths; clearly there would be no representative of death from violence under twenty in the parent generation, and we should have a most marked case of antedating, because the offspring generation would contain all the infant deaths from violence. "

In the case of insanity, is the man or woman who develops insanity at an early age as likely to become a parent as one who develops it at a later age? I think there is no doubt as to the answer to be given; those who become insane before twenty-five, even if they recover, are far less likely to become parents than those who become insane at late ages—many, indeed, of them considering the high death-rate of the insane, will die before they could become parents of large families. Now Dr Mott took 508 pairs of parents and offspring, "collected from the records of 464 insane parents whose 500 insane offspring had also been resident in the County Council Asylums," and ascertained the age of first attack. As at present advised, it seems to me that his data must indicate a most marked antedating of disease in the offspring, but an antedating which is wholly spurious. There is, I think, a further grievous fallacy involved in this method of considering the problem, but before discussing that I should like to see if my criticism of this method of approaching the problem of antedating can be met.

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November 11, 1912.

Dr Mott has referred to this letter in his *Report* for 1912*, but it will be more convenient to deal with his reply after we have examined the method by which his data have been collected and the use made of the data. Let us consider first of all how the data were obtained. Dr Mott in describing his material says that it consists of a collection of cases in the London County Asylums where two or more persons are related to one another. Thus Dr Mott has dealt—not with a series of complete pedigrees in which every member is included, whether insane or normal, but with a series of cases in which two or more members of a family are known to have been in London County Asylums. No notice is taken of those who are normal throughout their lives and no allowance is made for those who are normal at the time the record is made but who may afterwards become insane.

Do cases selected in this way provide a complete or impartial view of the facts? Some of Dr Mott's own comments on his data throw a considerable amount of light on this point. In his *Report* for 1909† he says: "From all the Asylums I have received valuable reports, but in the case of the older asylums it has been a matter of the utmost difficulty to trace the records of so many years back," and in his *Report* for 1910‡ he says, "Some of the asylum authorities have gone through their case books for a number of years back, but the results have not been satisfactory owing to the difficulty of obtaining particulars without a living representative of the family being resident in the asylum—for instance, 110 old cases

* *Annual Report of the London County Council* for 1912, Vol. II. p. 62.

† *Twentieth Annual Report of the Asylums Committee of the L.C.C.*, p. 90.

‡ *Twenty-first Report of the Asylums Committee of the L.C.C.*, p. 94.

reported from Bexley have been rejected as the relatives in the other London County Asylums could not be traced, for no instance has been included unless full particulars could be obtained."

It is thus clear that not all the cases could be traced and that there was special difficulty in tracing the older cases. What is the effect of a selection of this kind? A study of the following hypothetical cases may serve to throw some light on this point.

TABLE II.

Anticipation or Antedating in Insanity. Hypothetical Examples to show the Effect of Dr Mott's Selection of Cases.

						First Example	Second Example
Mother:	Born	...	...	...	...	1873	1833
	Married	...	...	...	...	1893	1853
	Became Insane and admitted to Asylum	...	...	...	...	1913	1873
	Age at First Attack	...	...	...	...	40	40
	Died	...	...	...	...	1914	1874
Son:	Born...	...	...	...	...	1894	1854
	Became Insane	...	...	...	...	1894*	1914
	Admitted to Asylum	...	...	...	...	1914	1914
	Age at First Attack	...	...	...	...	0	60

The mothers in those two examples have exactly parallel careers. In each case the mother became insane at the age of 40 and only lived one year in the asylum. In the first case the son was a congenital idiot but was only admitted to an asylum at the age of 20. The age of onset in this case is taken at 0 years and the case shows marked "anticipation." In the second case the mother also became insane at the age of 40, the son not till the age of 60, 40 years after his mother's death. The second example thus tells against the Law of Anticipation. Are these two cases equally likely to appear in Dr Mott's data?

In the first case mother and son are in the asylum at the same time and were admitted within a year of each other. It is very improbable that the relationship would escape notice and such a case is almost certain to be recorded. In the second case, however, the son is not admitted to an asylum till 40 years after his mother's death. Even if the family remained in the same area for 40 years after the mother's death, it would obviously be very difficult to connect the histories of mother and son. This case, which tells against the Law of Anticipation, is almost certain to escape notice. A spurious anticipation or antedating is thus inevitable owing to the method of collecting the data.

It has also been pointed out that Dr Mott has made no allowance for those who are mentally normal at the time the record is made but may subsequently

* Congenital Idiot.

become insane, and this introduces further spurious anticipation. Another hypothetical example will perhaps make this clear. Let us take the case of a mother with six children, five of whom have become insane as follows :

TABLE III.

	Mother	Children					
		1	2	3	4	5	6
Born	1830	1850	1852	1854	1856	1858	1860
Became Insane ...	1860	—	1872	1896	1914	1888	1860+
Age at Onset of Insanity	30	—*	20	42	58	30	0

The extent to which this family would show anticipation or antedating would depend very largely on the time at which the record was made as is shown in the following table.

TABLE IV.

Date of Record	Age of Onset of Insanity in		Average for Children	Amount of Anticipation
	Mother	Children		
1860	30	0	0	30
1872	30	0, 20	10	20
1888	30	0, 20, 30	16·7	13·3
1896	30	0, 20, 30, 42	23 0	7
1914	30	0, 20, 30, 42, 58	30	0

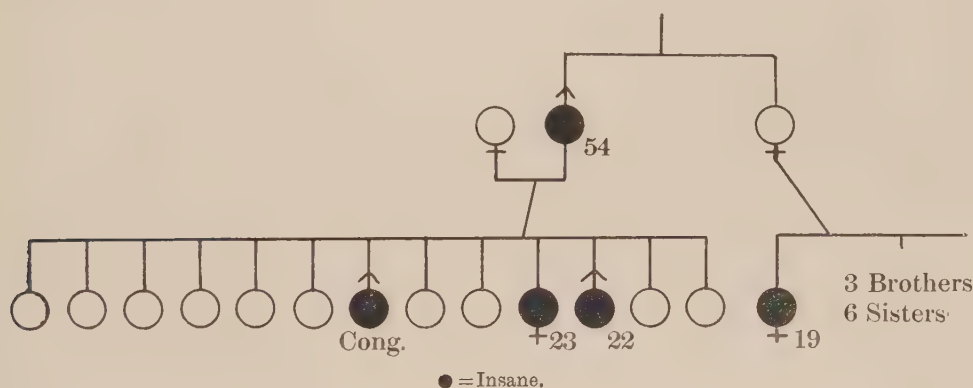
If the case were noted in 1860 then the age of onset of insanity in the mother is 30 years—of the child 0 years—a clear case of anticipation, and nothing would be known of the fact that four other children will afterwards become insane and will bring the average age of onset in the children up to 30 years—exactly the same as that of the mother. Nor is the record even now complete for if the eldest child ever becomes insane, the age of onset in his case must be at least 64 years and this will further increase the average age of onset in the children. It is thus clear that in dealing with incomplete families and ignoring the possibility that those who are normal at the time of record may afterwards become insane, Dr Mott has introduced a further spurious anticipation or antedating.

If we examine carefully the first pedigree given by Dr Mott at the Eugenics Congress[‡], we see clearly how probably much of the anticipation recorded by

* Alive, 64 years of age and still normal.
 † Congenital Idiot.
 ‡ *Problems in Eugenics*, p. 413.

Dr Mott has arisen. Unfortunately this is the only pedigree for which sufficient details have been given to enable its completeness to be tested. The pedigree and Dr Mott's description of it are as follows:

"A.B., an alien Jew, aged 54, was admitted to an asylum for the first time suffering from involuntional melancholia; he has a sister who has not been in an asylum, but, as events turned out, bore the latent seeds of insanity. The man is married to a healthy woman who bore him a large family; the first five are quite healthy, then comes a congenital imbecile epileptic (cong.)*, then two healthy children followed by a daughter who becomes insane at 23, then a son insane at 22, and lastly two children who are up to the present free from any taint. The sister of A.B. is married and has a family of ten, seven girls and three boys; one of the females was admitted to the asylum at the age of 19, and since this pedigree was constructed a brother of hers has been admitted aged 24. Half-black† circles are insane. The pedigree is instructive; it shows direct and collateral heredity; it also shows remarkably well the signal tendency to the occurrence of insanity at an early age in the children of an insane and potentially insane parent."



13 children: 9 Alive, 4 Sons, 5 Daughters. 4 Dead. 3 Insane.

Fig. 3. Pedigree to illustrate the effect of Dr Mott's selection of cases.

F. W. Mott: "Heredity and Eugenics in relation to Insanity." *Problems in Eugenics*, p. 413.

This pedigree was given as above in July 1912, and in an address previously delivered before the Manchester Medical Society on Oct. 4, 1911, Dr Mott gave the same pedigree, but without any reference to the nephew of A.B. (brother of the girl who became insane at 19) who became insane "since the pedigree was constructed," so that this man became insane between 1911 and 1912 and this serves to "date" the pedigree.

Now it should be noted that at least five of the children of A.B. are over 23 years of age and up to the present time healthy. But all these children are alive and if any one of them afterwards becomes insane, the average age of onset of insanity in the children will be raised—and it is clear that the more incomplete the pedigree the greater the amount of spurious anticipation. Again Dr Mott states that in

* This does not agree with Dr Mott's pedigree which gives the congenital case as the seventh instead of the sixth child.

† According to our usual custom, they are represented by full black circles in Fig. 3.

nephews and nieces the age of onset is earlier than in uncles and aunts. In 1911 this pedigree gives a case in which an uncle became insane at 54, his niece at 19—but one year later a nephew who became insane at 24 has to be added, thus raising the average and there are eight more children some at least of whom may become insane at later ages. As before the incompleteness of the pedigree introduces an artificial and spurious anticipation or antedating. The remedy is obvious; we must only deal with completed families.

A further fallacy involved in Dr Mott's method of work must now be noted. In directly comparing the age of onset in parent and child, Dr Mott has ignored the fact that in the parent the incidence of insanity is for all practical purposes limited to the age of 20 and over since cases of congenital defect and of adolescent insanity hardly ever marry. Among the general population of asylums, however, 12% become insane before the age of 20 and in Dr Mott's selected data the percentage rises to 27—or more than a quarter of the whole become insane before 20. This in itself causes a very marked spurious anticipation. As Professor Pearson has shown (p. 361 above) if we were to investigate the age at death in parent and child from accident or violence, we should find the same spurious anticipation.

There are thus three fallacies involved in Dr Mott's work. In the first place a spurious anticipation or antedating arises from the inclusion in the record of families whose history has not yet been completed, for those who become insane at late ages in the younger generation do not appear. Secondly, even with families whose history is completed, those cases in which the insanity of parent and child is contemporaneous are far more likely to be recorded than those in which the child becomes insane long after the parent*, and thus the cases which show anticipation are more likely to appear in the record than those which tell against Dr Mott's views. Thirdly, by directly comparing parent and child, he has practically limited one of the two groups which are being compared to ages at onset of over 20 years and has thus obtained further spurious anticipation.

Dr Mott also lays stress on the appearance of insanity in a more intense form in the younger generation. "I have proved," he says†, "that there is a signal tendency in the insane offspring of insane parents for the insanity to occur at an earlier age and in a more intense form in a large proportion of cases, for the form of insanity is usually either congenital imbecility, insanity of adolescence, or the more severe form of dementia praecox, the primary dementia of adolescence, which is generally an incurable disease." But we have already seen that Dr Mott's method of collecting his data is such that an enormous preponderance of early cases of insanity in the younger generation is inevitable and of course such cases are largely incurable. Type of disease is very closely related to the age of onset and

* Dr Mott states (*Archives of Neurology*, Vol. VI. p. 82) that "the main bulk of the cards (i.e. his records), however, refer to parents and offspring admitted to the asylums within the last fifteen years."

† *Archives of Neurology*, Vol. VI. p. 82.

by selecting the latter we can alter the proportion of any particular type of insanity. Dr Mott has obtained his material in such a way that, in the younger generation, cases of insanity coming on late in life are much less likely to be recorded than those which appear in early life, and hence the early cases are in a majority, but the change in age of onset, and consequently of the type, is entirely spurious and arises solely from the way in which the material has been obtained.

We can now deal with the reply Dr Mott has made to Professor Pearson's criticisms. In his *Annual Report* for 1912 (p. 62), Dr Mott says: "Professor Karl Pearson, writing to *Nature*, November 21, 1912, 'On an apparent fallacy in the statistical treatment of "Antedating" in the inheritance of pathological conditions,' criticises on mathematical grounds the evidence of anticipation. I do not feel myself competent to reply to the opinion of such an eminent authority on mathematics applied to biometrics, but it does not militate against my conclusions, nor explain away the fact that a large proportion of the insane offspring of insane parents are affected with imbecility or adolescent insanity; for granting the assumption that there is no antedating at all, we might rightly expect the ages at onset of insane offspring of insane parents to be comparable with the ages at onset of all the admissions to the asylums during the same period*. This is by no means the case, for amongst the insane offspring there is a far greater proportion affected early in life, as is shown in the following figures and curves" (they appear here as Fig. 4 and Table V).

According to these figures the onset of insanity among the *recorded* insane offspring of insane parents is considerably earlier than among the general admissions to asylums, but it has already been shown that this is due to the fact that the data have been selected in such a way that the early cases in the younger generation are the most likely to appear. Further, if Dr Mott's argument be a valid one, we might also expect the ages at onset of the insane parents of these insane offspring to be comparable with the ages at onset of all the admissions to asylums during the same period. This is by no means the case as is shown in Fig. 5 below (see also Tables I and V). We see here that the insanity of the parents comes on at a much *later* period than among the general admissions to asylums and that there is a far less proportion affected early in life. If Dr Mott's method of argument be sound, he has not only to deal with an antedating of insanity among the offspring but also a post-dating of insanity among the parents. Both are of course spurious and arise from the peculiar selection of the data and from the fact that, owing to differential death-rates, the ages at onset of "admissions" will never be the same as the ages at onset of the admitted—i.e. the asylum population—at any time.

* "We might rightly expect" these ages to be different, because "admissions" are not the same as the population in the country who have at one time or another been insane. The percentages of total cases of acute mania, of senile insanity, of congenital idiocy, and of melancholia, who reach the asylums, are not the same. The reader has to distinguish between the population of admissions, the population of admitted, and the insane population of the country. A sample of the latter may be reached from completed family histories, but not from records on admission or from records of an asylum population.

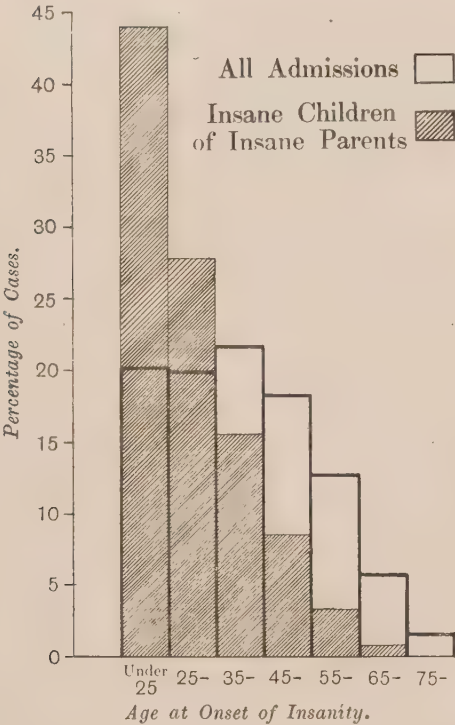


Fig. 4.

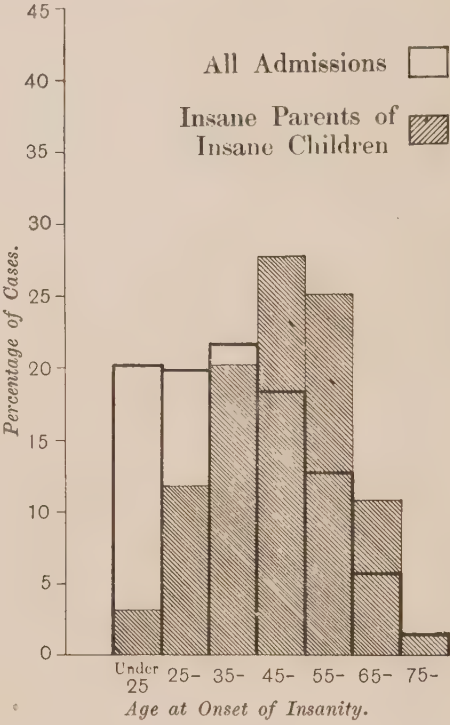


Fig. 5.

Diagram to illustrate the Distribution of Age at Onset of Insanity among:
(1) The Insane Offspring of Insane Parents.
(2) The Insane Parents of Insane Offspring.
(3) All Admissions to L.C.C. Asylums.

TABLE V. *Percentage Comparison of the Age at time of Onset of Insanity in the Insane Offspring of Insane Parents and the General Admissions to the London County Asylums.*

Age at Onset of Insanity	MALE		FEMALE		TOTAL	
	4482 direct admissions during last four years	274 insane offspring of insane parents	5097 direct admissions during last four years	389 insane offspring of insane parents	9579 direct admissions during last four years	663 insane offspring of insane parents
Under 25	20.0	43.8	20.2	44.2	20.1	44.0
25-34	19.9	27.7	19.9	28.0	19.9	27.9
35-44	21.9	13.8	21.5	16.7	21.7	15.5
45-54	17.7	10.2	18.6	7.4	18.2	8.5
55-64	13.3	3.6	12.4	2.8	12.7	3.2
65-74	5.7	0.7	5.9	0.8	5.8	0.7
75	1.5	—	1.6	—	1.5	—

41 male imbeciles out of 274 offspring
54 female " " 389 "
95 male and female " " 663 "

It is possible to illustrate the various fallacies which vitiate Dr Mott's conclusions regarding anticipation by considering the age at death of parent and child. I do not know whether it is generally recognised that it is exceedingly difficult to get any considerable body of data in which the ages at death of a parent and all his children are given, for of course the record is incomplete and biassed until the death of the last surviving member, and in some cases to get a complete record we must trace the history of a family for over 150 years. George the IIIrd, for instance, was born in 1738 and all but one of his 15 children were still alive in 1810, 72 years afterwards, and the last surviving son, Duke of Cumberland and King of Hanover, did not die till 1851, 113 years after his father's birth—and this is by no means an extreme case. In the material I am about to describe I found one case where the interval was 160 years.

Another difficulty which arises is the tendency in practically all family histories to omit infant deaths, so that we do not get a complete record. It seems probable that the deaths of minors are not represented in such records in anything like their true proportion and that the differences are greater than might be expected to arise from differences of physique and nurture due to class. Thus records of the Landed Gentry give 31 deaths per 1000 males under 20 years* while actual experience shows 163 to 197 per 1000†. But in the records of the reigning families of Europe we get a practically complete record of all members and therefore from von Behr's *Genealogie der in Europa regierenden Fürstenhäuser*‡, I have extracted particulars of the age at death of over 2000 individuals—all belonging to the 18th century. There was here no selection—every child was entered and every family had been traced from the birth of the parents till the death of the last survivor.

Now in Dr Mott's data we have already seen that cases in which the age at onset of insanity in parent and child is contemporaneous are most likely to be recorded. We can test the effect of a selection of this kind by investigating the effect of selecting, from our data regarding the age at death among those royal families, only those individuals who died within a certain number of years of their father's death, and the results are given below in Table VI, p. 370.

When we deal with the whole of the data, absolutely unselected, every family being complete and traced to the death of the last surviving member, we find that 680 out of 1829 or 37·2% died under 20 years of age. Let us now apply a very slight selection to the data and reject the 92 cases in which the interval between the deaths of father and child was at least 60 years. We find now that 680 out of the remaining 1737 died under 20 years of age—or 39·1%. Thus the effect of a selection of this kind is to cause a slight increase in the proportion of deaths at the early ages. If we make the selection slightly more stringent, by taking only those who died within 40 years of their father's death, the percentage of individuals dying under 20 years of age rises to 46·7 and if we go still further and consider

* See Pearson: *Proc. R. S.* Vol. 65, p. 291.

† *Statistics of Families*, p. 73.

‡ Tauchnitz, Leipzig, 1870.

TABLE VI.

Illustrating the Effect of Selection of Material on the Distribution of Age at Death.

(Reigning Houses in Europe—18th Century.)

Age at Death	All Cases Unselected Data		Children who died :							
			within 60 years of their father's death		within 40 years of their father's death		within 20 years of their father's death		in their father's lifetime	
	Numbers	%	Numbers	%	Numbers	%	Numbers	%	Numbers	%
Under 20	680	37·2	680	39·1	680	46·7	680	62·4	648	82·7
20—39	277	15·1	277	15·9	277	19·0	254	23·3	121	15·4
40—59	336	18·4	336	19·3	274	18·8	127	11·7	15	1·9
60—79	450	24·6	395	22·7	214	14·7	29	2·7	—	—
80 and over	86	4·7	49	2·8	10	·7	—	—	—	—
Totals	1829	—	1737	—	1455	—	1090	—	784	—
Average Age at Death*	35·9		33·7		26·9		16·2		7·7	

only those who died in their father's lifetime, then the percentage rises to 82·7 %. Looking at the matter in another way we find that the average age at death has fallen from 35·9 years to 7·7 years.

The same facts are given in Fig. 6, which shows that as the selection of cases becomes more stringent, there is a regular increase in the proportion of deaths at the younger ages. In exactly the same way, the fact that cases where the insanity of parent and child is contemporaneous are the most likely to appear in Dr Mott's records causes a spurious exaggeration of the cases of insanity at early ages in the younger generation and consequently a spurious exaggeration of the number of cases of imbecility and adolescent insanity.

We can also investigate directly the question of anticipation or antedating on this material. In order to avoid the heavy weighting of large families which would arise if every child were entered, I have taken only one child from each family. Let us consider first of all the distribution of age at death of Fathers and their First-born Children. The facts are given in Table VII.

We have altogether 294 cases in which we know the age at death of a father and his first-born child. None of the fathers died before 20 but of the children

* These averages were calculated, not from the five age groups given above, but from the same material classified in 15 age groups.

106 out of 294 or 36·1 % died before 20. The average age at death among the fathers is 61 years, but among the children it is only 36 years, so that there is an anticipation of 25 years. To borrow Dr Mott's words, the figures clearly show the signal tendency among the offspring to die at a much earlier age than their parents; that is to say, anticipation or antedating is the rule.

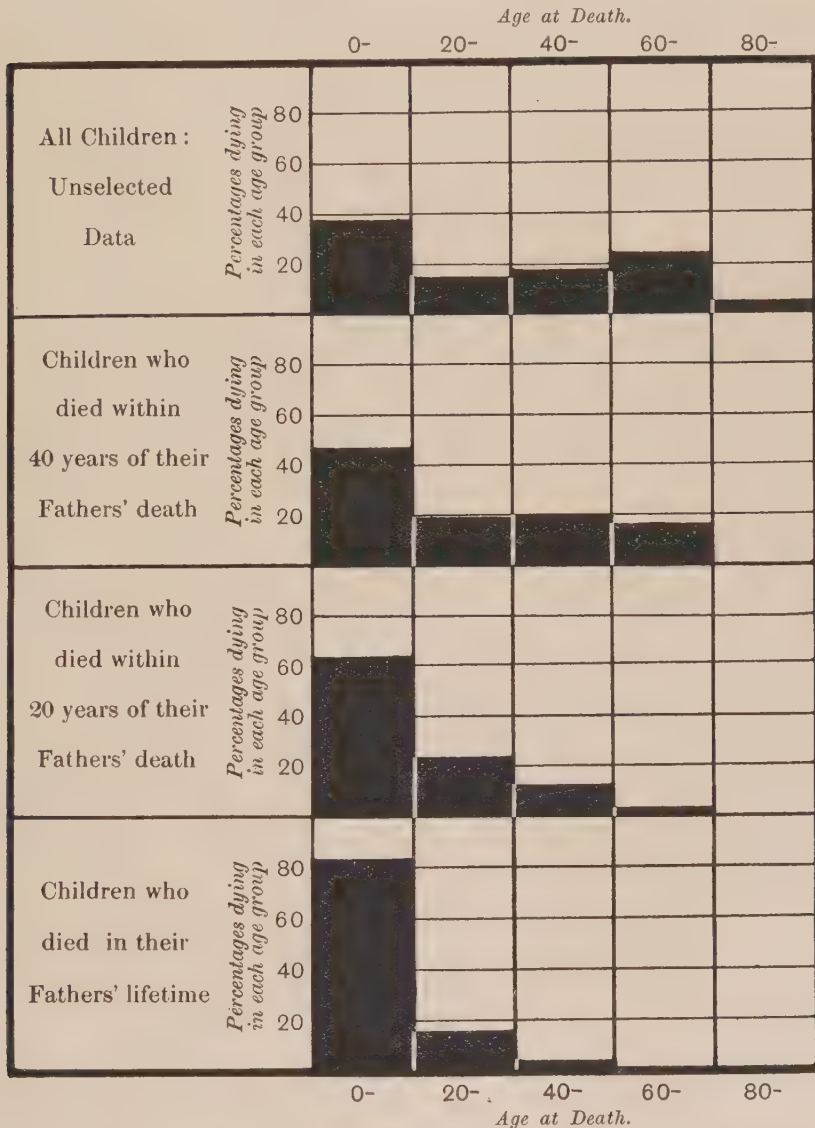


Fig. 6. Diagram to illustrate the Effect of Selection of Material upon the Distribution of Age at Death. (Reigning Houses in Europe, 18th Century.)

Now in this material there is no selection of families. Every family was taken and the age at death of every first-born is known, so that we are only left with the

TABLE VII.

Showing Anticipation in Age at Death. A. Fathers and Children.
(Reigning Houses in Europe—18th Century.)

Age at Death	Fathers	First-born Children	Fathers	First Sons who had children
0—9	—	95	—	—
10—19	—	11	—	—
20—29	6	21	4	8
30—39	16	18	8	15
40—49	45	31	31	39
50—59	70	34	54	39
60—69	77	37	58	44
70—79	62	33	46	54
80—89	18	14	12	13
90 and over	—	—	—	1
Totals	294	294	213	213
Percentage dying under 20	0	36.1	0	0
Average Age at Death ...	61	36	60	59
Anticipation	25		1	

third of Dr Mott's fallacies, in that no allowance has been made for the fact that the parental group is limited to ages over 20 while more than a third of the offspring die under 20. The effect of this selection can be removed almost entirely by taking instead of the first-born child, the first son who married and had at least one child. There are in all 213 such cases and we see that there is now no anticipation. The difference between the average ages at death is less than a year and by removing the artificial selection we have got rid of all anticipation or antedating.

These facts are also shown graphically in Figs. 7 and 8. The horizontal scale gives the age at death in 10-year groups while the vertical scale gives the actual numbers of parents and offspring dying in each age group. The diagram on the left shows marked anticipation, and should be compared with Dr Mott's diagram (Fig. 1) in which the ages at onset of insanity of father and child are compared. When, however, we get rid of the selection of cases by taking only sons who have had children, then there is no anticipation.

If we compare the distributions of age at death in mothers and children we get exactly the same results. The facts are shown in Table VIII.

We see that the first-born children died on an average 18 years before their mothers, but when we compare the age at death of mothers and the first son in

REIGNING HOUSES IN EUROPE — 18TH CENTURY.

AGE AT DEATH OF FATHER & OF FIRST BORN CHILD.

AVERAGE AGE AT DEATH OF —	
FATHERS	61
FIRST BORN CHILDREN	36
ANTICIPATION	25

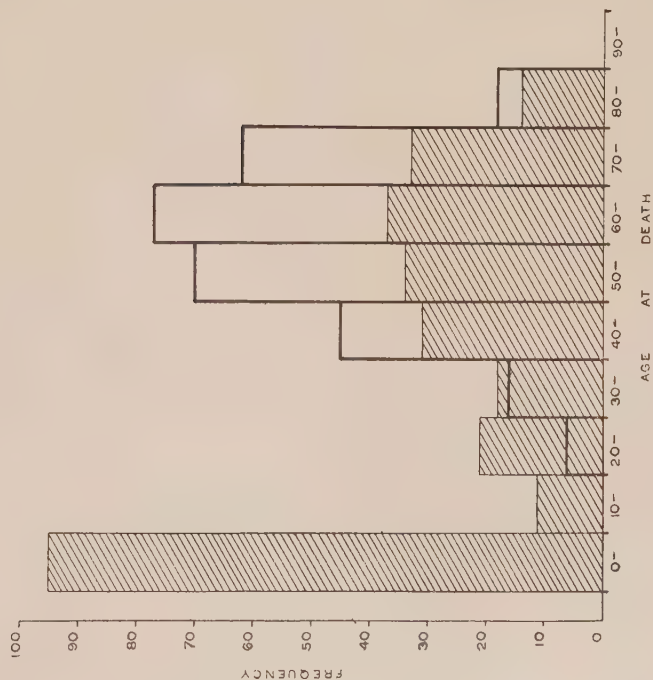


Fig. 7.

AGE AT DEATH OF FATHER & OF FIRST SON TO HAVE CHILDREN.

AVERAGE AGE AT DEATH OF	
FATHERS	60
SONS WITH CHILDREN	59
ANTICIPATION	1

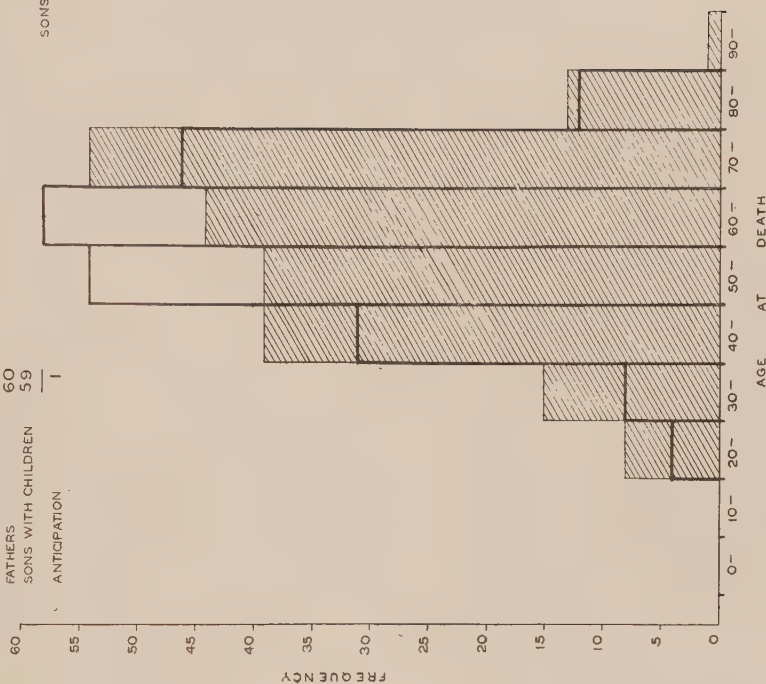


Fig. 8.

TABLE VIII.

Showing Anticipation in Age at Death. B. Mothers and Children.
(Reigning Houses in Europe—18th Century.)

Age at Death	Mothers	First-born Children	Mothers	First Sons to have Children
0—9	—	122	—	—
10—19	2	13	1	—
20—29	47	26	21	8
30—39	49	22	30	16
40—49	43	32	25	41
50—59	52	35	39	40
60—69	80	42	52	46
70—79	52	36	41	54
80—89	18	17	10	14
90 and over	2	—	1	1
Totals	345	345	220	220
Percentage dying under 20	·6	39·1	·5	0
Average Age at Death ...	53	35	55	59
Anticipation	18		—4	

each case to have children, then the sons live four years longer than their mothers. It would have been better in this case to have compared the mothers with the first daughters to have children but unfortunately von Behr gives very little information regarding the female lives, except in special cases. The figures show a marked anticipation in age at death when we directly compare, as Dr Mott has done, mother and child, but this vanishes when we remove the arbitrary selection. The same facts are shown graphically in Figs. 9 and 10.

If we combine these figures we can compare the age at death of parent and child and the results are shown graphically in Figs. 11 and 12.

Fig. 11 shows that Dr Mott's limitation of one of the two generations he is comparing to adults, without imposing a similar limitation on the other generation, introduces an artificial and spurious anticipation. The average age at death of the parents is 56 years and of their first-born children only 35 years—so that we get an anticipation of 21 years. If, however, we make the two generations almost directly comparable by dealing only with sons who have children—there is no significant difference between the two averages (58 against 59 years).

In these cases we have dealt only with completed families and have taken every family without selection. If, however, we consider only the cases in which

REIGNING HOUSES IN EUROPE — 18TH CENTURY.

AGE AT DEATH OF MOTHER & OF FIRST BORN CHILD.

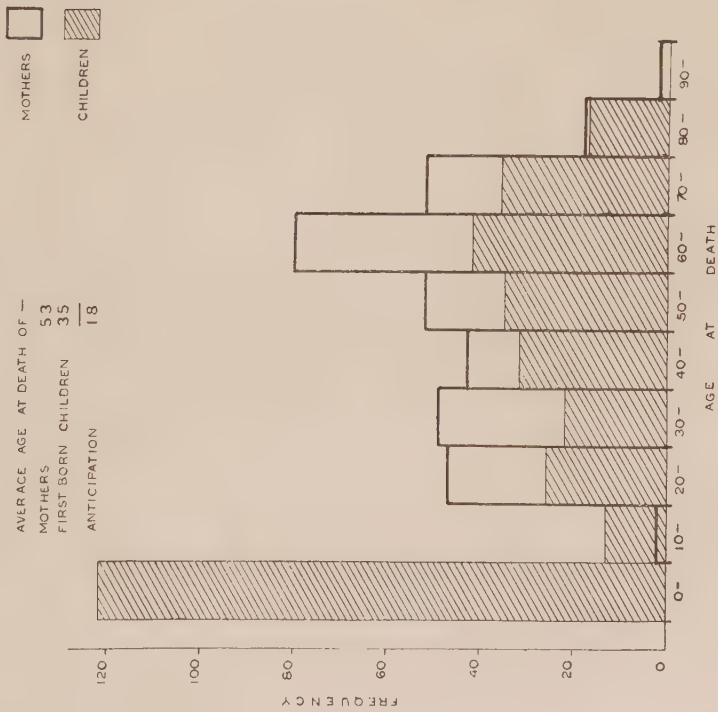


Fig. 9.

AGE AT DEATH OF MOTHER & OF FIRST SON TO HAVE CHILDREN.

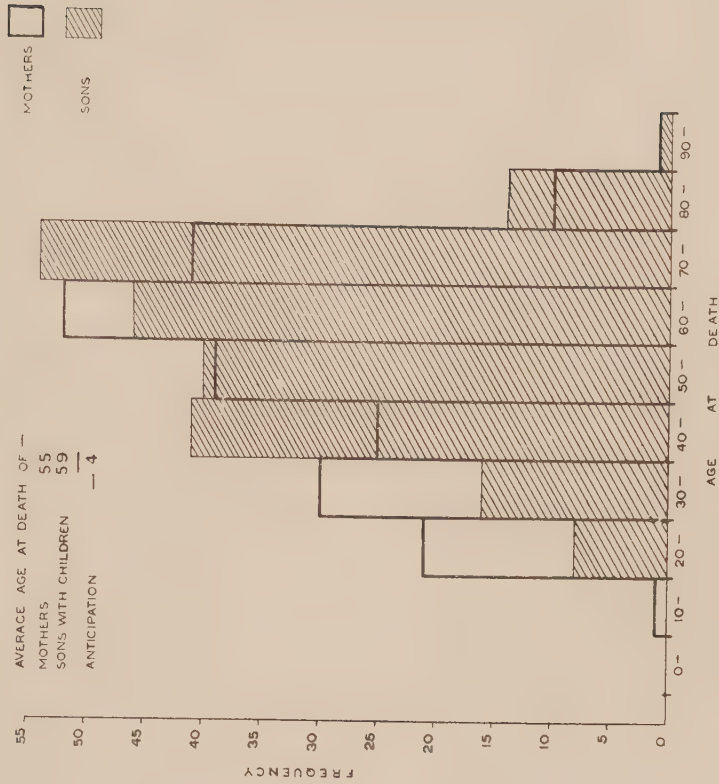


Fig. 10.

REIGNING HOUSES IN EUROPE — 18TH CENTURY.

AGE AT DEATH OF PARENT & OF FIRST BORN CHILD.

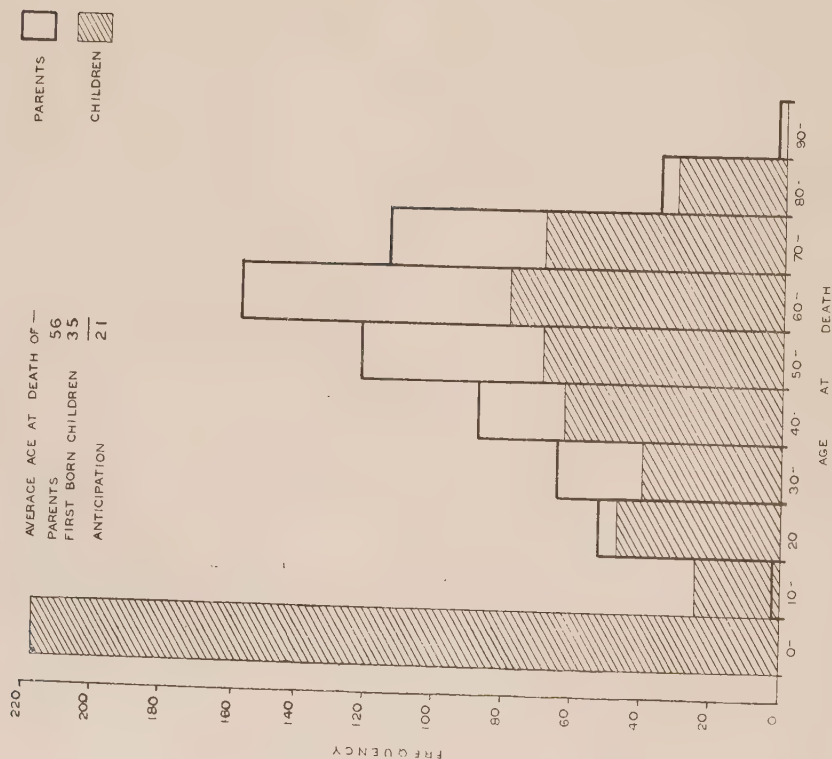


Fig. 11.

AGE AT DEATH OF PARENT & OF FIRST SON TO HAVE CHILDREN.

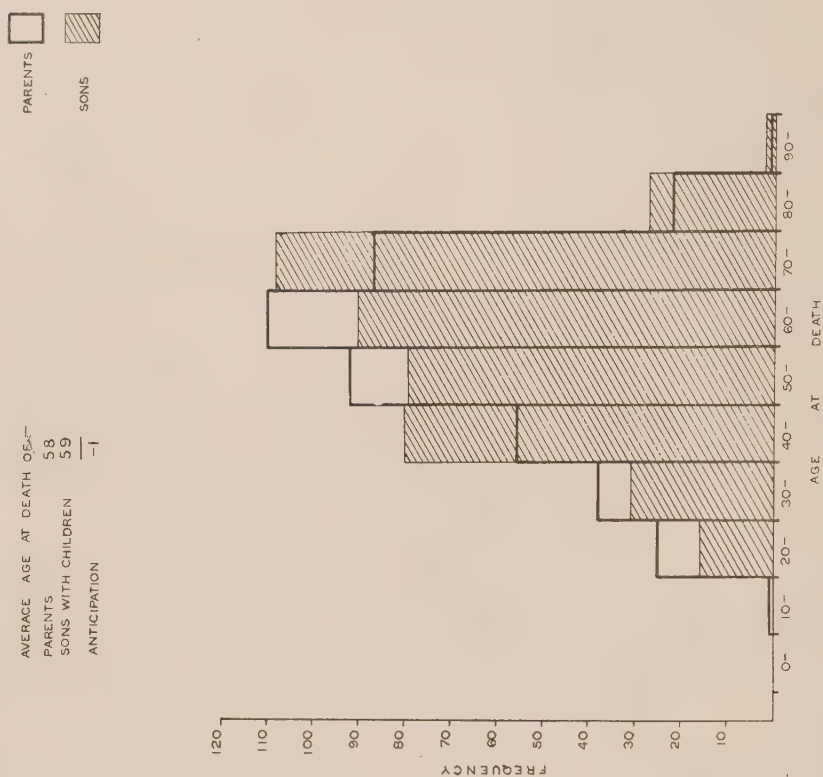


Fig. 12.

the eldest child died in his father's lifetime the amount of anticipation is greatly increased. The facts are shown in Table IX and in Fig. 13.

REIGNING HOUSES IN EUROPE — 18TH CENTURY.

AGE AT DEATH OF FATHERS & OF FIRST BORN CHILDREN,
WHO DIED IN THEIR FATHERS' LIFETIME.

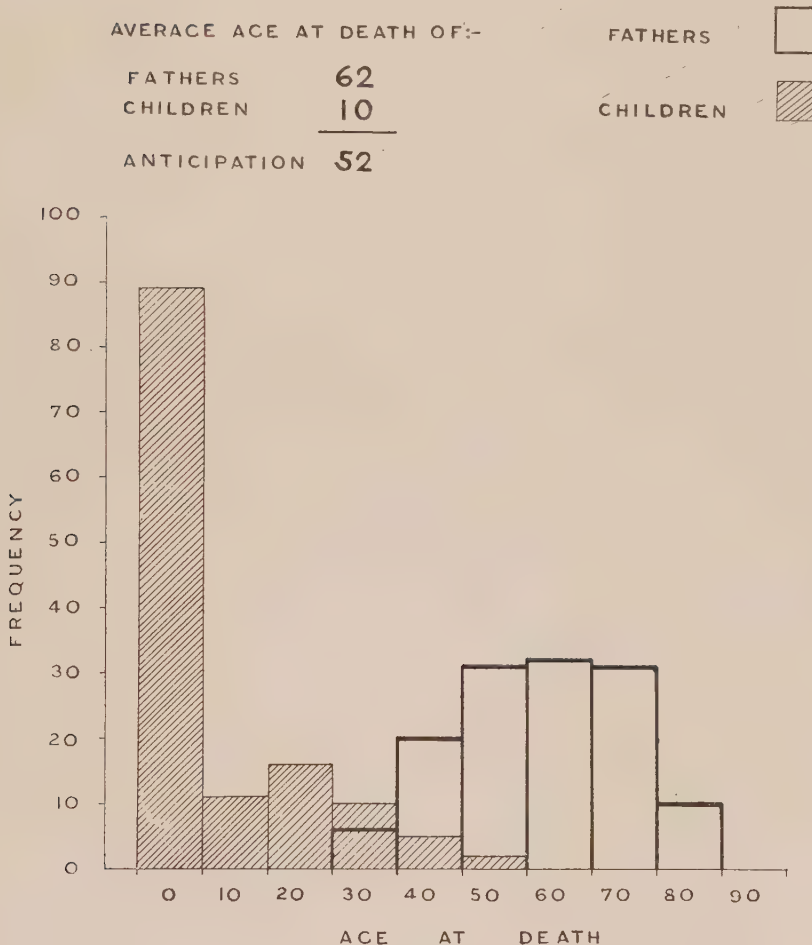


Fig. 13.

We see here that among the fathers none died under 30 while 87% of their children died under 30; the average age at death among the fathers was 62-- among the children only 10, showing an anticipation of 52 years.

TABLE IX.

Showing Anticipation in Age at Death. C. Fathers and First-born Children who died in their Fathers' Lifetime.

(Reigning Houses of Europe: 18th Century.)

Age at Death	Fathers	First-born Children dying in their Fathers' lifetime
0—9	—	89
10—19	—	11
20—29	—	16
30—39	6	10
40—49	20	5
50—59	32	2
60—69	33	—
70—79	32	—
80—89	10	—
Totals	133	133
Percentage dying under 20	0	75.2
Average Age at Death ...	62	10
Anticipation		52

It is now possible to illustrate the effect of the principal fallacies which vitiate Dr Mott's conclusions. In the first place he has dealt with families which are largely incomplete and has collected his material in such a way that cases in which the insanity of parent and child is contemporaneous are the most likely to be recorded; in the second place he has directly compared parent and child without allowing for the fact that practically no *parent* can become insane before 20, while there is no limitation of this kind among the offspring of these insane parents.

In Table IX and Fig. 13 we see the effect of dealing with incomplete families in which the children died in their fathers' lifetime. There we get an anticipation of 52 years. If we get rid of the first and second fallacies involving a selection of cases by dealing with every family, as shown in Table VII and Fig. 7, the anticipation falls to 26 years. If we get rid of the third source of fallacy also, by comparing the fathers with the first sons who have children, as in Table VII and Fig. 8, then the anticipation falls to less than a year. The Law of Anticipation or Antedating has thus in Dr Mott's case no foundation, in fact it is a spurious result of the mode of collecting and interpreting data.

Now Dr Mott has not only asserted that this "Law" applies to insanity but has also drawn the conclusion that the offspring of insane parents if still normal

at the age of 25 may safely marry. In an address delivered before the First International Eugenics Congress*, he said: "You will observe that 47·8% of the 500 offspring had their first attack (of insanity) at or before the age of 25 years and as you see in the curves of parents and offspring, the liability of the child of an insane parent becoming insane tends rapidly to fall. Now besides the fact that this shows Nature's method of eliminating unsound elements of a stock, it has another important bearing, for it shows that after twenty-five there is a greatly decreasing liability of the offspring of insane parents to become insane and therefore in the question of advising marriage of the offspring of an insane parent this is of great importance. Sir George Savage recently said that this statistical proof [*sic*!] of mine entirely accorded with his own experiences, and that if an individual who had such an hereditary history had passed twenty-five and never previously shown any signs (of insanity) he would probably be free and he would offer no objection to marriage."

Now I entirely fail to understand how anyone could recommend marriage in such cases, even on Dr Mott's own figures; for if it be true that 48% become insane before 25, it must be equally true that 52% become insane after that age and this very important point seems to have been forgotten. These figures, however, are taken from Dr Mott's selected data, selected in such a way that the early cases are enormously exaggerated. Until Dr Mott publishes a series of *complete* pedigrees, it will be safer to assume that the age at onset of insanity among the offspring of insane parents does not differ widely from that of all admissions to Asylums and there we find that only 21% become insane before 25, and 79% after 25.

But surely at a Eugenics Congress of all places some thought might have been given to the mental condition of the children resulting from such matings, before advising marriage. It would not have been difficult for Dr Mott to have extracted all the available cases of this kind from his collection of pedigrees, i.e. all cases in which an individual had an insane parent and was normal at the age of 25, and so have discovered the probable fate of the offspring from such matings.

Unfortunately the details given by Dr Mott regarding his pedigrees are usually so scanty that little use of them can be made, but two at least show the danger of the matings Sir George Savage and he sanctioned; these two pedigrees were given by Dr Mott in his lecture on *Heredity in Relation to Insanity*, delivered to the members of the London County Council. The first is shown in Fig. 14. (It appeared as Fig. 11, p. 18 of Dr Mott's lecture.) In the first generation a man who became insane at 70 had four children. The eldest, a girl, became insane at 68 and was therefore normal long after the age of 25. Dr Mott does not state whether the marriage of this woman preceded or followed the onset of insanity in her father, but even if her father had become insane before her marriage, Dr Mott

* *Problems in Eugenics*, p. 425. This is one of many illustrations of the evil done by that Congress; attention was directed and much weight given to hasty statements and ill-digested material.

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would have raised no objection to the marriage since the woman herself was not insane. There were in all six children from this marriage of which Dr Mott would have approved. Two became insane, three were blind and five are said to have been paupers.

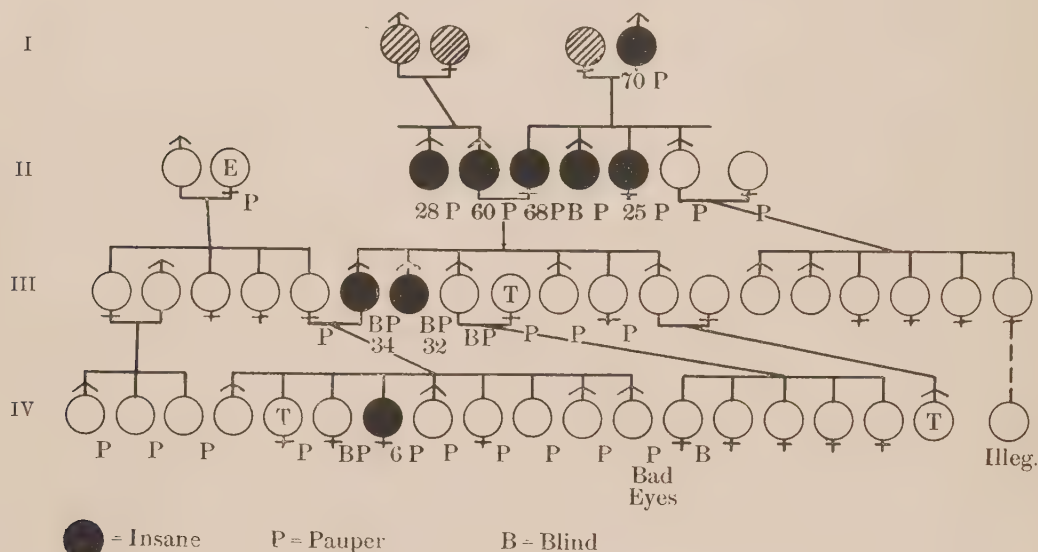


Fig. 14.

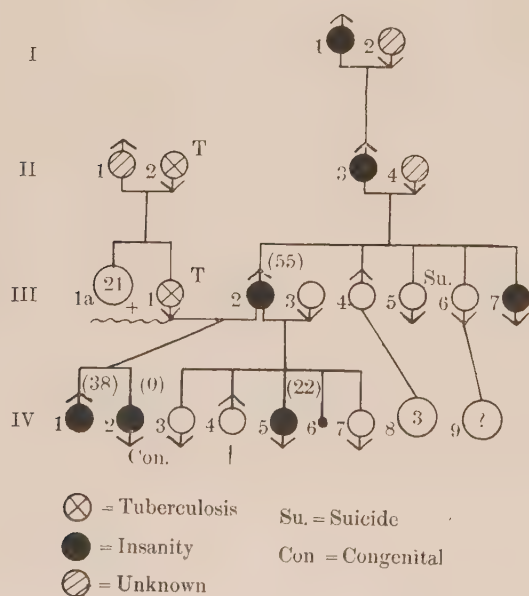


Fig. 15.

The eldest child remained normal till the age of 34 and although both his parents became insane Dr Mott apparently would not have objected to his marrying. He did so and one of his children became insane and eight out of nine are said to be paupers. These nine children are apparently still young so that their ultimate fate is still uncertain.

The second pedigree I shall quote was given as Fig. 28, p. 33 of Dr Mott's lecture, and appears here as Fig. 15.

A man who had an insane father and an insane grandfather became insane at the age of 55. He was therefore normal at the age of 25* and Dr Mott would have sanctioned marriage in his case. He actually married twice. His first wife was tuberculous but not insane; they had two children, both insane. His second wife was normal and it is definitely stated that there was no insanity in her family; they had five children and one of these became insane. Yet Dr Mott would permit the children of insane parents to marry if only they are normal at the age of 25!

Again, Dr Mott has stated that it is useless to attempt to limit the fertility of the insane since most of their children are born before the onset of insanity, and therefore before any action can be taken. From his statistics of relatives in L.C.C. Asylums, Dr Mott has calculated the proportion of offspring who were born after the first attack of insanity in the parent and found that "Forty-six offspring out of 581 were born after the first attack of insanity in the parent, i.e., 7.9%. That is to say in the case of 529 insane parents, *the birth of only one-twelfth of their 581 insane children would have been prevented by sterilisation or life segregation of the parent after the first attack of insanity.* These figures refer to the offspring which become insane, *but there are a large number of offspring which do not become insane and these would be cut off if life segregation or sterilisation were adopted†.*"

But here again Dr Mott is using the data obtained from his index of relatives which shows a greatly exaggerated number of cases at the earlier ages among the offspring, and he thus greatly exaggerates the number of cases in which the children were born *before* the onset of insanity. No conclusion can be drawn from any but complete records of families. But apart altogether from this, many of these parents are themselves the children of the insane and much could be done to discourage such marriages. Unfortunately as we have seen Dr Mott directly sanctions marriage to those who remain normal till the age of 25.

In further support of his view Dr Mott has stated that out of 642 females admitted to three London County Asylums in 1911, 148 were recurrent cases and of these 32 (21%) had children between their respective dates of admission. "The inference that can be drawn," he says, "is that about one-fifth of the recurrent cases, or approximately one-twentieth of the female admissions have

* If the term "age at onset" has any real meaning.

† The italics are Dr Mott's.

children after their first attack of insanity and of 31 such cases examined, 73 children were born after the first attack of insanity in the parent."

But have these 148 recurrent cases been followed up to the end of the reproductive period? Not at all. No ages are given and the cases are merely those which were admitted to Asylums in 1911, Dr Mott's remarks being made in June 1912, so that no attempt has been made to follow them up. There is no justification for Dr Mott's advice.

There are many other points in Dr Mott's work which deserve detailed examination, but time will not permit more than a brief account of a few of them.

It should be noted, for instance, that Dr Mott has used his index of relatives in London County Asylums as an argument in favour of the importance of the inheritance factor in insanity. His argument is as follows:

"At the present time in the London County Asylums there are 725 individuals so closely related as parents and offspring, brothers and sisters. *A priori*, this, to my mind, is striking proof of the importance of heredity in relation to insanity, for we cannot suppose that 20,000 of the $4\frac{1}{2}$ millions of people in London brought together from some random cause would show such a large number closely related as 3.6 %."

But Dr Mott has not attempted to give, and I doubt if he ever will be able to give, a satisfactory estimate of the number of relatives in even a random sample of the population, and the population of asylums is far from being a random sample of the general population—there is for instance an extraordinary divergence in age. Yet without definite information on this point it would be impossible to say whether insanity is inherited or not—that is if we had to depend solely on Dr Mott's data.

It should also be noted that in these cases Dr Mott has clubbed together every form of insanity, from congenital idiocy to senile dementia, except of course cases due to specific infections or trauma. I myself think that course is the only possible one. To anyone who has studied even a few pedigrees of mental defect, nothing is more striking than the extraordinary number of different forms of mental defect that may appear in the same family.

Seven years ago, in a *First Study of the Statistics of Insanity and of the Inheritance of the Insane Diathesis**, I was confronted with the same problem, and after a full consideration of all the available data and of the opinions of those medical men who were best qualified to express an opinion came to the conclusion that the only possible course was to group all forms of insanity together, with, of course, the exceptions I have already indicated. The whole question was discussed very fully in my paper and it was there suggested that an even broader classification might be of service. This point of view met with some criticism at the time but nothing has occurred to alter it, and the study of the inheritance of

* *Galton Memoirs*, No. II. (Dulau and Co.)

insanity in general or of an even broader degeneracy must always remain the first object of our studies.

Any investigation of the inheritance of special types of insanity or degeneracy can only be carried out however on unselected material—on the records of complete families. The type of insanity is so closely related to the age of onset that any tendency to exaggerate the number of early cases, as in Dr Mott's material, will entirely vitiate the conclusions drawn. Thus Dr Schuster's conclusions as to the inheritance of special types of insanity based upon Dr Mott's data* must also be rejected on the above grounds.

Dr Mott's index of relatives in London County Asylums is unfortunately of very little value in the study of inheritance in insanity. Progress can only come from the study of complete pedigrees in which every member of the family is entered, whether insane or normal, and the ages of the normal at the time the record was made are just as important as the age at onset of insanity in the insane members, for a statement that a young man of 20 has not been insane is of a very different degree of importance from the statement that a man of 70 has not been insane.

In the papers I have cited the children of the insane if normal at 25 are advised to marry, and it is asserted that it is useless to attempt to discourage the reproduction of the insane since most of their children are born before the onset of insanity, and that we should rely on the Law of Anticipation to end or mend degenerate stocks.

I have shown, I think, that the Law of Anticipation as applied to the insane has no foundation in the facts provided and that the advice given as to the marriage of the insane and of their normal offspring is fundamentally unsound and directly cacogenic. Much yet remains to be learnt regarding the inheritance of the insane diathesis, but no one who has studied the family histories of the insane can doubt that in inheritance we have by far the most important element in the production of insanity, and in view of all the facts it is the obvious duty of the Eugenist to discourage, rather than to encourage, procreation by the insane and even by those of their offspring who appear to be normal.

* *Report on the Statistical Investigation of Relative Cards*, 21st Annual Report of the London County Council Asylums Committee (1910), p. 95.

ON THE PROBABLE ERROR OF THE BI-SERIAL EXPRESSION FOR THE CORRELATION COEFFICIENT.

By H. E. SOPER, M.A. Biometric Laboratory, University of London.

IN a recent paper* Professor Pearson shows that where one character is in multiple graded grouping and the other in alternative categories, greater or less than a given magnitude, the correlation coefficient admits of simple expression; the assumptions being that the unmeasured character, B , has a normal distribution and that the measured character, A , has linear regression upon B . Under these conditions the data required are the numerical ratio of the alternative groups, the standard deviation of the measured character and the deviation from the general mean of this character of the mean of one of the groups.

This expression is subject to greater fluctuations of value in samples of N of the population than is the product moment form, especially where one of the groups is relatively small; and it is proposed to find formulae for the mean and second moment of the errors from this mean to a first approximation, that is to terms in $1/N$. These will appear in terms of the correlation coefficient, r , of the original population (which will be supposed *normally* correlated) and the fractional frequency, f , in that population of the group possessing the greater or positive intensity of the character put into two classes.

Let y be the graded character and x the alternative character the intenser value of which is possessed by the fraction f of the population. Let $\bar{x}$, $\bar{y}$ be the general means and σ_x , σ_y the standard deviations of x and y . Then it is shown in the paper that if $\bar{x}'$, $\bar{y}'$ are the means of the group f ,

$$r = \frac{\bar{y}' - \bar{y}}{\sigma_y} \times \frac{\sigma_x}{\bar{x}' - \bar{x}} \dots\dots\dots(1),$$

on the assumption that the regression of y upon x is linear; and that if x be normally distributed this is equivalent to

$$r = \frac{\bar{y}' - \bar{y}}{\sigma_y} \times \frac{f}{z} \dots\dots\dots(2),$$

* *Biometrika*, Vol. VII. (1909), p. 96: "On a New Method of Determining Correlation between a Measured Character A , and a Character B , of which only the Percentage of Cases wherein B exceeds (or falls short of) a given intensity is recorded for each grade of A ."

where z is the ordinate of the normal curve cutting off the area

$$f = \frac{1}{2}(1 - \alpha) \quad \text{or} \quad \frac{1}{2}(1 + \alpha)$$

as defined in Sheppard's Tables of the Probability Integral.

Now if p_0, p_1, p_2 , etc.

be the moment coefficients of the whole population with respect to the character y defined by

$$p_v = S(n_s y_s^v) / N \dots\dots\dots (3),$$

[see bi-serial table below which is here to be looked upon as representing the general population] and

$$p'_0, p'_1, p'_2, \text{ etc.}$$

are moment coefficients of the group $n' (=fN)$ defined by

$$p'_v = S(n'_s y_s^v) / N \dots\dots\dots (4),$$

we may write $f = p'_0, f\bar{y}' = p'_1, \bar{y} = p_1, \sigma_y = \sqrt{(p_2 - p_1^2)}$ and

$$r = \frac{p'_1 - p'_0 p_1}{\sqrt{(p_2 - p_1^2)}} \times \frac{1}{z} \dots\dots\dots (5).$$

→ Grades of y in Bi-serial Table.

↓ Grades of x .	n''_1	n''_2	n''_3	...	n''_s	...	n''	
	n'_1	n'_2	n'_3	...	n'_s	...	n'	
	n_1	n_2	n_3	...	n_s	...	N	(6).
	$n' / N = f$							

In samples of N the frequencies n_s, n'_s, n''_s and consequently the moment coefficients p and p' and the ordinate z are subject to fluctuation and the values of the correlation coefficient calculated from this formula will have a distribution of errors. Let $\bar{r}$ be the mean value in such samples, δr the deviation from this mean value in any sample when $\delta p_0, \delta p_1, \delta p'_0$, etc. δz are the deviations in moments and ordinate. Then

$$\bar{r} + \delta r = \frac{p'_1 + \delta p'_1 - (p'_0 + \delta p'_0)(p_1 + \delta p_1)}{\sqrt{\{p_2 + \delta p_2 - (p_1 + \delta p_1)^2\}} \times (z + \delta z)} \dots\dots\dots (7).$$

To express δz in terms of deviations of the moments we have

$$z = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a^2} \dots\dots\dots (8),$$

where a is the abscissa of the point of section of the normal curve, defined by

$$f = \int_a^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \dots\dots\dots (9).$$

Hence to second order terms in powers and products of deviations

$$\begin{aligned}
 z + \delta z &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(a + \delta a)^2} \\
 &= z \left\{ 1 - a\delta a - \frac{1}{2}(1 - a^2)(\delta a)^2 \right\} \\
 -\delta f &= \int_a^{a+\delta a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx \\
 &= \int_0^{\delta a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(a+\xi)^2} d\xi \\
 &= z \int_0^{\delta a} \left\{ 1 - a\xi - \frac{1}{2}(1 - a^2)\xi^2 + \dots \right\} d\xi \\
 &= z \left\{ \delta a - \frac{1}{2}a(\delta a)^2 \right\}.
 \end{aligned}$$

It follows that

$$\begin{aligned}
 z + \delta z &= z + a\delta f - \frac{1}{2z}(\delta f)^2 \\
 &= z + a\delta p'_0 - \frac{1}{2z}(\delta p'_0)^2 \dots\dots\dots(10).
 \end{aligned}$$

At the same time that this value is put in (7) we may simplify the expression and subsequent algebra by supposing the graded character y to be measured from its universal mean value as origin and in terms of its standard deviation as unit of measurement in which case

$$p_1 = 0, \quad p_2 = \sigma_y^2 = 1, \quad \text{and by (5)} \quad p'_1 = zr \dots\dots\dots(11),$$

and since $p'_0 = f$ (7) becomes,

$$\bar{r} + \delta r = \frac{zr + \delta p'_1 - f\delta p_1 - \delta p'_0\delta p_1}{\sqrt{\{1 + \delta p_2 - (\delta p_1)^2\}} \times \left\{ z + a\delta p'_0 - \frac{1}{2z}(\delta p'_0)^2 \right\}} \dots\dots\dots(12).$$

Expanding to second order of deviations we find,

$$\bar{r} + \delta r = r + \delta_1 + \delta_2 \dots\dots\dots(13),$$

where δ_1, δ_2 are the first order and second order expressions

$$\begin{aligned}
 \delta_1 &= \frac{1}{z} \{ \delta p'_1 - f\delta p_1 - \frac{1}{2}zr\delta p_2 - ar\delta p'_0 \}, \\
 \delta_2 &= \frac{1}{z} \left\{ \frac{1}{2}ar\delta p_2\delta p'_0 - \frac{a}{z}\delta p'_1\delta p'_0 + \frac{fa}{z}\delta p_1\delta p'_0 - \frac{1}{2}\delta p'_1\delta p_2 + \frac{1}{2}f\delta p_1\delta p_2 \right. \\
 &\quad \left. - \delta p'_0\delta p_1 + \frac{1}{2}zr(\delta p_1)^2 + \frac{3}{8}zr(\delta p_2)^2 + (1 + 2a^2)\frac{r}{2z}(\delta p'_0)^2 \right\} \dots\dots(14).
 \end{aligned}$$

Taking mean values

$$\bar{r} = r + \text{mean } \delta_2 \dots\dots\dots(15),$$

mean δ_1 being zero since by (3), (4)

$$\begin{aligned}
 \text{mean } \delta p_v &= S(\text{mean } \delta n_s y_s^v) / N = 0, \\
 \text{mean } \delta p'_v &= S(\text{mean } \delta n'_s y_s^v) / N = 0.
 \end{aligned}$$

Mean δ_3 is to be evaluated by the formulae

$$\left. \begin{aligned} \text{mean } \delta p_u \delta p_v &= (p_{u+v} - p_u p_v) / N \\ \text{mean } \delta p_u' \delta p_v' &= (p'_{u+v} - p_u' p_v') / N \\ \text{mean } \delta p_u \delta p_v' &= (p'_{u+v} - p_u p_v') / N \end{aligned} \right\} \dots\dots\dots (16),$$

of which the first two are well known* and the third may be proved thus:

$$N \delta p_u = S(\delta n_s y_s^u) = S(\delta n_s' y_s^u) + S(\delta n_s'' y_s^u),$$

$$N \delta p_v' = S(\delta n_s' y_s^v),$$

$$\therefore N^2 \delta p_u \delta p_v' = S\{(\delta n_s')^2 y_s^{u+v}\} + S\{\delta n_s' \delta n_s' y_s^u y_s^v\} + S\{\delta n_s' \delta n_s'' y_s^v y_s^u\},$$

where in the third sum s' may or may not equal s .

$$\begin{aligned} \text{But} \quad \text{mean } (\delta n_s')^2 &= n_s' (1 - n_s' / N), \\ \text{mean } (\delta n_s' \delta n_s') &= -n_s' n_s' / N, \\ \text{mean } (\delta n_s' \delta n_s'') &= -n_s' n_s'' / N, \end{aligned}$$

the last whether $s = s'$ or not. Thus we find†

$$\text{mean } \delta p_u \delta p_v' = (p'_{u+v} - p_u' p_v' - p_u'' p_v') / N = (p'_{u+v} - p_u p_v') / N.$$

Evaluating mean δ_3 by these formulae we find,

$$\begin{aligned} \bar{r} = r + \frac{1}{Nz} \left\{ \frac{1}{2} ar (p_2' - p_2 p_0') - \frac{a}{z} (p_1' - p_1' p_0') \right. \\ \left. + \frac{fa}{z} (p_1' - p_1 p_0') - \frac{1}{2} (p_3' - p_1' p_2) + \frac{1}{2} f (p_3 - p_1 p_2) \right. \\ \left. - (p_1' - p_0' p_1) + \frac{1}{2} zr (p_2 - p_1^2) + \frac{3}{8} zr (p_4 - p_2^2) + (1 + 2a^2) \frac{r}{2z} (p_0' - p_0'^2) \right\} \dots (17), \end{aligned}$$

in which the undashed moments, being those of a normal curve with unit standard deviation, about its mean, have the values

$$p_1 = 0, \quad p_2 = 1, \quad p_3 = 0, \quad p_4 = 3,$$

and the dashed moments beyond the first two,

$$p_0' = f, \quad p_1' = zr,$$

have values depending upon the nature of the frequency distribution of y and x .

Assuming x, y normally distributed‡

$$p_2' = \int_a^\infty \int_{-\infty}^\infty y^2 \cdot \frac{1}{2\pi \sqrt{(1-r^2)}} e^{-\frac{1}{2(1-r^2)} \{x^2 - 2rxy + y^2\}} dx dy,$$

$$p_3' = \int_a^\infty \int_{-\infty}^\infty y^3 \cdot \frac{1}{2\pi \sqrt{(1-r^2)}} e^{-\frac{1}{2(1-r^2)} \{x^2 - 2rxy + y^2\}} dx dy.$$

* See *Biometrika*, Vol. II. (1903), p. 275: "On the Probable Errors of Frequency Constants." K. Pearson. The second follows in exactly the same manner as the first, since the constancy of the total frequency dealt with is only involved, in deducing the relations (i) and (ii) of p. 274, of that paper.

† $p_u'' = S(n_s'' y_s^u) / N$.

‡ Since the moments appear in the term containing $1/N$ any errors in their calculation due to incorrect assumption of normality will not affect the present approximate formulae provided such errors are of the order $1/N$.

388 *Probable Error of the Bi-serial Correlation Coefficient*

Putting $y = rx + \eta$ and integrating with respect to η for constant x ,

$$p_2' = \int_a^\infty \{(rx)^2 + (1 - r^2)\} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

$$= f' + ar^2 \dots\dots\dots(18),$$

$$p_3' = \int_a^\infty \{(rx)^3 + 3(rx)(1 - r^2)\} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

$$= z(3 - r^2 + a^2r^2)r \dots\dots\dots(19).$$

When these values are put in (17), and terms collected, the mean value of the bi-serial correlation coefficient in samples of N is found to be

$$\bar{r} = r \left\{ 1 + \frac{1}{N} \left[\frac{1}{4} + \frac{1}{2} \frac{ff''}{z^2} - \left(1 - \frac{fa}{z} \right) \left(1 + \frac{f'a}{z} \right) + \frac{1}{2} r^2 \right] \right\} \dots\dots\dots(20),$$

where $f' = 1 - f = n''/N$.

In the work of obtaining this approximation all powers and products of deviations above the second order have been neglected. The means of such terms in samples of N involve second and higher powers of $1/N^*$ and the present result is correct to the first approximation.

Again squaring (13) and taking mean values and subtracting the square of (15) we find to the same approximation as before

$$\sigma_r^2 = \text{mean } (\delta r)^2 = \text{mean } \delta_1^2 \dots\dots\dots(21).$$

The evaluation of mean δ_1^2 being carried out precisely in the same way as mean δ_2 , the result is the second moment of deviations of the bi-serial correlation coefficient in samples of N ,

$$\sigma_r^2 = \frac{1}{N} \left[\frac{ff''}{z^2} - \left\{ \frac{3}{2} + \left(1 - \frac{fa}{z} \right) \left(1 + \frac{f'a}{z} \right) \right\} r^2 + r^4 \right] \dots\dots\dots(22).$$

Writing the two results (20) and (22)

$$\bar{r} = r \left\{ 1 + \frac{1}{N} (\phi_a + \frac{1}{2} r^2) \right\},$$

$$\sigma_r^2 = \frac{1}{N} [\chi_a^2 - \psi_a r^2 + r^4] \dots\dots\dots(23),$$

the values of ϕ_a , χ_a^2 † and ψ_a for values of $\frac{1}{2}(1 - \alpha)$ [= the smaller of n'/N , n''/N] from .50 to .01 are to be found in table (24).

* See *Biometrika*, Vol. ix. (1913), pp. 97—99.

† χ_a for $\frac{1}{2}(1 + \alpha)$ was tabled in *Biometrika*, Vol. ix. p. 27, and the table is reproduced in *Tables for Statisticians and Biometricians*, p. 35, Cambridge University Press.

$\frac{1}{2}(1-a)$	ϕ_a	χ_a^2	ψ_a	$\frac{1}{2}(1-a)$	ϕ_a	χ_a^2	ψ_a
.50	.0354	1.5708	2.5000	.20	— .0871	2.0414	2.8578
.49	.0353	1.5711	2.5003	.19	— .1001	2.0898	2.8951
.48	.0350	1.5722	2.5011	.18	— .1146	2.1437	2.9364
.47	.0346	1.5741	2.5024	.17	— .1308	2.2035	2.9825
.46	.0339	1.5766	2.5043	.16	— .1490	2.2703	3.0341
.45	.0331	1.5799	2.5068	.15	— .1696	2.3453	3.0923
.44	.0321	1.5839	2.5098	.14	— .1931	2.4303	3.1582
.43	.0309	1.5886	2.5134	.13	— .2201	2.5272	3.2337
.42	.0295	1.5943	2.5177	.12	— .2513	2.6389	3.3208
.41	.0279	1.6007	2.5225	.11	— .2880	2.7687	3.4224
.40	.0260	1.6079	2.5279	.10	— .3317	2.9221	3.5427
.39	.0240	1.6161	2.5341	.095	— .3568	3.0095	3.6115
.38	.0217	1.6251	2.5409	.090	— .3844	3.1057	3.6873
.37	.0191	1.6351	2.5484	.085	— .4153	3.2119	3.7713
.36	.0163	1.6461	2.5568	.080	— .4499	3.3299	3.8648
.35	.0132	1.6582	2.5659	.075	— .4887	3.4620	3.9696
.34	.0098	1.6714	2.5759	.070	— .5324	3.6110	4.0879
.33	.0062	1.6858	2.5868	.065	— .5822	3.7806	4.2224
.32	.0021	1.7015	2.5986	.060	— .6403	3.9748	4.3777
.31	— .0023	1.7186	2.6115	.055	— .7083	4.2002	4.5584
.30	— .0070	1.7371	2.6256	.050	— .7897	4.4652	4.7723
.29	— .0122	1.7573	2.6409	.045	— .8868	4.7829	5.0283
.28	— .0179	1.7791	2.6575	.040	— 1.0053	5.1715	5.3410
.27	— .0241	1.8028	2.6755	.035	— 1.1577	5.6568	5.7362
.26	— .0308	1.8286	2.6952	.030	— 1.3556	6.2859	6.2485
.25	— .0382	1.8567	2.7166	.025	— 1.6308	7.1347	6.9481
.24	— .0462	1.8874	2.7399	.020	— 2.0272	8.3600	7.9572
.23	— .0550	1.9208	2.7654	.015	— 2.5263	10.3024	9.4275
.22	— .0647	1.9574	2.7933	.010	— 3.8889	13.9393	12.6086
.21	— .0753	1.9974	2.8240				

..... (24).

The bi-serial value of the correlation coefficient has the standard deviation

$$\sigma_r = \sqrt{(\chi_a^2 - \psi_a r^2 + r^4)} / \sqrt{N},$$

whilst that of the product moment value is

$$\sigma_r = (1 - r^2) / \sqrt{N}.$$

In table (25) a comparison of the values of the numerator is made for five values of r , for divisions at 0, .5, ...2.5 times the standard deviation from the mean of the ungraded character.

r	Values of (1 - r^2)	Values of $\sqrt{(\chi_a^2 - \psi_a r^2 + r^4)}$ for $\frac{1}{2}(1-a) =$					
		.500	.309	.159	.067	.023	.006
.00	1.00	1.25	1.31	1.51	1.93	2.76	4.5
.25	.9375	1.19	1.25	1.45	1.86	2.68	4.3
.50	.750	1.00	1.06	1.26	1.65	2.42	4.0
.75	.4375	.69	.75	.94	1.30	1.95	3.2
1.00	.00	.27	.33	.49	.74	1.13	1.8

..... (25).

Thus the effect of grouping and applying the bi-serial value of the correlation coefficient is to add 25% to the probable error in the most accordant case where r is zero and the division equal, whilst if r is as large as .5 and one group as small as 10% of the whole the probable error is nearly doubled. For higher values of r the errors of sampling, in the case of the product moment formula, grow smaller and ultimately vanish when $r=1$; but the bi-serial values are not invariable in samples drawn from a perfectly correlated population but possess a variability as high as $\cdot 27/\sqrt{N}$ in the most favourable case when the grouping is equal.

If the standard deviation be calculated from the approximate formula,

$$\sigma_r = (\sqrt{f\bar{f}}/z - r^2)/\sqrt{N} \dots\dots\dots(26),$$

which may be written*

$$\sigma_r = (\chi_a - r^2)/\sqrt{N} \dots\dots\dots(27),$$

the error of computation will not be great for values of f and r commonly met with as the following table compared with the last will show:

r	Values of $\chi_a - r^2$ for $\frac{1}{2}(1-a)=$					
	.500	.309	.159	.067	.023	.006
.00	1.25	1.31	1.51	1.93	2.76	4.5
.25	1.19	1.25	1.45	1.86	2.70	4.4
.50	1.00	1.06	1.26	1.68	2.51	4.2
.75	.69	.75	.95	1.36	2.20	3.9
1.00	.25	.31	.51	.93	1.76	3.5

.....(28).

The difference between the two expressions only reaches 5% when the smaller group is less than 7% of the whole.

It will not be necessary, excepting in small samples, to apply a correction to the bi-serial formula for r in virtue of the mean of samples differing from the population value. The correction is less than $1/N$ th part of the value calculated unless one of the alternative classes is as small as 4% of the whole.

I have to thank fellow members of the Staff for assistance in calculating the tables.

* For a table of χ_a see *Tables for Statisticians and Biometricians*, p. 37.

ON THE PARTIAL CORRELATION RATIO.

PART I. THEORETICAL.

By L. ISSERLIS, B.A.

§ 1. The theory of non-linear regression in the case of two correlated variables is due to Prof. Karl Pearson*. He shows that regression ceases to be linear when the correlation ratio η differs sensibly from the correlation coefficient r and establishes criteria for parabolic, cubic and higher forms of regression.

The present paper deals with the regression surface of three correlated variables x, y, z , where, though the regression of z on x, y cannot be adequately represented by an equation of type

$$\frac{\bar{z}_{xy} - \bar{z}}{\sigma_z} = \gamma_3 \frac{x - \bar{x}}{\sigma_x} + \gamma_3' \frac{y - \bar{y}}{\sigma_y} \dots\dots\dots(1),$$

the regression of z on x for a constant y and of z on y for a constant x is linear. A large proportion of the non-linear cases that occur in practice fall into this class. It will be remembered that $\bar{z}_{xy}$ in (1) denoting the mean of the array of z 's for a given x and y the coefficients γ_3, γ_3' are partial regression coefficients and it will appear that just as it is necessary to introduce the correlation ratio η for an adequate description of non-linear regression of two variables, there must be introduced multiple or partial η 's for the description of such regression in the case of more than two variables.

We recall the definition and principal properties of ${}_x\eta_y$ —the correlation ratio of y on x , σ_{a_y} being the square root mean weighted square standard deviations of the arrays of y :

$$(1 - \eta^2) \sigma_y^2 = \sigma_{a_y}^2 = \frac{S(n_x \sigma_{n_x}^2)}{N} = \frac{SS \{n_{xy} (y - \bar{y}_x)^2\}}{N} \dots\dots\dots(2),$$

$$\eta^2 = \frac{\sigma_{m_y}^2}{\sigma_y^2} = \frac{S \{n_x (\bar{y}_x - \bar{y})^2\}}{N \sigma_y^2} \dots\dots\dots(3),$$

and

$$N (\eta^2 - r^2) \sigma_y^2 = S \{n_x (y_{n_x} - Y)^2\} \dots\dots\dots(4).$$

* *Drapers' Company Research Memoirs. Mathematical Contributions to the Theory of Evolution.*
XIV. "On the General Theory of Skew Correlation and Non-linear Regression." 1905.

Here we are dealing with N pairs of two characters A and B . n_x of these have the character x of A . $\bar{y}_x$ is the mean of this x -array of B 's. σ_{n_x} is the standard deviation of this array, σ_{m_y} is the weighted standard deviation of the means of the arrays and Y is the value of y given by the regression straight line, i.e.

$$Y = \bar{y} + r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \dots\dots\dots (5).$$

This is the "best fitting" straight line (in the Gaussian sense) to the means of the arrays, and is the regression line when the regression is linear.

§ 2. Consider now three correlated characters A, B, C . If N combinations of A, B, C are taken, we may denote by n_x the number of these which have the character $A = x$ and by n_{xy} the number in which A has the value x and B has the value y . Let $\bar{x}, \bar{y}, \bar{z}$ be the mean values of the total population, and let $\bar{z}_{xy}$ be the mean of z for a given x and y . The frequency of $A = x, B = y, C = z$ is n_{xyz} .

We define the correlation ratio of z on x and y , which may be denoted by ${}_{xy}H_z$, or if no confusion is likely to arise by H_z , by the equation

$$\sigma_z^2 [1 - {}_{xy}H_z^2] = \frac{SSS \{n_{xyz} (z - \bar{z}_{xy})^2\}}{N} \dots\dots\dots (6).$$

The triple sum in the definition can be written

$$\begin{aligned} & SSS \{n_{xyz} (z - \bar{z} + \bar{z} - \bar{z}_{xy})^2\} \\ &= SS \{n_{xy} (\bar{z} - \bar{z}_{xy})^2\} + 2SS \{(\bar{z} - \bar{z}_{xy})\} \times S \{n_{xyz} (z - \bar{z})\} + SSS \{n_{xyz} (z - \bar{z})^2\} \\ &= SS \{n_{xy} (\bar{z} - \bar{z}_{xy})^2\} - 2SS \{n_{xy} (\bar{z} - \bar{z}_{xy})\} + N\sigma_z^2. \end{aligned}$$

$$\text{Hence } {}_{xy}H_z^2 = \frac{SS \{n_{xy} (\bar{z} - \bar{z}_{xy})^2\}}{N\sigma_z^2} \dots\dots\dots (7)$$

This is a generalisation of the property of ${}_{x\eta_y}$ given by (3).

Further, the "best fitting" plane to the means $\bar{z}_{xy}$ is given by

$$\frac{z - \bar{z}}{\sigma_z} = \gamma_3 \frac{x - \bar{x}}{\sigma_x} + \gamma_3' \frac{y - \bar{y}}{\sigma_y} \dots\dots\dots (8),$$

where

$$\gamma_3 = \frac{r_{xz} - r_{yz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots (9),$$

$$\gamma_3' = \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots (10).$$

Let ${}_{xy}R_z$ denote as usual the maximum correlation of z with any linear function of x and y , then

$${}_{xy}R_z^2 = \gamma_3 r_{zx} + \gamma_3' r_{zy} \dots\dots\dots (11),$$

$$= \frac{r_{yz}^2 + r_{zx}^2 - 2r_{yz}r_{xz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots (12).$$

Subtract (11) from (7) after replacing r_{zx} and r_{zy} by the appropriate sums, and we obtain

$$\begin{aligned} & [{}_{xy}H_z^2 - {}_{xy}R_z^2] N\sigma_z^2 \\ &= SS \left[n_{xy}(\bar{z} - \bar{z}_{xy})^2 - S \left\{ n_{xyz} \frac{\sigma_z}{\sigma_x} (z - \bar{z})(x - \bar{x}) \gamma_3 \right\} - S \left\{ n_{xyz} \frac{\sigma_z}{\sigma_y} (z - \bar{z})(y - \bar{y}) \gamma_3' \right\} \right] \\ &= SS \left[n_{xy}(\bar{z} - \bar{z}_{xy})^2 - n_{xy} \frac{\sigma_z}{\sigma_x} (\bar{z}_{xy} - \bar{z})(x - \bar{x}) \gamma_3 - n_{xy} \frac{\sigma_z}{\sigma_y} (\bar{z}_{xy} - \bar{z})(y - \bar{y}) \gamma_3' \right]. \end{aligned}$$

Using (8) this can be written

$$\begin{aligned} [{}_{xy}H_z^2 - {}_{xy}R_z^2] N\sigma_z^2 &= SS \{ n_{xy}(\bar{z}_{xy} - \bar{z})(\bar{z}_{xy} - Z) \} \\ &= SS \{ n_{xy}(\bar{z}_{xy} - Z)^2 \} + SS \{ n_{xy}(Z - \bar{z})(\bar{z}_{xy} - Z) \} \dots (13). \end{aligned}$$

But $SS \{ n_{xy}(Z - \bar{z})(\bar{z}_{xy} - Z) \}$

$$\begin{aligned} &= SS \left\{ n_{xy} \left(\gamma_3 \frac{x - \bar{x}}{\sigma_x} \sigma_z + \gamma_3' \frac{y - \bar{y}}{\sigma_y} \sigma_z \right) (\bar{z}_{xy} - \bar{z} - \gamma_3 \frac{x - \bar{x}}{\sigma_x} \sigma_z - \gamma_3' \frac{y - \bar{y}}{\sigma_y} \sigma_z) \right\} \\ &= N\sigma_z (\gamma_3 \sigma_z r_{zx} - \gamma_3^2 \sigma_z - \gamma_3 \gamma_3' r_{xy} \sigma_z + \gamma_3' \sigma_z r_{yz} - \gamma_3 \gamma_3' r_{xy} \sigma_z - \gamma_3'^2 \sigma_z) \\ &= N\sigma_z^2 \{ \gamma_3(r_{zx} - \gamma_3 - \gamma_3' r_{xy}) + \gamma_3'(r_{yz} - \gamma_3' - \gamma_3 r_{xy}) \} \dots (14). \end{aligned}$$

Using the values of γ_3 and γ_3' given by (9) and (10) we see immediately that

$$r_{zx} - \gamma_3 - \gamma_3' r_{xy} = r_{yz} - \gamma_3' - \gamma_3 r_{xy} = 0.$$

Hence (13) becomes

$$[{}_{xy}H_z^2 - {}_{xy}R_z^2] N\sigma_z^2 = SS \{ n_{xy}(\bar{z}_{xy} - Z)^2 \} \dots (15).$$

This is the generalisation of (4). We deduce that ${}_{xy}H_z = {}_{xy}R_z$ if and only if the regression is strictly linear, that otherwise ${}_{xy}H_z^2 > {}_{xy}R_z^2$ and by (6) that ${}_{xy}H_z^2 < 1$.

§ 3. These properties and definitions can be extended to the case of m variables $x_1, x_2, \dots, x_m$. We now use ${}_{2\dots m}\bar{x}_1$ for the mean of x_1 when $x_2, x_3, \dots, x_m$ are given and denote by $S_{1\dots m}$ a summation extending to the variables $x_1, x_2, \dots, x_m$.

If we define the correlation ratio of x_1 on $x_2, x_3, \dots, x_m$ by the equation

$$N\sigma_1^2(1 - {}_{2,3\dots m}H_1^2) = S_{1\dots m} (x_1 - {}_{2,3\dots m}\bar{x}_1)^2 n_{1\dots m} \dots (16).$$

We can deduce in the same way as in § 2, the relation

$$N\sigma_{1,2,3\dots m}^2 H_1^2 = S_{2\dots m} \{ (\bar{x}_1 - {}_{2,3\dots m}\bar{x}_1)^2 n_{2\dots m} \} \dots (17).$$

In order to generalise (15) we recall that the "best fitting" linear function of the variables $x_2, x_3, \dots, x_m$ to the mean ${}_{2,3\dots m}\bar{x}_1$ is

$$\frac{X_1}{\sigma_1} = \frac{\bar{x}_1}{\sigma_1} - b_{12} \frac{x_2 - \bar{x}_2}{\sigma_2} - b_{13} \frac{x_3 - \bar{x}_3}{\sigma_3} - \dots - b_{1m} \frac{x_m - \bar{x}_m}{\sigma_m} \dots (18),$$

where $b_{1t} = \frac{R_{1t}}{R_{11}}$ and R_{pq} is the minor with its proper sign of the element in the p th row and q th column of the determinant

$$R = \begin{vmatrix} 1 & r_{12} & \dots & r_{1m} \\ r_{21} & 1 & \dots & r_{2m} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & \dots & 1 \end{vmatrix} \dots \dots \dots (19),$$

while the maximum correlation between any linear function of $x_2, x_3, \dots, x_m$ and x_1 is ${}_{2,3,\dots,m}R_1$ where

$$-{}_{2,3,\dots,m}R_1^2 = \frac{R - R_{11}}{R_{11}} = b_{12}r_{12} + b_{13}r_{13} \dots + b_{1m}r_{1m} \dots \dots \dots (20),$$

$$= b_{12} \frac{{}_{12}S \{n_{12}(x_1 - \bar{x}_1)(x_2 - \bar{x}_2)\}}{N\sigma_1\sigma_2} + b_{13} \frac{{}_{13}S \{n_{13}(x_1 - \bar{x}_1)(x_3 - \bar{x}_3)\}}{N\sigma_1\sigma_3} \\ + \dots + b_{1m} \frac{{}_{1m}S \{n_{1m}(x_1 - \bar{x}_1)(x_m - \bar{x}_m)\}}{N\sigma_1\sigma_m} \dots \dots \dots (21).$$

Subtract (21) from (17), noting that $n_{12} = \frac{S}{3\dots m} \{n_{12\dots m}\}$, etc., we obtain*

$$({}_{2,3,\dots,m}H_1^2 - {}_{2,3,\dots,m}R_1^2)N\sigma_1^2 \\ = \frac{S}{2\dots m} \{n_{2\dots m}(\bar{x}_1 - {}_{2\dots m}\bar{x}_1)^2\} + S \left\{ n_{12} \frac{\sigma_1}{\sigma_2} b_{12} (x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \right\} \\ + \dots + S \left\{ n_{1m} \frac{\sigma_1}{\sigma_m} b_{1m} (x_1 - \bar{x}_1)(x_m - \bar{x}_m) \right\} \\ = \frac{S}{2\dots m} \{n_{2\dots m}(\bar{x}_1 - {}_{2\dots m}\bar{x}_1)^2\} + S \left\{ n_2 \frac{\sigma_1}{\sigma_2} b_{12} (x_2 - \bar{x}_2)({}_{2\dots m}\bar{x}_1 - \bar{x}_1) \right\} - \dots \\ + S \left\{ n_m \frac{\sigma_1}{\sigma_m} b_{1m} (x_m - \bar{x}_m)({}_{2\dots m}\bar{x}_1 - \bar{x}_1) \right\} \\ = \frac{S}{2\dots m} \left[n_{2\dots m}(\bar{x}_1 - {}_{2\dots m}\bar{x}_1)^2 + n_{2\dots m} \frac{\sigma_1}{\sigma_2} b_{12} (x_2 - \bar{x}_2)({}_{2\dots m}\bar{x}_1 - \bar{x}_1) - \dots \right] \\ = \frac{S}{2\dots m} \left[n_{2\dots m}({}_{2\dots m}\bar{x}_1 - \bar{x}_1) \left\{ {}_{2\dots m}\bar{x}_1 - \bar{x}_1 + b_{12} \frac{\sigma_1}{\sigma_2} (x_2 - \bar{x}_2) - \dots \right. \right. \\ \left. \left. + b_{1m} \frac{\sigma_1}{\sigma_m} (x_m - \bar{x}_m) \right\} \right] \dots \dots \dots (22),$$

using (18) this equation becomes

$$({}_{2,3,\dots,m}H_1^2 - {}_{2,3,\dots,m}R_1^2)N\sigma_1^2 = \frac{S}{2\dots m} \{n_{2\dots m}({}_{2\dots m}\bar{x}_1 - \bar{x}_1)({}_{2\dots m}\bar{x}_1 - X_1)\} \\ = \frac{S}{2\dots m} \{n_{2\dots m}({}_{2\dots m}\bar{x}_1 - X_1)^2\} + \frac{S}{2\dots m} \{n_{2\dots m}(X_1 - \bar{x}_1)({}_{2\dots m}\bar{x}_1 - X_1)\} \\ \dots \dots \dots (23).$$

* By an extension of the notation described at the beginning of this section $S_{3\dots m}$ denotes a summation with regard to the variables $x_3, x_4, \dots, x_m$; $n_{1,2,\dots,m}$ is the frequency of a particular combination of the characters $x_1, x_2, \dots, x_m$ while n_{12} is the frequency of the combination x_1, x_2 .

$$\text{But } S_{2...m} \{n_{2...m} (X_1 - \bar{x}_1) ({}_{2...m}\bar{x}_1 - X_1)\} \\ = S_{2...m} \left\{ n_{2...m} \left(-b_{12} \frac{x_2 - \bar{x}_2}{\sigma_2} \sigma_1 - b_{13} \frac{x_3 - \bar{x}_3}{\sigma_3} \sigma_1 - \dots \right) \left({}_{2...m}\bar{x}_1 - \bar{x}_1 + b_{12} \frac{x_2 - \bar{x}_2}{\sigma_2} \sigma_1 + \dots \right) \right\} \quad (24),$$

$$\text{and } N\sigma_1\sigma_2r_{12} = S_{1...m} \{n_{1...m} (x_1 - \bar{x}_1) (x_2 - \bar{x}_2)\} \\ = S_{2...m} \{n_{2...m} (x_2 - \bar{x}_2) ({}_{2...m}\bar{x}_1 - \bar{x}_1)\}$$

with similar values for r_{13} , r_{23} , etc.

$\therefore$ the right hand side of (24)

$$\begin{aligned} &= -b_{12}\sigma_1^2r_{12} - b_{12}^2\sigma_1^2 - b_{13}b_{12}\sigma_1^2r_{23} \dots \\ &\quad - \sigma_1^2b_{13}r_{13} - b_{13}b_{12}\sigma_1^2r_{23} - b_{13}^2\sigma_1^2 - b_{13}b_{14}\sigma_1^2 \dots \\ &\quad - \dots \\ &= \sigma_1^2b_{12}(-r_{12} - b_{12} - b_{13}r_{23} - b_{14}r_{24} - \dots - b_{1m}r_{2m}) \\ &\quad + \sigma_1^2b_{13}(-r_{13} - b_{13} - b_{14}r_{34} - \dots) \\ &\quad + \dots \dots \dots (25). \end{aligned}$$

Each line in (25) is identically zero from the definition of the b 's and the properties of the determinant in (19).

$$\text{Hence } {}_{2...m}H_1^2 - {}_{2...m}R_1^2 = \frac{S_{2...m} \{n_{2...m} ({}_{2...m}\bar{x}_1 - X_1)^2\}}{N\sigma_1^2} \dots \dots \dots (26),$$

so that the fundamental properties proved by Professor Pearson in connection with the correlation ratio η , hold for the generalised H defined in this section. In particular equation (26) shows that a necessary and sufficient condition for linear regression in multiple correlation of m variables is that

$${}_{2,3,\dots,m}H_1^2 = {}_{2,3,\dots,m}R_1^2.$$

For in this case the mean value of any array of x_1 will lie on the "best fitting" m dimensional plane.

§ 4. The regression surface of z on xy being assumed of any particular type the constants in the equation may be determined (i) by the method of least squares, i.e. by making the sum of the squares of the deviations $\bar{z}_{xy} - \phi(xy)$ a minimum, $z = \phi(x, y)$ being the regression surface, or (ii) by giving such values to the constants that the correlation between z and $\phi(x, y)$ shall be a maximum. When $\phi(x, y)$ is of the second degree the two methods lead to identical equations for the determination of the coefficients.

The same equations are also obtained if the surface be "fitted" to the means by the method of moments. There is, however, a distinction to be observed. The equation $z = \phi(x, y)$ when the regression surface is of specified degree contains a definite number of constants and the first two methods will give exactly as many independent equations as there are constants to be determined. The method of moments will give as many equations as we please if sufficiently high moments

are used, including of course the equations given by the "least squares" method or maximum correlation method. Even without introducing high moments, when there are three variable characters new equations may be obtained by the method of moments, by combinations of characters which do not arise in the other methods. The method of moments is most convenient for our purpose, but we shall only employ those equations which can also be justified by (say) the method of least squares.

For convenience let the origin be taken at the mean of the three characters so that $\bar{x} = \bar{y} = \bar{z} = 0$, let $q_{x^s y^t z^u}$ denote

$$\frac{SSS \{n_{xyz} x^s y^t z^u\}}{N \sigma_x^s \sigma_y^t \sigma_z^u} = \frac{p_{s,t,u}}{\sigma_x^s \sigma_y^t \sigma_z^u} \dots\dots\dots (27).$$

With this notation r_{xy} and $q_{x^1 y^1 z^0}$ are identical; when z does not appear in the product, it is sufficient to write $q_{x^s y^t}$.

The most reasonable next approximation to make when a linear function $\phi(x, y)$ does not adequately represent the statistics is $\phi(x, y) = a$ quadratic function of x, y .

$$\text{Let } \frac{z}{\sigma_z} = d + \frac{ax}{\sigma_x} + \frac{by}{\sigma_y} + \frac{cxy}{\sigma_x \sigma_y} + \frac{ex^2}{\sigma_x^2} + \frac{fy^2}{\sigma_y^2} \dots\dots\dots (28).$$

Multiply (28) by n_{xyz} , sum for all values of x, y, z and divide by N

$$0 = d + cr_{xy} + e + f \dots\dots\dots (29).$$

Multiply (28) in turn by $\frac{n_{xyz}}{N}$ times $\frac{x}{\sigma_x}, \frac{y}{\sigma_y}, \frac{xy}{\sigma_x \sigma_y}, \frac{x^2}{\sigma_x^2}, \frac{y^2}{\sigma_y^2}$ and sum as before, and we obtain

$$r_{xy} = a + br_{xy} + cq_{x^2 y} + eq_{xy^2} + fq_{xy^2} \dots\dots\dots (30),$$

$$r_{yz} = b + ar_{xy} + cq_{xy^2} + eq_{x^2 y} + fq_{y^3} \dots\dots\dots (31),$$

$$q_{xyz} = dr_{xy} + aq_{x^2 y} + bq_{xy^2} + cq_{x^2 y} + eq_{x^3 y} + fq_{xy^3} \dots\dots\dots (32),$$

$$q_{x^2 z} = d + aq_{x^3} + bq_{x^2 y} + cq_{x^3 y} + eq_{x^4} + fq_{x^2 y^2} \dots\dots\dots (33),$$

$$q_{y^2 z} = d + aq_{xy^2} + bq_{y^3} + cq_{xy^3} + eq_{x^2 y} + fq_{y^4} \dots\dots\dots (34).$$

Actual numerical fitting shows that in many cases e and f are small compared with c^* . This is the case when the regression of z on x for a constant y and of z on y for a constant x is linear. We shall therefore confine ourselves in the present preliminary paper to the case where we may write

$$\frac{z}{\sigma_z} = d + \frac{ax}{\sigma_x} + \frac{by}{\sigma_y} + \frac{cxy}{\sigma_x \sigma_y} \dots\dots\dots (35).$$

Here for constant x or y the regression of z on y or of z on x is linear.

* Cf. *Census of Scotland*, 1911, Vol. III. p. XLVII. where Mr G. Rae obtains by moments the regression of fertility on age of husband and wife. Let W = age of wife, H = age of husband, C = number of children in completed marriages. He finds

$$C_{WH} = 20.149493 - 0.555812W - 0.173804H - 0.002846W^2 - 0.003494H^2 + 0.012675WH.$$

See also the paper by E. M. Elderton in the current part of this Journal, pp. 291—295.

Equations (29) to (32) become, when the regression surface is given by (35),

$$0 = d + cr_{xy} \dots\dots\dots(36),$$

$$r_{xy} = a + br_{xy} + cq_{x^2y} \dots\dots\dots(37),$$

$$r_{yz} = b + ar_{xy} + cq_{xy} \dots\dots\dots(38),$$

$$q_{xyz} = dr_{xy} + aq_{x^2y} + bq_{xy^2} + cq_{x^2y^2} \dots\dots\dots(39),$$

$$= -cr_{xy}^2 + aq_{x^2y} + bq_{xy^2} + cq_{x^2y^2} \text{ by (36).}$$

Solving these equations we obtain

$$\begin{array}{c} a \\ \left| \begin{array}{ccc} r_{xy} & q_{x^2y} & r_{xz} \\ 1 & q_{xy^2} & r_{yz} \\ q_{xy^2} & q_{x^2y^2} - r_{xy}^2 & q_{xyz} \end{array} \right| \end{array} = \begin{array}{c} b \\ \left| \begin{array}{ccc} q_{x^2y} & 1 & r_{xz} \\ q_{xy^2} & r_{xy} & r_{yz} \\ q_{x^2y^2} - r_{xy}^2 & q_{x^2y} & q_{xyz} \end{array} \right| \end{array} \\ = \begin{array}{c} c \\ \left| \begin{array}{ccc} 1 & r_{xy} & r_{xz} \\ r_{xy} & 1 & r_{yz} \\ q_{x^2y} & q_{xy^2} & q_{xyz} \end{array} \right| \end{array} = \begin{array}{c} 1 \\ \left| \begin{array}{ccc} 1 & r_{xy} & q_{x^2y} \\ r_{xy} & 1 & q_{xy^2} \\ q_{x^2y} & q_{xy^2} & q_{x^2y^2} - r_{xy}^2 \end{array} \right| \end{array} \dots\dots(40),$$

we have already denoted the partial regression coefficients by γ_3 and γ_3' so that

$$\gamma_3 = \frac{r_{xz} - r_{yz}r_{xy}}{1 - r_{xy}^2} \text{ and } \gamma_3' = \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots(9) \text{ and } (10).$$

In addition let

$$\frac{r_{xy}q_{xy^2} - q_{x^2y}}{1 - r_{xy}^2} = \theta \dots\dots\dots(41),$$

and

$$\frac{r_{xy}q_{x^2y} - q_{xy^2}}{1 - r_{xy}^2} = \phi \dots\dots\dots(42).$$

After some reductions the determinants in (40) yield

$$a = \gamma_3 + c\theta \dots\dots\dots(43),$$

$$b = \gamma_3' + c\phi \dots\dots\dots(44),$$

$$c = \frac{\theta r_{xz} + \phi r_{yz} + q_{xyz}}{\theta q_{x^2y} + \phi q_{xy^2} + q_{x^2y^2} - r_{xy}^2} \dots\dots\dots(45),$$

also

$$d = -c\gamma_{xy} \dots\dots\dots(36).$$

Note that

$$\gamma_3 q_{x^2y} + \gamma_3' q_{xy} \equiv -(\theta r_{zx} + \phi r_{zy}) \dots\dots\dots(46),$$

and

$$\gamma_3 r_{zx} + \gamma_3' r_{zy} = {}_{xy}R_x^2 \text{ (cf. eqn. 11).}$$

$$\S 5. \text{ By definition } (1 - {}_{xy}H_z^2) = \frac{S \{n_{xyz} (z - \bar{z}_{xy})^2\}}{N\sigma_z^2},$$

$\therefore$ using (35)

$$\begin{aligned} (1 - {}_{xy}H_z^2) &= S \left\{ \frac{n_{xyz}}{N} \left(\frac{z}{\sigma_z} - d - \frac{ax}{\sigma_x} - \frac{by}{\sigma_y} - \frac{cxy}{\sigma_x\sigma_y} \right)^2 \right\} \\ &= 1 - 2S \left\{ \frac{z}{\sigma_z} \left(d + \frac{ax}{\sigma_x} + \frac{by}{\sigma_y} + \frac{cxy}{\sigma_x\sigma_y} \right) \frac{n_{xyz}}{N} \right\} \\ &\quad + d^2 + a^2 + b^2 + c^2 q_{x^2y^2} + 2dcr_{xy} + 2abr_{xy} + 2acq_{x^2y} + 2bcq_{xy^2}, \end{aligned}$$

$$\therefore -{}_{xy}H_z^2 = -2ar_{zx} - 2br_{yz} - 2cq_{xyz} + a^2 + b^2 + d^2 + c^2q_{x^2y^2} + 2cdr_{xy} + 2abr_{xy} + 2acq_{x^2y} + 2bcq_{xy^2} \dots (47),$$

$$\begin{aligned} &= a(-r_{zx} + a + br_{xy} + cq_{x^2y}) \\ &\quad + b(-r_{yz} + b + ar_{xy} + cq_{xy^2}) \\ &\quad + c(-q_{xyz} + dr_{xy} + aq_{x^2y} + bq_{xy^2} + cq_{x^2y^2}) \\ &\quad + d(d + cr_{xy}) \\ &\quad - ar_{zx} - br_{yz} - cq_{xyz} \dots (48). \end{aligned}$$

The first four terms vanish by equs. (36)...(39),

$$\therefore {}_{xy}H_z^2 = ar_{zx} + br_{yz} + cq_{xyz} \dots (49).$$

If we now insert the values of a, b, c from (43)...(45) in (49)

$$\begin{aligned} {}_{xy}H_z^2 &= (\gamma_3 + c\theta)r_{zx} + (\gamma_3' + c\phi)r_{zy} + cq_{xyz} \\ &= \gamma_3r_{zx} + \gamma_3'r_{yz} + c(\theta r_{zx} + \phi r_{yz} + q_{xyz}) \\ &= {}_{xy}R_z^2 + \frac{(\theta r_{zx} + \phi r_{yz} + q_{xyz})^2}{q_{x^2y^2}\theta + q_{xy^2}\phi + q_{x^2y^2} - r_{xy}^2}, \end{aligned}$$

$$\text{or } {}_{xy}H_z^2 - {}_{xy}R_z^2 = \frac{\{q_{xyz}(1 - r_{xy}^2) - q_{xy}(r_{zx}r_{xy} - r_{yz}) - q_{x^2y}(r_{yz}r_{xy} - r_{xz})\}^2}{\left[(q_{x^2y^2} - r_{xy}^2) - \left\{\frac{q_{x^2y}^2 + q_{xy^2}^2 - 2q_{x^2y}q_{xy^2}r_{xy}}{1 - r_{xy}^2}\right\}\right](1 - r_{xy}^2)} \dots (50).$$

It follows from (50) and (15) that

$$q_{x^2y^2} - r_{xy}^2 > \frac{q_{x^2y}^2 + q_{xy^2}^2 - 2q_{x^2y}q_{xy^2}r_{xy}}{1 - r_{xy}^2} \dots (51).$$

If we eliminate q_{xyz} (which is a triple moment troublesome to calculate) between equations (45) and (49) we have

$$\begin{aligned} {}_{xy}H_z^2 &= ar_{zx} + br_{yz} + cq_{xyz} \\ &= ar_{zx} + br_{yz} + c^2(q_{x^2y^2} - r_{xy}^2) + acq_{x^2y} + bcq_{xy^2} \\ &= r_{zx}(\gamma_3 + c\theta) + r_{zy}(\gamma_3' + c\phi) + c^2(q_{x^2y^2} - r_{xy}^2) \\ &\quad + (\gamma_3c + c^2\theta)q_{x^2y} + (\gamma_3'c + c^2\phi)q_{xy^2} \\ &= c^2[q_{x^2y^2} - r_{xy}^2 + \theta q_{x^2y} + \phi q_{xy^2}] + c[\theta r_{zx} + \phi r_{zy} + \gamma_3q_{x^2y} + \gamma_3'q_{xy^2}] \\ &\quad + \gamma_3r_{zx} + \gamma_3'r_{zy} \\ &= {}_{xy}R_z^2 + c^2[q_{x^2y^2} - r_{xy}^2 + \theta q_{x^2y} + \phi q_{xy^2}] \text{ by (46) and (11),} \\ \therefore {}_{xy}H_z^2 - {}_{xy}R_z^2 &= c^2 \left[q_{x^2y^2} - r_{xy}^2 - \frac{q_{x^2y}^2 + q_{xy^2}^2 - 2q_{x^2y}q_{xy^2}r_{xy}}{1 - r_{xy}^2} \right]. \end{aligned}$$

$$\text{Hence } c^2 = \frac{{}_{xy}H_z^2 - {}_{xy}R_z^2}{q_{x^2y^2} - r_{xy}^2 - \frac{q_{x^2y}^2 + q_{xy^2}^2 - 2q_{x^2y}q_{xy^2}r_{xy}}{1 - r_{xy}^2}} \dots (52).$$

This value of c^2 is positive by (51).

Equation (52) shows that ${}_{xy}H_z^2 = {}_{xy}R_z^2$ is a necessary condition for linear regression, which we have already proved in equation (26).

The regression surface of z on x, y is, with the values we have now obtained for the constants

$$\begin{aligned} \frac{Z - \bar{z}}{\sigma_z} = & \frac{r_{xz} - r_{yz}r_{xy}}{1 - r_{xy}^2} \frac{x - \bar{x}}{\sigma_x} + \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xy}^2} \frac{y - \bar{y}}{\sigma_y} \\ & + \frac{\sqrt{{}_{xy}H_z^2 - {}_{xy}R_z^2}}{\sqrt{{}_{xy}q_{x^2y^2} - r_{xy}^2 - \frac{{}_{xy}q_{x^2y}^2 + {}_{xy}q_{y^2x}^2 - 2q_{x^2y}q_{xy^2}r_{xy}}{1 - r_{xy}^2}}} \left\{ \frac{r_{xy}q_{x^2y} - q_{x^2y}}{1 - r_{xy}^2} \frac{x - \bar{x}}{\sigma_x} + \frac{r_{xy}q_{xy^2} - q_{xy^2}}{1 - r_{xy}^2} \frac{y - \bar{y}}{\sigma_y} \right. \\ & \left. + \frac{(x - \bar{x})(y - \bar{y})}{\sigma_x \sigma_y} - r_{xy} \right\} \dots \dots \dots (53). \end{aligned}$$

The terms in the first line give the ordinary regression plane. In most cases the regression does not differ widely from linearity so that ${}_{xy}H_z^2 - {}_{xy}R_z^2$ is small.

§ 6. We must now get some idea of the relative magnitudes of $q_{x^2y^2}, q_{x^2y}, q_{x^2y^2}$ and moments of lower order.

First with regard to q_{x^2y} which is equal to $\frac{\bar{p}_{21}}{\sigma_x^2 \sigma_y}$,

$$q_{x^2y} = \frac{S(n_{xy}x^2y)}{N\sigma_x^2\sigma_y} = \frac{S(n_x\bar{y}_x x^2)}{N\sigma_x^2\sigma_y} \dots \dots \dots (54).$$

If the regression of y on x be linear

$$q_{x^2y} = S \frac{\left(n_x \frac{\sigma_y}{\sigma_x} x^2 r_{xy} \right)}{N\sigma_x^2\sigma_y} = r_{xy} \sqrt{\beta_1},$$

so that q_{x^2y} is zero if the regression is linear and the frequency of x symmetrical. In fact $q_{x^2y} - r_{xy} \sqrt{\beta_1} = 0$ is the same as Pearson's criterion for linear regression given by $\bar{\epsilon} = 0$ (*Skew Correlation and Non-linear Regression*, p. 30, Eqn. (lxix)).

We may obtain a good approximation for q_{x^2y} by considering the regression of y on x to be parabolic. This is a natural assumption to make if the regression surfaces of x on z, y and y on z, x be also of the hyperboloid type we are discussing for the regression of z on x, y .

For with origin at mean we may write

$$\frac{\bar{x}_{yz}}{\sigma_x} = e + \frac{fy}{\sigma_y} + \frac{gz}{\sigma_z} + \frac{hyz}{\sigma_y \sigma_z}.$$

Hence keeping y constant and summing for the z 's

$$\frac{\bar{x}_y}{\sigma_x} = e + \frac{fy}{\sigma_y} + \frac{g\bar{z}_y}{\sigma_z} + \frac{hy\bar{z}_y}{\sigma_y \sigma_z}.$$

But
$$\frac{\bar{z}_y}{\sigma_z} = r_{yz} \frac{y}{\sigma_y} \pm \frac{\sqrt{{}_y\eta_z^2 - r_{yz}^2}}{\sqrt{\beta_2'' - \beta_1'' - 1}} \left\{ \frac{y^2}{\sigma_{y^2}} - \sqrt{\beta_1''} \frac{y}{\sigma_y} - 1 \right\}^*.$$

* See Pearson: *l.c.* Eqn. (lxv) (where Y_{px} is a misprint for X_p), and β_2'', β_1'' refer to the distribution of z .

$\therefore \frac{x_y}{\sigma_x}$ is a quadratic expression in y if we remember that h being of order $\sqrt{{}_{yz}H_x^2 - {}_{yz}R_x^2}$ is of the same order as $\sqrt{{}_y\eta_z^2 - r_{yz}^2}^*$.

But the relation $\phi_2(x\eta_y^2 - r_{xy}^2) - \bar{\epsilon} = 0$ is satisfied when the regression of y on x is parabolic†.

Here $\phi_2 = \beta_2 - \beta_1 - 1$, $\bar{\epsilon} = \epsilon - r_{xy}\sqrt{\beta_1}$, $\epsilon = q_{x^2y}^\dagger$.

Hence $q_{x^2y} - r_{xy}\sqrt{\beta_1} = \sqrt{{}_x\eta_y^2 - r_{xy}^2}\sqrt{\beta_2 - \beta_1 - 1} \dots\dots\dots(55)$.

Similarly $q_{xy^2} - r_{xy}\sqrt{\beta_1'} = \sqrt{{}_y\eta_x^2 - r_{xy}^2}\sqrt{\beta_2' - \beta_1' - 1} \dots\dots\dots(56)$.

The use of these approximations will save the direct calculation of q_{x^2y} and q_{xy^2} provided we can determine the signs to be attached to $\sqrt{{}_x\eta_y^2 - r_{xy}^2}$ and $\sqrt{{}_y\eta_x^2 - r_{xy}^2}$. This is often easily done by inspection of the regression curve whose equation is

$$\frac{\bar{y}_x}{\sigma_y} = r \frac{x}{\sigma_x} \pm \sqrt{\frac{{}_x\eta_y^2 - r_{xy}^2}{\beta_2 - \beta_1 - 1}} \left\{ \frac{x^2}{\sigma_x^2} - \sqrt{\beta_1} \frac{x}{\sigma_x} - 1 \right\} \S.$$

We can approximate to $q_{x^2y^2}$ as follows||:

$$q_{x^2y^2} = \frac{SS(n_{xy}x^2y^2)}{N\sigma_x^2\sigma_y^2} = \frac{S\{n_x x^2(\sigma_{Yx}^2 + (Y_x - \bar{y})^2)\}}{N\sigma_x^2\sigma_y^2},$$

where Y_x is the value given by the regression straight line and $\sigma_{Yx}^2 \times n_x$ the second moment of the array of y 's for a given x , about the point Y_x .

$$\begin{aligned} \text{But } \frac{S\{n_x x^2(Y_x - \bar{y})^2\}}{N\sigma_x^2\sigma_y^2} &= S(n_x x^4 r_{xy}^2 \sigma_y^2) / \sigma_x^2 \\ &= \beta_2 r_{xy}^2. \end{aligned}$$

$$\text{Thus } q_{x^2y^2} = \frac{S(n_x x^2 \sigma_{Yx}^2)}{N\sigma_x^2\sigma_y^2} + \beta_2 r_{xy}^2,$$

$$\text{Similarly } q_{x^2y^2} = \frac{S(n_y y^2 \sigma_{Xy}^2)}{N\sigma_x^2\sigma_y^2} + \beta_2' r_{xy}^2;$$

or, so far *without approximation*

$$q_{x^2y^2} = \frac{S(n_x x^2 \sigma_{Yx}^2) + S(n_y y^2 \sigma_{Xy}^2)}{2N\sigma_x^2\sigma_y^2} + \frac{1}{2}(\beta_2 + \beta_2') r_{xy}^2.$$

* It is noteworthy that the hypothesis that regression of z on x, y although of 2nd degree is such that regression of z on x for a constant y is linear leads to the result that the total regression of z on x is *parabolic*.

† Pearson, *l.c.* p. 28, Eqn. (lxiii).

‡ Pearson, *l.c.* Eqns. (li), (xlv) and (lxiii).

§ Pearson, *l.c.* Eqn. (lxv).

|| It can be found fairly directly by tabling to the squares of the variates, when we need a simple product moment. In a later part of this paper some comparisons of actual and approximate values for numerical cases will be found.

Now the mean values of σ_{Yx}^2 and σ_{Xy}^2 are known to be

$$\sigma_y^2(1 - r_{xy}^2) \text{ and } \sigma_x^2(1 - r_{xy}^2),$$

and the deviations from these are usually somewhat irregular. Rarely can we do anything better than assume them to vary with a *slight* linear variation from the mean. For example

$$\sigma_{Yx}^2 = \sigma_y^2(1 - r_{xy}^2)(1 + \lambda x),$$

where λ is small. In such a case

$$\frac{S(n_x x^2 \sigma_{Yx}^2)}{N \sigma_x^2 \sigma_y^2} = (1 - r_{xy}^2)(1 + \lambda \sqrt{\beta_1}),$$

or to a fair degree of approximation*, we may put

$$q_{x^2y^2} = 1 - r_{xy}^2 + \frac{1}{2}(\beta_2 + \beta_2') r_{xy}^2,$$

and thus write

$$\begin{aligned} q_{x^2y^2} - r_{xy}^2 &= 1 + \frac{1}{2}(\beta_2 + \beta_2' - 4) r_{xy}^2 \\ &= 1 + r_{xy}^2 + \frac{1}{2}(\beta_2 - 3 + \beta_2' - 3) r_{xy}^2. \end{aligned}$$

The latter part of this expression vanishes if the frequency of the x and y variates be mesokurtic. It can of course be retained if desired but its product with $\sqrt{x_y H_z^2 - x_y R_z^2}$ will usually be of the second order. If we write

$$\psi = \frac{1}{2}(\beta_2 - 3 + \beta_2' - 3),$$

we find the approximate regression surface

$$\begin{aligned} \frac{Z - \bar{z}}{\sigma_z} &= \frac{r_{xz} - r_{yz} r_{xy}}{1 - r_{xy}^2} \frac{(x - \bar{x})}{\sigma_x} + \frac{r_{yz} - r_{xz} r_{xy}}{1 - r_{xy}^2} \frac{(y - \bar{y})}{\sigma_y} \\ &+ \sqrt{\frac{x_y H_z^2 - x_y R_z^2}{1 + (1 + \psi) r_{xy}^2}} \left\{ \frac{(x - \bar{x})}{\sigma_x} \frac{(y - \bar{y})}{\sigma_y} - r_{xy} \right\} \dots\dots (57). \end{aligned}$$

This equation (57) enables us to express approximately the multiple $_{xy}H_z$ in terms of the simple ${}_y\eta_z$, ${}_x\eta_z$, ${}_x\eta_y$, ${}_y\eta_x$.

§ 7. To obtain this connection between the multiple $_{xy}H_z$ and the simple η 's we may proceed as follows:

$$\frac{\bar{z}_{xy}}{\sigma_z} = d + \frac{ax}{\sigma_x} + \frac{by}{\sigma_y} + \frac{cxy}{\sigma_x \sigma_y}, \text{ origin at mean.}$$

Hence keeping y constant and summing for the x 's

$$\frac{\bar{z}_y}{\sigma_z} = d + \frac{a\bar{x}_y}{\sigma_x} + \frac{by}{\sigma_y} + \frac{c\bar{x}_y y}{\sigma_x \sigma_y} \dots\dots\dots (58).$$

But ${}_y\eta_z^2 = \frac{SSS \{n_{xyz} (\bar{z}_y - \bar{z})^2\}}{N \sigma_z^2} = \frac{SSS (n_{xyz} \bar{z}_y^2)}{N \sigma_z^2}$ if $\bar{z} = 0$,

and $_{xy}H_z^2 = \frac{SSS \{n_{xyz} (\bar{z}_{xy} - \bar{z})^2\}}{N \sigma_z^2} = \frac{SSS \{n_{xyz} \bar{z}_{xy}^2\}}{N \sigma_z^2},$

* I.e. we are neglecting terms of the second order as $\lambda \sqrt{\beta_1}$.

$$\begin{aligned}
 \therefore \quad x_y H_z^2 - y \eta_z^2 &= SSS_{x y z} \left[\frac{n_{xyz}}{N} \left(\frac{a(x - \bar{x}_y)}{\sigma_x} + \frac{cy(x - \bar{x}_y)}{\sigma_y \sigma_y} \right) \left(2d + a \frac{x + \bar{x}_y}{\sigma_x} + 2 \frac{by}{\sigma_y} \right. \right. \\
 &\quad \left. \left. + \frac{cy}{\sigma_y} \frac{x + \bar{x}}{\sigma_x} \right) \right] \\
 &= SS_{x y} \left\{ \frac{n_{xy}}{N} \left(\frac{x - \bar{x}_y}{\sigma_x} \right) \left(a + \frac{cy}{\sigma_y} \right) \left[2 \left(d + \frac{by}{\sigma_y} \right) + \left(a + \frac{cy}{\sigma_y} \right) \left(\frac{x + \bar{x}_y}{\sigma_x} \right) \right] \right\} \\
 &= 2S_y \left\{ \left(a + \frac{cy}{\sigma_y} \right) \left(d + \frac{by}{\sigma_y} \right) S_x \frac{x - \bar{x}_y}{\sigma_x} \frac{n_{xy}}{N} \right\} + SS_{x y} \left\{ \frac{x^2 - \bar{x}_y^2}{\sigma_x^2} \left(a + \frac{cy}{\sigma_y} \right)^2 \frac{n_{xy}}{N} \right\} \\
 &= 0 + S_y \left\{ \left(a + \frac{cy}{\sigma_y} \right)^2 \left(S_x \frac{n_{xy}}{N} \frac{x^2 - \bar{x}_y^2}{\sigma_x^2} \right) \right\}.
 \end{aligned}$$

Now $S_x \{n_{xy} x^2\} = S_x \{(x - \bar{x}_y + \bar{x}_y)^2 n_{xy}\} = n_y (\sigma_{x_y}^2 + 0 + \bar{x}_y^2),$

$$\begin{aligned}
 \therefore \quad x_y H_z^2 - y \eta_z^2 &= S_y \left\{ \left(a + \frac{cy}{\sigma_y} \right)^2 \frac{\sigma_{x_y}^2 n_y}{\sigma_x^2 N} \right\} \\
 &= S_y \left\{ \left(a + \frac{cy}{\sigma_y} \right)^2 (1 - y \eta_x^2) \frac{n_y}{N} \right\} \\
 &= (1 - y \eta_x^2) S_y \left\{ \frac{n_y}{N} \left(a^2 + \frac{2acy}{\sigma_y} + \frac{c^2 y^2}{\sigma_y^2} \right) \right\}, \\
 \therefore \quad x_y H_z^2 - y \eta_z^2 &= (1 - y \eta_x^2) (a^2 + c^2) \dots \dots \dots (59).
 \end{aligned}$$

Similarly $x_y H_z^2 - x \eta_z^2 = (1 - x \eta_y^2) (b^2 + c^2) \dots \dots \dots (60).$

Remembering that $a = \gamma_3 + c\theta$, $b = \gamma_3' + c\phi$ we get from (59) and (60)

$$\begin{aligned}
 \gamma_3' \phi \left(\frac{x_y H_z^2 - y \eta_z^2}{1 - y \eta_x^2} \right) - \gamma_3 \theta \left(\frac{x_y H_z^2 - x \eta_z^2}{1 - x \eta_y^2} \right) &= \gamma_3 \gamma_3' (\gamma_3 \phi - \gamma_3' \theta) \\
 &\quad + c^2 \{ \gamma_3' \phi - \gamma_3 \theta - \theta \phi (\gamma_3 \phi - \gamma_3' \theta) \} \dots \dots (61).
 \end{aligned}$$

From the values of γ_3 , γ_3' , θ , ϕ in (9), (10), (41), (42) we obtain easily

$$\begin{aligned}
 \gamma_3 \phi - \gamma_3' \theta &= \frac{r_{yz} q_{x^2 y} - r_{xz} q_{xy^2}}{1 - r_{xy}^2}, \\
 \gamma_3' \phi - \gamma_3 \theta &= \frac{r_{xz} q_{x^2 y} - r_{yz} q_{xy^2}}{1 - r_{xy}^2}, \\
 \gamma_3' \phi - \gamma_3 \theta - \theta \phi (\gamma_3 \phi - \gamma_3' \theta) &= \frac{r_{xz} q_{x^2 y} - r_{yz} q_{xy^2}}{1 - r_{xy}^2} - \frac{(r_{xy} q_{xy^2} - q_{x^2 y})(r_{xy} q_{x^2 y} - q_{xy^2})(r_{yz} q_{x^2 y} - r_{xz} q_{xy^2})}{(1 - r_{xy}^2)^3}
 \end{aligned}$$

and c^2 is given by (52). Hence (61) may be written

$$\begin{aligned}
 (x_y H_z^2 - x_y R_z^2) &\left\{ \frac{(r_{yz} - r_{xz} r_{xy})(r_{xy} q_{xy^2} - q_{x^2 y})}{(1 - r_{xy}^2)^2 (1 - y \eta_x^2)} - \frac{(r_{xz} - r_{yz} r_{xy})(r_{xy} q_{xy^2} - q_{x^2 y})}{(1 - r_{xy}^2)^2 (1 - x \eta_y^2)} \right. \\
 &\quad - \frac{(r_{xz} q_{x^2 y} - r_{yz} q_{xy^2}) - (r_{xy} q_{xy^2} - q_{x^2 y})(r_{xy} q_{x^2 y} - q_{xy^2})(r_{yz} q_{x^2 y} - r_{xz} q_{xy^2})}{(1 - r_{xy}^2)^3} \\
 &\quad \left. - \frac{(q_{x^2 y}^2 - r_{xy}^2) - \frac{(q_{x^2 y}^2 + q_{xy^2}^2 - 2q_{xy^2} q_{x^2 y} r_{xy})}{1 - r_{xy}^2}}{(1 - r_{xy}^2)^2} \right\}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})_y\eta_z^2}{(1 - r_{xy}^2)^2(1 - y\eta_x^2)} - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})_x\eta_z^2}{(1 - r_{xy}^2)^2(1 - y\eta_x^2)} \\
&\quad + \frac{(r_{xz} - r_{yz}r_{xy})(r_{yz} - r_{xz}r_{xy})(r_{yz}q_{x^2y} - r_{xz}q_{xy^2})}{(1 - r_{xy}^2)^3} \\
&- x_y R_z^2 \left\{ \frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^2(1 - y\eta_x^2)} - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^2(1 - x\eta_y^2)} \right\} \dots\dots(62).
\end{aligned}$$

In general the square of any correlation ratio (ordinary or generalised) differs little from the square of the corresponding correlation coefficient; also we have seen in (55) that an approximate value for q_{x^2y} is

$$r_{xy} \sqrt{\beta_1 - \sqrt{x\eta_y^2 - r_{xy}^2}} \sqrt{\beta_2 - \beta_1 - 1}.$$

Now β_1 is itself in general small, so that without making any assumption as to the relative order of magnitude of β_1 and $x\eta_y^2 - r_{xy}^2$ we may safely treat q_{x^2y} and q_{xy^2} as small quantities.

Thus it appears that (62) is an equation in which all the terms involved are small, and a certain amount of care is required in deducing from it an approximation to the value of $xyH_z^2 - xyR_z^2$.

We have

$$\begin{aligned}
\frac{1}{1 - y\eta_x^2} &\equiv \frac{1}{1 - r_{xy}^2} + \frac{y\eta_x^2 - r_{xy}^2}{(1 - y\eta_x^2)(1 - r_{xy}^2)} = \frac{1}{1 - r_{xy}^2} + \xi_1 \text{ say} \\
\frac{1}{1 - x\eta_y^2} &\equiv \frac{1}{1 - r_{xy}^2} + \frac{x\eta_y^2 - r_{xy}^2}{(1 - x\eta_y^2)(1 - r_{xy}^2)} = \frac{1}{1 - r_{xy}^2} + \xi_1' \\
\frac{y\eta_z^2}{1 - y\eta_x^2} &= r_{zy}^2 \left(\frac{1}{1 - r_{xy}^2} + \xi_1 \right) + (y\eta_z^2 - r_{zy}^2) \left(\frac{1}{1 - r_{xy}^2} + \xi_1 \right) \\
&= \frac{r_{zy}^2}{1 - r_{xy}^2} + \lambda_1 + \lambda_2 \dots\dots\dots(64),
\end{aligned}$$

$$\text{where } \lambda_1 = \frac{r_{zy}^2(y\eta_x^2 - r_{xy}^2)}{(1 - y\eta_x^2)(1 - r_{xy}^2)} + \frac{y\eta_z^2 - r_{zy}^2}{1 - r_{xy}^2} \text{ and } \lambda_2 = \frac{(y\eta_z^2 - r_{zy}^2)(y\eta_x^2 - r_{xy}^2)}{(1 - y\eta_x^2)(1 - r_{xy}^2)},$$

$$\text{while } \frac{x\eta_z^2}{1 - x\eta_y^2} = \frac{r_{zx}^2}{1 - r_{xy}^2} + \lambda_1' + \lambda_2' \dots\dots\dots(64)',$$

where λ_1', λ_2' may be obtained from λ_1, λ_2 by an interchange of x and y in the suffixes.

$$\text{Finally } \frac{1}{q_{x^2y^2} - r_{xy}^2 - \frac{q_{x^2y}^2 + q_{x^2y} - 2q_{xy^2}q_{x^2y}r_{xy}}{1 - r_{xy}^2}} \equiv \frac{1}{q_{x^2y^2} - r_{xy}^2} + \kappa_2 \dots\dots\dots(65),$$

where

$$\kappa_2 = \frac{q_{x^2y}^2 + q_{x^2y} - 2q_{xy^2}q_{x^2y}r_{xy}}{(1 - r_{xy}^2)(q_{x^2y^2} - r_{xy}^2) \left(q_{x^2y^2} - r_{xy}^2 - \frac{q_{x^2y}^2 + q_{x^2y} - 2q_{xy^2}q_{x^2y}r_{xy}}{1 - r_{xy}^2} \right)}.$$

The suffixes of $\xi_1, \xi_1', \lambda_1, \lambda_2, \lambda_1', \lambda_2'$ and κ_2 denote the order of smallness of these terms.

We make the corresponding substitutions in equation (62), including the value $\frac{r_{yz}^2 + r_{zx}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2}$ for ${}_{xy}R_z^2$ on the right-hand side and obtain the following accurate formula for ${}_{xy}H_z^2 - {}_{xy}R_z^2$:

$$\begin{aligned}
 & ({}_{xy}H_z^2 - {}_{xy}R_z^2) \left[\frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^2} \left(\frac{1}{1 - r_{xy}^2} + \xi_1 \right) \right. \\
 & \quad \left. - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^2} \left(\frac{1}{1 - r_{xy}^2} + \xi_1' \right) \right. \\
 & \quad \left. - \left\{ \frac{r_{xz}q_{x^2y} - r_{yz}q_{xy^2}}{1 - r_{xy}^2} - \frac{(r_{yz}q_{x^2y} - r_{xz}q_{xy^2})(r_{xy}q_{xy^2} - q_{x^2y})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^3} \right\} \right. \\
 & \quad \left. \times \left(\frac{1}{q_{x^2y^2} - r_{xy}^2} + \kappa_2 \right) \right] \\
 & = \frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^2} \left(\frac{r_{xy}^2}{1 - r_{xy}^2} + \lambda_1 + \lambda_2 \right) \\
 & \quad - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^2} \left(\frac{r_{xy}^2}{1 - r_{xy}^2} + \lambda_1' + \lambda_2' \right) \\
 & \quad + \frac{(r_{yz}q_{x^2y} - r_{xz}q_{xy^2})(r_{xz} - r_{yz}r_{xy})(r_{yz} - r_{xz}r_{xy})}{(1 - r_{xy}^2)^3} \\
 & \quad - \frac{r_{yz}^2 + r_{xz}^2 - 2r_{xz}r_{yz}r_{xy}}{(1 - r_{xy}^2)} \left[\frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^2} \left(\frac{1}{1 - r_{xy}^2} + \xi_1 \right) \right. \\
 & \quad \left. - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^2} \left(\frac{1}{1 - r_{xy}^2} + \xi_1' \right) \right] \dots\dots(66).
 \end{aligned}$$

The right-hand side of (66) apparently contains terms of the first order, while on the left the lowest order occurring is the second. But the coefficient of q_{x^2y} in the first order terms is

$$\begin{aligned}
 & \frac{1}{(1 - r_{xy}^2)^3} \{ (r_{yz} - r_{xz}r_{xy})r_{xy}r_{yz}^2 + (r_{xz} - r_{yz}r_{xy})r_{yz}^2 + r_{yz}(r_{xz} - r_{yz}r_{xy})(r_{yz} - r_{xz}r_{xy}) \} \\
 & - \frac{1}{(1 - r_{xy}^2)^4} \{ (r_{yz} - r_{xz}r_{xy})r_{xy} + (r_{xz} - r_{yz}r_{xy}) \} (r_{yz}^2 + r_{xz}^2 - 2r_{yz}r_{xy}r_{xz}),
 \end{aligned}$$

and vanishes identically. Similarly the coefficient of q_{xy^2} is zero and thus equation (66) is a relation between terms of second and higher orders.

A first approximation then may be obtained by equating second order terms. This gives

$$\begin{aligned}
 & ({}_{xy}H_z^2 - {}_{xy}R_z^2) \left[\frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2}) - (r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^3} \right. \\
 & \quad \left. - \frac{r_{xz}q_{x^2y} - r_{yz}q_{xy^2}}{(1 - r_{xy}^2)(q_{x^2y^2} - r_{xy}^2)} \right] \\
 & = \frac{(r_{yz} - r_{xz}r_{xy})(r_{xy}q_{x^2y} - q_{xy^2})}{(1 - r_{xy}^2)^2} \left[\lambda_1 - \frac{r_{yz}^2 + r_{xz}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2} \xi_1 \right] \\
 & \quad - \frac{(r_{xz} - r_{yz}r_{xy})(r_{xy}q_{xy^2} - q_{x^2y})}{(1 - r_{xy}^2)^2} \left[\lambda_1' - \frac{r_{yz}^2 + r_{xz}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2} \xi_1' \right] \dots\dots(67).
 \end{aligned}$$

The coefficient of $_{xy}H_z^2 - _{xy}R_z^2$ on the left reduces to

$$\frac{1}{(1 - r_{xy}^2)^2} [q_{x^2y}r_{xz} - q_{xy^2}r_{yz}] \left(1 - \frac{1 - r_{xy}^2}{q_{x^2y^2} - r_{xy}^2}\right),$$

$$\lambda_1 - \frac{r_{yz}^2 + r_{xz}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2} \xi_1 = \frac{_{y}\eta_z^2 - r_{zy}^2}{1 - r_{xy}^2} - \xi_1 \frac{(r_{yz}r_{xy} - r_{xz})^2}{1 - r_{xy}^2}$$

$$= \frac{_{y}\eta_z^2 - r_{zy}^2}{1 - r_{xy}^2} - \frac{(_{y}\eta_x^2 - r_{xy}^2)}{1 - _{y}\eta_x^2} \left(\frac{r_{yz}r_{xy} - r_{xz}}{1 - r_{xy}^2} \right)^2 \dots\dots (68).$$

Similarly

$$\lambda_1' - \frac{r_{yz}^2 + r_{xz}^2 - 2r_{xz}r_{yz}r_{xy}}{1 - r_{xy}^2} \xi_1' = \frac{x\eta_z^2 - r_{zx}^2}{1 - r_{xy}^2} - \frac{x\eta_y^2 - r_{xy}^2}{1 - x\eta_y^2} \left(\frac{r_{xz}r_{xy} - r_{yz}}{1 - r_{xy}^2} \right)^2 \dots (69).$$

We shall still be correct to second order terms if when using (68) and (69) in (67) we replace $1 - _{y}\eta_x^2$ and $1 - x\eta_y^2$ occurring in denominators by $1 - r_{xy}^2$; so that

$$(_{xy}H_z^2 - _{xy}R_z^2) \frac{q_{x^2y^2} - 1}{q_{x^2y^2} - r_{xy}^2}$$

$$= \frac{r_{xy}q_{x^2y} - q_{xy^2}}{q_{x^2y}r_{xz} - q_{xy^2}r_{yz}} \cdot \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xy}^2} \left[(_{y}\eta_z^2 - r_{zy}^2) - \left(\frac{r_{yz}r_{xy} - r_{xz}}{1 - r_{xy}^2} \right)^2 (_{y}\eta_x^2 - r_{xy}^2) \right]$$

$$+ \frac{r_{xy}q_{xy^2} - q_{x^2y}}{q_{xy^2}r_{yz} - q_{x^2y}r_{xz}} \cdot \frac{r_{xz} - r_{yz}r_{xy}}{1 - r_{xy}^2} \left[(x\eta_z^2 - r_{zx}^2) - \left(\frac{r_{xz}r_{xy} - r_{yz}}{1 - r_{xy}^2} \right)^2 (x\eta_y^2 - r_{xy}^2) \right]$$

$$\dots\dots (70).$$

§ 8. This result is of importance, as it shows that the heavy labour of the direct calculation of the generalised correlation ratio can be replaced by the calculation of four simple correlation ratios.

The coefficients involved are the ordinary coefficients of linear regression denoted above by γ_s , γ_s' and expressions involving product moments of orders 3 and 4. To these latter we may approximate by the methods of § 6.

A good approximation for $\frac{q_{x^2y^2} - 1}{q_{x^2y^2} - r_{xy}^2}$ is $\frac{2r_{xy}^2}{1 + r_{xy}^2}$. If greater accuracy be needed we may use

$$\frac{q_{x^2y^2} - 1}{q_{x^2y^2} - r_{xy}^2} = \frac{(\frac{1}{2}(\beta_2 + \beta_2' - 6) + 2)r_{xy}^2}{1 + r_{xy}^2 + \frac{1}{2}(\beta_2 + \beta_2' - 6)r_{xy}^2}.$$

We saw in § 6 (equation 55) that

$$q_{x^2y} = r_{xy} \sqrt{\beta_1} + \sqrt{x\eta_y^2 - r_{xy}^2} \sqrt{\beta_2 - \beta_1 - 1},$$

$$q_{xy^2} = r_{xy} \sqrt{\beta_1'} + \sqrt{_{y}\eta_x^2 - r_{xy}^2} \sqrt{\beta_2' - \beta_1' - 1},$$

approximately. β_1 and β_1' , which are zero in normal correlation, will in general be very small compared with $x\eta_y^2 - r_{xy}^2$ and $_{y}\eta_x^2 - r_{xy}^2$ so that

$$\frac{r_{xy}q_{x^2y} - q_{xy^2}}{q_{x^2y}r_{xz} - q_{xy^2}r_{yz}} = \frac{r_{xy} \sqrt{(\beta_2 - 1)(x\eta_y^2 - r_{xy}^2)} - \sqrt{(\beta_2' - 1)(_{y}\eta_x^2 - r_{xy}^2)}}{r_{xz} \sqrt{(\beta_2 - 1)(x\eta_y^2 - r_{xy}^2)} - r_{yz} \sqrt{(\beta_2' - 1)(_{y}\eta_x^2 - r_{xy}^2)}},$$

and

$$\frac{r_{xy}q_{xy^2} - q_{x^2y}}{q_{xy^2}r_{yz} - q_{x^2y}r_{xz}} = \frac{r_{xy} \sqrt{(\beta_2' - 1)(_{y}\eta_x^2 - r_{xy}^2)} - \sqrt{(\beta_2 - 1)(x\eta_y^2 - r_{xy}^2)}}{r_{yz} \sqrt{(\beta_2' - 1)(_{y}\eta_x^2 - r_{xy}^2)} - \sqrt{(\beta_2 - 1)(x\eta_y^2 - r_{xy}^2)}}.$$

In the important case $\beta_1 = \beta'_1 = 0$ and ${}_y\eta_x = {}_x\eta_y = r_{xy}$ these approximations fail, and so does the process by which (70) was obtained as θ and ϕ vanish and (61) is indeterminate.

We must then fall back on equation (59)

$$\begin{aligned} {}_{xy}H_z^2 - {}_y\eta_z^2 &= (a^2 + c^2)(1 - {}_y\eta_x^2) \\ &= (\gamma_3^2 + c^2)(1 - r_{xy}^2) \text{ since } \theta = 0 \text{ and } {}_y\eta_x = r_{xy} \\ &= \gamma_3^2(1 - r_{xy}^2) + \frac{({}_{xy}H_z^2 - {}_{xy}R_z^2)(1 - r_{xy}^2)}{q_{x^2y^2} - r_{xy}^2}(1 + \kappa_2), \end{aligned}$$

or
$$({}_{xy}H_z^2 - {}_{xy}R_z^2) \left(1 - \frac{1 - r_{xy}^2}{q_{x^2y^2} - r_{xy}^2} \right) = {}_y\eta_z^2 - {}_{xy}R_z^2 + \gamma_3^2(1 - r_{xy}^2),$$

neglecting 3rd order terms.

The right-hand side reduces to ${}_y\eta_z^2 - r_{yz}^2$, so that

$$\begin{aligned} {}_{xy}H_z^2 - {}_{xy}R_z^2 &= \left(\frac{q_{x^2y^2} - r_{xy}^2}{q_{x^2y^2} - 1} \right) ({}_y\eta_z^2 - r_{yz}^2) \dots\dots\dots(71) \\ &= \frac{2r_{xy}^2}{1 + r_{xy}^2} ({}_y\eta_z^2 - r_{yz}^2) \text{ approximately; } \end{aligned}$$

of course in these circumstances (60) would lead to the value

$$\frac{2r_{xy}^2}{1 + r_{xy}^2} ({}_x\eta_z^2 - r_{zx}^2) \dots\dots\dots(72),$$

showing that if $\beta_1 = \beta'_1 = 0$ and if $({}_x\eta_y^2 - r_{yx}^2) - ({}_y\eta_x^2 - r_{xy}^2)$ is of higher order than the first then $({}_x\eta_z^2 - r_{zx}^2) - ({}_y\eta_z^2 - r_{zy}^2)$ is also of higher order than the first.

We shall now seek relations between the six correlation ratios of three "hyperbolic" variates.

From (59) and (60) we get, on eliminating ${}_{xy}H_z^2$,

$$\begin{aligned} {}_y\eta_z^2 - {}_x\eta_z^2 &= b^2 - a^2 + c^2({}_y\eta_x^2 - {}_x\eta_y^2)^2 + a^2{}_y\eta_x^2 - b^2{}_x\eta_y^2 \\ &= \gamma_3'^2 - \gamma_3^2 + \gamma_3^2{}_y\eta_x^2 - \gamma_3'^2{}_x\eta_y^2 + 2(\gamma_3\theta{}_y\eta_x^2 - \gamma_3'\phi{}_x\eta_y^2) \\ &\quad + c^2(\theta^2 - \phi^2 + {}_y\eta_x^2 - {}_x\eta_y^2 + \theta^2{}_y\eta_x^2 - \phi^2{}_x\eta_y^2) \\ &= (\gamma_3'^2 - \gamma_3^2)(1 - r_{xy}^2) + \gamma_3^2({}_y\eta_x^2 - r_{xy}^2) - \gamma_3'^2({}_x\eta_y^2 - r_{xy}^2) \\ &\quad - 2c(\gamma\theta - \gamma'\phi)(1 - r_{xy}^2) \\ &\quad + \{2c\gamma_3\theta({}_y\eta_x^2 - r_{xy}^2) - 2c\gamma_3'\phi({}_x\eta_y^2 - r_{xy}^2) \\ &\quad + c^2(\theta^2 - \phi^2 + {}_y\eta_x^2 - {}_x\eta_y^2 + \theta^2{}_y\eta_x^2 - \phi^2{}_x\eta_y^2)\}. \end{aligned}$$

The terms in the second line are second order terms. Neglecting these and noting that

$$(\gamma_3'^2 - \gamma_3^2)(1 - r_{xy}^2) \equiv r_{yz}^2 - r_{zx}^2 \text{ and } (\gamma_3\theta - \gamma_3'\phi)(1 - r_{xy}^2) \equiv r_{yz}q_{xy^2} - r_{xz}q_{x^2y},$$

we obtain the following equation for c :

$$\begin{aligned} ({}_y\eta_z^2 - r_{zy}^2) - ({}_x\eta_z^2 - r_{zx}^2) &- \gamma_3^2({}_y\eta_x^2 - r_{xy}^2) + \gamma_3'^2({}_x\eta_y^2 - r_{xy}^2) \\ &= 2c(r_{xz}q_{x^2y} - r_{yz}q_{xy^2}) \dots\dots(73). \end{aligned}$$

Now $({}_xH_z^2 - {}_{xy}R_z^2) \frac{q_{x^2y^2} - 1}{q_{x^2y^2} - r_{xy}^2} = 2c^2r_{xy}^2$ if we neglect second order terms. If we use the value of c given by (73) in (70), and adopt the notation ${}_xU_z$ for ${}_x\eta_z^2 - r_{zx}^2$, we have

$$r_{xy}^2 [{}_yU_z - {}_xU_z - \gamma_3^2 {}_yU_x + \gamma_3'^2 {}_xU_y]^2 \\ = 2(r_{xz}q_{x^2y} - r_{yz}q_{xy^2}) \{ (r_{xy}q_{x^2y} - q_{xy^2}) \gamma_3' ({}_yU_z - \gamma_3^2 {}_yU_x) \\ - (r_{xy}q_{xy^2} - q_{x^2y}) \gamma_3 ({}_xU_z - \gamma_3'^2 {}_xU_y) \} \dots\dots(74).$$

Let us write ${}_x\chi_z$ for ${}_xU_z - \gamma_3'^2 {}_xU_y$ and ${}_y\chi_z$ for ${}_yU_z - \gamma_3^2 {}_yU_x$, then (74) becomes

$$({}_x\chi_z - {}_y\chi_z)^2 r_{xy}^2 = 2(r_{xz}q_{x^2y} - r_{yz}q_{xy^2}) \{ \gamma_3' (r_{xy}q_{x^2y} - q_{xy^2}) {}_y\chi_z - \gamma_3 (r_{xy}q_{xy^2} - q_{x^2y}) {}_x\chi_z \} \\ \dots\dots(75).$$

(75) is a relation between second order terms and it is sufficient to use equation (55) for q_{x^2y} with β_1 replaced by zero and β_x by 3, so that

$$q_{x^2y} = \sqrt{2} {}_x\overline{U}_y, \\ q_{xy^2} = \sqrt{2} {}_y\overline{U}_x, \\ \therefore ({}_x\chi_z - {}_y\chi_z)^2 r_{xy}^2 = 4(r_{xz}\sqrt{{}_x\overline{U}_y} - r_{yz}\sqrt{{}_y\overline{U}_x}) \{ \gamma_3' {}_y\chi_z (r_{xy}\sqrt{{}_x\overline{U}_y} - \sqrt{{}_y\overline{U}_x}) \\ - \gamma_3 {}_x\chi_z (r_{xy}\sqrt{{}_y\overline{U}_x} - \sqrt{{}_x\overline{U}_y}) \} \dots\dots(76).$$

This identity between ${}_x\eta_z$, ${}_y\eta_z$, ${}_x\eta_y$ and ${}_y\eta_x$ is symmetrical in x, y . Two more such identities may be obtained by interchanging the letters x, y, z in cyclic order*. There are therefore three identities between the six correlation ratios:

$${}_y\eta_x, \quad {}_x\eta_y, \quad {}_y\eta_z, \quad {}_z\eta_y, \quad {}_x\eta_z, \quad {}_z\eta_x.$$

I have not so far succeeded in reducing them to simpler forms, although possibly such exist. In special cases simplifications result. These are illustrated in the following section.

§ 9. We defined γ_3, γ_3' the regression coefficients of z on x, y by the equations

$$\gamma_3 = \frac{r_{xz} - r_{yz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots(9), \quad \gamma_3' = \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xy}^2} \dots\dots\dots(10).$$

Let us now introduce the remaining regression coefficients

$$\left. \begin{aligned} \gamma_1 &= \frac{r_{yx} - r_{zx}r_{yz}}{1 - r_{yz}^2}, & \gamma_1' &= \frac{r_{zx} - r_{yx}r_{yz}}{1 - r_{yz}^2} \\ \gamma_2 &= \frac{r_{yz} - r_{xz}r_{xy}}{1 - r_{xz}^2}, & \gamma_2' &= \frac{r_{xy} - r_{yz}r_{zx}}{1 - r_{xz}^2} \end{aligned} \right\} \dots\dots\dots(77).$$

It will simplify the algebra if we use $\lambda^2, \mu^2, \nu^2, \lambda'^2, \mu'^2, \nu'^2$ for ${}_yU_x, {}_zU_y, {}_xU_z, {}_zU_x, {}_xU_y, {}_yU_z$ respectively and P, Q, R, P', Q', R' for ${}_y\chi_x, {}_z\chi_y, {}_x\chi_z, {}_z\chi_x, {}_x\chi_y, {}_y\chi_z$ respectively so that

$$\left. \begin{aligned} P &= \lambda^2 - \gamma_1'^2 \nu'^2, & Q &= \mu^2 - \gamma_2'^2 \lambda'^2, & R &= \nu^2 - \gamma_3'^2 \mu'^2 \\ P' &= \lambda'^2 - \gamma_1^2 \mu^2, & Q' &= \mu'^2 - \gamma_2^2 \nu^2, & R' &= \nu'^2 - \gamma_3^2 \lambda^2 \end{aligned} \right\} \dots\dots\dots(78).$$

* We are supposing here that the regression surfaces of x on y, z and of y on z, x are also hyperboloids of type similar to (35).

The three identities connecting the six simple η 's become in this notation

$$(R - R')^2 r_{xy}^2 = 4(r_{xz}\mu' - r_{yz}\lambda)\{\gamma_3' R'(r_{xy}\mu' - \lambda) - \gamma_3 R(r_{xy}\lambda - \mu')\} \dots (79),$$

$$(P - P')^2 r_{yz}^2 = 4(r_{yx}\nu' - r_{zx}\mu)\{\gamma_1' P'(r_{yz}\nu' - \mu) - \gamma_1 P(r_{yz}\mu - \nu')\} \dots (80),$$

$$(Q - Q')^2 r_{zx}^2 = 4(r_{zx}\lambda' - r_{xy}\nu)\{\gamma_2' Q'(r_{zx}\lambda' - \nu) - \gamma_2 Q(r_{zx}\nu - \lambda')\} \dots (81).$$

(i) We can deduce from (79) that if the correlations of both x and y on z be linear and equal, then the correlation ratio of x on y and y on x , i.e. ${}_x\eta_y$ and ${}_y\eta_x$ are equal. Thus in biparental correlation, if the regression of the child on each parent be linear, then the correlation ratio of the father on the mother is equal to that of the mother on the father.

For under the conditions stated

$${}_x\eta_z = {}_y\eta_z = r_{zx} = r_{zy}.$$

Hence $\gamma_3 = \gamma_3' = \frac{r_{zx}}{1 + r_{xy}}$ and (79) becomes

$$\gamma_3^4 [\lambda^2 - \mu'^2]^2 r_{xy}^2 = 4r_{yz}(\mu' - \lambda)\{-\gamma_3^3 \lambda^2 (r_{xy}\mu' - \lambda) + \gamma_3^3 \mu'^2 (r_{xy}\lambda - \mu')\},$$

$$\text{or } \gamma_3(\lambda - \mu')^2(\lambda + \mu')^2 r_{xy}^2 = 4r_{yz}(\lambda - \mu')^2 \{r_{xy}\lambda\mu' - \lambda^2 - \lambda\mu' - \mu'^2\},$$

$$\text{or } (\lambda - \mu')^2 \left[\frac{r_{xy}^2}{1 + r_{xy}} (\lambda + \mu')^2 - 4(r_{xy}\lambda\mu' - \lambda\mu' - \lambda^2 - \mu'^2) \right] = 0,$$

which reduces to

$$(\lambda - \mu')^2 \left\{ \frac{[(r_{xy} + 2)^2 \lambda + (2 - r_{xy}^2) \mu']^2 + 4\mu'^2 (2r_{xy} + 3)(r_{xy} + 1)^2}{(r_{xy} + 1)(r_{xy} + 2)^2} \right\} = 0.$$

But r_{xy} is numerically < 1 . $\therefore$ the factor in curved brackets is positive. Hence $\lambda = \mu'$, i.e. ${}_yU_x = {}_xU_y$. $\therefore {}_y\eta_x = {}_x\eta_y$.

(ii) An interesting deduction from the identities (79)–(81) is the following: "If any four of the six regression lines that occur in the mutual variation of three variables are linear, so are the other two."

We have to prove that if any four of the six quantities $\lambda, \mu, \nu, \lambda', \mu', \nu'$ vanish, then the remaining two vanish as well.

First let $\lambda = \mu = \nu = \lambda' = 0$.

$$(79) \text{ gives } r_{xy}^2 (\gamma_3' \mu'^2 + \nu'^2)^2 = 4r_{xz}\mu' \{\gamma_3' \nu'^2 r_{xy}\mu' - \gamma_3 \gamma_3' \mu'^3\},$$

$$(80) \quad \gamma_1'^4 \nu'^4 r_{yz}^2 = -4r_{xy}\nu'^4 \gamma_1 \gamma_1'^2,$$

$$(81) \quad \mu'^4 r_{xz}^2 = 0.$$

$$\text{From (80)} \quad \gamma_1'^3 \nu'^4 [4\gamma_1 r_{yx} + \gamma_1'^3 r_{yz}^2] = 0.$$

$$\text{But } 4\gamma_1 r_{xy} + \gamma_1'^3 r_{yz}^2 = \left(\frac{r_{yz}r_{xz} + r_{yz}^2 r_{xy} - 2r_{xy}}{1 - r_{yz}} \right)^2 > 0.$$

$$\therefore \nu' = 0,$$

and

$$\mu' = 0,$$

and these satisfy (79).

There are three cases of this type.

The case $\lambda' = \mu' = \nu' = \lambda = 0$ is proved in the same way. There are three cases of this type as well.

Next take the three cases of type

$$\lambda = \mu = 0,$$

$$\lambda' = \mu' = 0.$$

Equations (79)–(81) become

$$(79) \quad R = R', \text{ i.e. } \nu^2 = \nu'^2,$$

$$(80) \quad \gamma_1'^4 \nu'^4 r_{yz}^2 = 4r_{xy} \nu' \{ \gamma_1 \gamma_1'^2 \nu'^2 (-\nu') \},$$

$$(81) \quad \gamma_2^4 \nu^4 r_{xz}^2 = 4(-r_{xy} \nu) \{ -\gamma_2' \gamma_2^2 \nu^2 (-\nu) \},$$

$$(80) \text{ leads to } (\gamma_1'^2 r_{yz}^2 + 4r_{xy} \gamma_1) \gamma_1'^2 \nu'^4 = 0.$$

We have already shown that the first factor is positive,

$$\therefore \nu' = 0,$$

and hence

$$\nu = 0,$$

and these values satisfy (81).

The three cases of type $\lambda = \mu = \mu' = \nu' = 0$ lead to

$$(79) \quad \nu^4 r_{xy}^2 = 0,$$

$$(80) \quad \lambda'^4 r_{yz}^2 = 0,$$

$$(81) \quad (\gamma_2^2 \nu^2 - \gamma_2'^2 \lambda'^2) r_{yz}^2 = 4(r_{yz} \lambda' - r_{xy} \nu) \{ -\gamma_2' \gamma_2^2 \nu^2 (r_{xz} \lambda' - \nu) + \gamma_2 \gamma_2'^2 \lambda'^2 (r_{xz} \nu - \lambda') \},$$

whence $\lambda' = \nu = 0$.

There remain the three cases of type

$$\lambda = \nu = 0,$$

$$\mu' = \nu' = 0.$$

Here (79) is satisfied identically.

$$(80) \text{ becomes } (\lambda'^2 - \gamma_1^2 \mu^2) r_{yz}^2 = -4r_{yz} \mu \{ \gamma_1' (\lambda'^2 - \gamma_1^2 \mu^2) (-\mu) \},$$

$$(81) \quad (\mu^2 - \gamma_2'^2 \lambda'^2) r_{xz}^2 = 4r_{yz} \lambda' \{ -\gamma_2 (\mu^2 - \gamma_2'^2 \lambda'^2) (-\lambda') \},$$

which reduce to

$$(\lambda'^2 - \gamma_1^2 \mu^2) \{ r_{yz}^2 \lambda'^2 - (\gamma_1^2 r_{yz}^2 + 4\gamma_1' r_{xz}) \mu^2 \} = 0,$$

$$(\mu^2 - \gamma_2'^2 \lambda'^2) \{ r_{xz}^2 \mu^2 - (\gamma_2'^2 r_{xz}^2 + 4\gamma_2 r_{yz}) \lambda'^2 \} = 0.$$

The only common solution of these equations is

$$\lambda' = \mu = 0.$$

We have thus accounted for all the fifteen possible cases.

(iii) *Three regression curves linear.*

In six cases out of the possible 20 cases the linearity of three only of the regression curves involves the linearity of the remaining three.

Let $\lambda = \mu' = 0$ and either ν or $\nu' = 0$.

It follows from (79) that $R = R'$, i.e. $\nu = \nu'$.

$\therefore$ both ν and ν' are zero. We have now four linear regression curves, $\therefore$ all six are linear.

Let $\mu = \nu' = 0$ and $\mu' = 0$.

Since $\mu = \nu' = 0$, $\therefore P = P'$ or $\lambda = \lambda'$, so that

$$\begin{aligned} P &= \lambda^2, & Q &= -\gamma_2'^2 \lambda^2, & R &= \nu^2, \\ P' &= \lambda^2, & Q' &= \mu'^2 - \gamma_2^2 \nu^2, & R' &= -\gamma_3^2 \lambda^2. \end{aligned}$$

(79) becomes $(\nu^2 + \gamma_3^2 \lambda^2)^2 r_{xy}^2 = 4r_{yz} \gamma_3 \lambda^2 (r_{xy} \nu^2 - \gamma_3 \gamma_3' \lambda^2)$,

(80) becomes

$$(\gamma_2^2 \nu^2 - \gamma_2'^2 \lambda^2)^2 r_{yz}^2 = 4(r_{yz} \lambda - r_{xy} \nu) \gamma_2 \gamma_2' \nu^2 [(\gamma_2 \nu - \gamma_2' \lambda) + r_{xz} (\gamma_2' \nu - \gamma_2 \lambda)].$$

The first reduces to

$$\left\{ \frac{\lambda^2 \gamma_3 (r_{xz} + r_{yz} r_{xy}) r_{xy} - 2r_{yz}}{1 - r_{xy}^2} + r_{xy} \nu^2 \right\}^2 = 0,$$

and the second to

$$(\gamma_2 \nu^2 - \gamma_2' \lambda^2)^2 r_{xz}^2 + 4\gamma_2 \gamma_2' (r_{yz} \lambda - r_{xy} \nu)^2 \nu^2 = 0.$$

Hence either $\lambda = \nu = 0$ or there must be a very special relation between r_{xy} , r_{yz} , r_{zx} .

If instead of $\mu' = 0$ we take $\nu = 0$ we get similar results, i.e. in general the vanishing μ , ν' , μ' or μ , ν' , ν involves that of λ , ν , λ' or λ , μ' , λ' .

This accounts for six more cases.

There are eight left. Of these six are typified by

$$\mu' = \nu' = 0 = \nu \quad \text{or} \quad \mu' = \nu' = 0 = \mu,$$

and lead to the same conclusions.

The remaining two are

$$\lambda = \mu = \nu = 0 \quad \text{or} \quad \lambda' = \mu' = \nu' = 0.$$

The first supposition, $\lambda = \mu = \nu = 0$ gives

$$\begin{aligned} P &= -\gamma_1'^2 \nu'^2, & Q &= -\gamma_2'^2 \lambda'^2, & R &= -\gamma_3'^2 \mu'^2, \\ P' &= \lambda'^2, & Q' &= \mu'^2, & R' &= \nu'^2, \end{aligned}$$

leading to

$$(79) \quad (\nu'^2 + \gamma_3'^2 \mu'^2)^2 r_{yx}^2 = 4r_{xz} \mu'^2 \gamma_3' \{r_{xy} \nu'^2 - \gamma_3 \gamma_3' \mu'^2\},$$

$$(80) \quad (\lambda'^2 + \gamma_1'^2 \nu'^2)^2 r_{yz}^2 = 4r_{xy} \nu'^2 \gamma_1' \{r_{yz} \lambda'^2 - \gamma_1 \gamma_1' \nu'^2\}$$

$$(81) \quad (\mu'^2 + \gamma_2'^2 \lambda'^2)^2 r_{zx}^2 = 4r_{yz} \lambda'^2 \gamma_2' \{r_{xz} \mu'^2 - \gamma_2 \gamma_2' \lambda'^2\},$$

which give $\lambda' = \mu' = \nu' = 0$ or a very special condition to be satisfied by the correlation coefficients r_{xy} , r_{yz} , r_{zx} .

We may conclude then that in general the *linearity of any three of the six regression lines involves that of the remaining three.*

(iv) If the regression surface of z on x, y reduces to a plane, the regression curves of x on y and y on x reduce to straight lines.

We have as in § 7

$$\frac{\bar{z}_y}{\sigma_z} = d + a \frac{\bar{x}_y}{\sigma_x} + \frac{by}{\sigma_y} + c \frac{\bar{x}_y y}{\sigma_x \sigma_y} \dots\dots\dots (58).$$

But
$$\frac{\bar{x}_y}{\sigma_x} = r_{xy} \frac{y}{\sigma_y} \pm \sqrt{\frac{y\eta_{x^2}^2 - r_{xy}^2}{\beta_2 - \beta_1 - 1}} \left\{ \frac{y^2}{\sigma_y^2} - \sqrt{\beta_1} \frac{y}{\sigma_y} - 1 \right\},$$

$$\begin{aligned} \therefore \frac{\bar{z}_y}{\sigma_z} &= d \pm a \sqrt{\frac{y\eta_{x^2}^2 - r_{xy}^2}{\beta_2 - \beta_1 - 1}} + \frac{y}{\sigma_y} \left\{ b + ar_{xy} \mp (a\sqrt{\beta_1} + c) \sqrt{\frac{y\eta_{x^2}^2 - r_{xy}^2}{\beta_2 - \beta_1 - 1}} \right\} \\ &+ \frac{y^2}{\sigma_y^2} \left\{ \left(cr_{xy} \mp c \sqrt{\frac{\beta_1(y\eta_{x^2}^2 - r_{xy}^2)}{\beta_2 - \beta_1 - 1}} \right) \pm a \sqrt{\frac{y\eta_{x^2}^2 - r_{xy}^2}{\beta_2 - \beta_1 - 1}} \right\} \\ &+ \text{terms of higher order.} \end{aligned}$$

Now the regression of z on y for a constant x is linear. Therefore the coefficient of y^2 is zero. To first order terms we may put $\beta_1 = 0$ and $\beta_2 = 3$, and thus

$$cr_{xy} \pm a \sqrt{\frac{y\eta_{x^2}^2 - r_{xy}^2}{2}} = 0.$$

Similarly

$$cr_{xy} \pm b \sqrt{\frac{x\eta_{y^2}^2 - r_{xy}^2}{2}} = 0,$$

But c vanishes when ${}_{xy}H_z^2 = {}_{xy}R_z^2$ by (52).

Hence if

$${}_{xy}H_z^2 = {}_{xy}R_z^2,$$

it follows that

$$x\eta_y = y\eta_x = r_{xy}.$$

We thus see that if the three generalised correlation ratios ${}_{xy}H_z$, ${}_{yz}H_x$, ${}_{zx}H_y$ are equal to ${}_{xy}R_z$, ${}_{yz}R_x$, ${}_{zx}R_y$ respectively, the six correlation ratios $x\eta_y$, $y\eta_x$, $x\eta_z$, $z\eta_x$, $z\eta_y$, $y\eta_z$ reduce to the corresponding correlation coefficients r_{xy} , r_{zx} , r_{yz} and that the "linearity" of the three regression surfaces involves the linearity of the six regression lines.

MISCELLANEA.

I. On Spurious Values of Intra-class Correlation Coefficients arising from Disorderly Differentiation within the Classes.

By J. ARTHUR HARRIS, PH.D. Carnegie Institution of Washington, U.S.A.

WHEN the constants of the x and y characters of the population in r_{xy} are quite indistinguishable symmetrical tables* may be used, but not otherwise.

Primarily and for the most part, however, the use of symmetrical tables has been restricted to cases in which the degree of interdependence between the measures of all possible pairs† drawn from a considerable series of associated individuals—in short to intra-class correlations‡—is sought.

The dangers of spurious correlation due to the artificial symmetry of the surface is then much greater§. Pearson|| long ago pointed out that when intra-class differentiation exists, for example, because of age in the case of characters determined upon the members of a fraternity, or of position on the axis in the case of serial organs, the values of r may be to some extent spurious.

In the cases considered by Pearson differentiation is an orderly phenomenon, i.e. the magnitudes under consideration increase or decrease with age, position on the axis, or some other extrinsic characteristic with such regularity that the relationship can be expressed by an equation which may be used in correcting the raw values of r .

In other cases, the problem is not so simple. Differentiation within the class may exist, but it may be difficult or impossible to arrange the individual measurements by any character outside of themselves to obtain the constants necessary for determining the true correlations from the spurious values deduced from the tables.

Illustration I. The correlation between yields of wheat in variety testing.

In variety testing, the experimenter seeks (or should seek), among other things, to determine the correlation between yields of varieties in different years. If this correlation be 0 (and regression be linear) it is clear that the yield of a variety in one year furnishes no basis for prediction

* R. Pearl, *Biometrika*, Vol. v. pp. 249—297, 1907; H. S. Jennings, *Journ. Exp. Zool.* Vol. xi. pp. 1—134, 1911; J. Arthur Harris, *Biometrika*, Vol. vii. pp. 325—328, 1910.

† K. Pearson and others, *Phil. Trans.*, A, Vol. cxcvii. pp. 285—379, 1901; K. Pearson and A. Barrington, *Eugenics Laboratory Memoirs*, No. V, 1909.

‡ *Biometrika*, Vol. ix. pp. 446—472, 1913.

§ With only one pair of measures the probability of spurious correlation is, in cautious work, very slight, for the possibility of differentiation can be easily tested by the critical comparison of the physical constants.

|| Pearson, K., "On Homotypy in Homologous but Differentiated Organs." *Roy. Soc. Proc.* Vol. LXXI. pp. 288—313, 1903.

concerning its yield in any subsequent year. If, on the other hand, the correlation be high, prediction from a few years' test may be made with great probability of certainty.

Given a measure of the "performance" of a series of varieties during a number of years it would at first seem quite allowable to form symmetrical tables or to use the intra-class formulae of a former paper* to determine the intra-varietal correlation, and to regard this as a satisfactory measure of the differentiation of the varieties and of the average prediction value of a year's test. Such is, however, not the case, for while there may be no orderly change in yield throughout the period under consideration, the individual years differ greatly in their average yield for all the varieties. The influence of this "disorderly differentiation" upon r is admirably shown by A. D. Hall's† table of the yield in bushels of wheat in the Rothamsted experiments.

Let b = yield in bushels per acre of any one of m varieties in any one of n years, y_1, y_2 be the "first" and the "second" years of a symmetrical intra-varietal correlation surface, v_1, v_2 be the "first" and "second" varieties of a symmetrical intra-annual correlation surface. Then $r_{b_{y_1} b_{y_2}}$ will be a (spurious) measure of the (persistent) differentiation of varieties, $r_{b_{v_1} b_{v_2}}$, a (spurious) measure of the differentiation (in the yield of all the varieties) of years. Applying formulae (v)—(ix) of *Biometrika*, Vol. ix. p. 450, to these data, I find

$$\begin{aligned} S[n(n-1)] &= 2128, \\ S[(n-1)\Sigma(b')] &= 83122\cdot5, \quad S[(n-1)\Sigma(b'^2)] = 3483626\cdot4, \\ S[\Sigma(b')^2] &= 3610204\cdot57, \quad S[\Sigma(b'^2)] = 370820\cdot13, \\ \bar{b} &= 39\cdot0613, \quad \sigma_b^2 = 111\cdot257328, \\ r_{b_{y_1} b_{y_2}} &= -\cdot032. \end{aligned}$$

The result is obviously spurious, for mere inspection of the entries in the table shows that some varieties regularly give heavier yields than others. The source of the spurious value is to be seen in the fact that an intra-class coefficient has been calculated from a symmetrical surface formed from classes (varieties) represented by a series of yields *differentiated by annual variations in the growing conditions*. By correcting for this source of differentiation by expressing each yield as a deviation from the mean yield of all the varieties for the particular year, i.e. $b'' = b - \bar{b}_y$, where the bar denotes a mean and the subscript y that it is for all the yields of a year, I have found‡

$$r_{b''_{y_1} b''_{y_2}} = \cdot266.$$

Measuring the differentiation of years in terms of intra-annual correlation (intra-class correlation in which each class is defined by the year and its individuals are the yields of the different varieties grown), I find from Hall's table

$$\begin{aligned} S[m(m-1)] &= 4440, \\ S[(m-1)\Sigma(b')] &= 174129\cdot2, \quad S[(m-1)\Sigma(b'^2)] = 7317531\cdot92, \\ S[\Sigma(b')^2] &= 7586436\cdot21, \quad S[\Sigma(b'^2)] = 370820\cdot13, \\ \bar{b} &= 39\cdot2183, \quad \sigma_b^2 = 110\cdot017719, \\ r_{b_{v_1} b_{v_2}} &= \cdot791. \end{aligned}$$

Since the varieties have been shown to be differentiated, this result must also be spurious. Let $b'' = b - \bar{b}_v$, where the v indicates that the mean denoted by the bar is for the yield of the

* *Biometrika*, Vol. ix. pp. 446—472, 1913.

† Hall, A. D., *The Book of the Rothamsted Experiments*, p. 66, 1905.

‡ *Science*, N. S. Vol. xxxvi. pp. 318—320, 1912. Probably a better method of dealing with such cases will sometime be found. So far I have not succeeded.

TABLE I.
Yield of Varieties of Wheat in Different Years.

Variety	1871	1872	1873	1874	1875	1876	1877	1878	1879	1880	1881	Lots	Σ (y)	Σ (y ²)
Rivet (Red) ...	—	—	48.1	67.0	48.4	42.5	49.6	66.1	16.0	22.4	52.2	9	412.3	21263.39
White Chaff (Red) ...	—	45.8	40.6	55.1	40.2	49.5	48.4	59.0	22.8	28.1	54.5	9	398.2	18853.92
Club Wheat (Red) ...	36.0	49.8	47.5	59.6	46.6	47.6	49.5	61.0	23.5	16.4	43.4	11	476.9	22515.39
Golden Drop (Red), Hallett's	39.5	42.8	44.2	51.8	38.1	48.4	49.5	52.8	21.0	18.9	50.8	11	464.8	21088.28
Bole's Prolific (Red) ...	33.6	46.5	45.2	48.1	43.8	41.4	44.8	52.8	31.0	24.5	46.5	11	454.5	19468.23
Hardcastle (White) ...	—	—	42.0	49.6	33.9	44.0	42.1	54.0	21.5	24.4	45.6	10	403.6	17297.00
Red Rostock ...	37.0	—	46.3	53.8	37.4	40.0	46.4	57.0	8.5	28.4	45.8	10	400.6	17784.30
Red Langham ...	30.8	43.8	34.1	53.1	34.9	42.5	42.9	50.8	25.8	28.6	48.5	11	435.8	18130.66
Bristol Red ...	29.4	44.4	39.5	53.4	31.6	42.4	44.1	52.1	21.6	30.6	46.2	11	435.3	18240.43
Red Wonder ...	31.2	43.8	37.1	55.1	33.2	44.2	41.6	52.1	22.0	28.2	45.9	11	434.4	18191.20
Red Chaff (White) ...	32.8	37.0	35.3	48.8	34.3	43.8	41.0	—	—	—	—	7	273.0	10848.30
Browick (Red) ...	35.3	40.5	38.5	51.1	38.5	39.1	40.9	49.5	24.0	19.6	47.3	11	424.3	17311.37
Casey's White ...	29.9	42.1	37.5	52.1	39.0	45.5	43.0	47.8	15.4	24.1	42.9	11	419.3	17170.55
Red Nursery ...	34.1	45.3	27.1	41.1	39.0	37.5	40.6	47.8	30.9	27.5	46.0	11	416.9	16326.03
Woolly Ear (White) ...	31.2	42.8	37.0	51.3	36.1	46.6	37.5	48.3	20.0	21.0	44.1	11	415.9	16805.69
Burwell (Old Red Lammas)	31.1	41.3	35.1	47.3	38.5	38.4	39.0	46.3	27.0	27.0	44.8	11	415.8	16228.74
Golden Rough Chaff (Red) ...	33.0	39.3	38.5	52.1	38.8	38.4	36.4	46.8	14.4	31.3	41.6	11	410.6	16242.96
Chubb Wheat (Red) ...	28.4	40.0	35.8	50.5	38.3	40.3	41.5	55.1	20.8	14.9	—	10	365.6	14742.34
Original Red (Hallett's) ...	30.0	35.3	36.4	43.6	26.0	40.1	44.4	—	—	—	—	7	255.8	9627.38
Victoria White (Hallett's) ...	33.8	45.3	38.3	44.3	33.8	41.1	42.6	43.9	14.9	15.8	44.0	11	397.8	15605.18
White Chiddam ...	26.9	38.8	31.8	42.0	32.4	37.5	37.6	49.8	11.9	27.4	47.1	11	383.2	14464.88
Hunter's White (Hallett's) ...	26.9	39.8	38.0	45.4	26.4	43.5	40.0	42.3	17.4	22.8	—	10	342.5	13613.91
Number of Lots ...	19	19	22	22	22	22	22	20	20	20	18	226	—	370820.13
Total Yields, Σ (y) ...	610.9	804.4	853.9	1116.2	809.2	934.3	943.4	1035.3	410.4	481.9	837.2	—	8837.1	—

variety for all the years it was grown. Correcting for the influence of the differentiation of varieties in this way I have found*

$$r_{b''v_1b''v_2} = .837.$$

Thus season is a far more important factor than variety in determining an individual yield.

Illustration II. Influence of Personal Equation upon the Correlation between the Grades assigned to the Same Paper by a Series of Instructors.

Stripped of the verbiage in which it has been clothed in discussions among pedagogues, one of the chief problems concerning the reliability of the grades assigned in examinations resolves itself unto the statistical question: What is the correlation between the grades assigned to the same paper by different instructors?

Let g be the grade assigned to any one of m papers by any one of n instructors, let i_1, i_2 be the "first" and "second" instructor (of a symmetrical intra-class table) passing judgment upon a paper, p_1, p_2 the "first" and "second" paper graded by the same instructor. Then from Table I of D. Starch† I deduce, by the intra-class formulae (v)—(1x) of *Biometrika*, Vol. ix. p. 450,

$$r_{gi_1gi_2} = .659, \quad r_{gp_1gp_2} = .071.$$

By using the deviation method as illustrated above, I have found

$$r_{g''i_1g''i_2} = .732, \quad r_{g''p_1g''p_2} = .386.$$

TABLE II.

Grades of Papers Assigned by Various Instructors.

Instructors.

	1	2	3	4	5	6	7	8	9	10	$\Sigma(g)$
1	85	86	88	85	75	80	88	87	85	87	846
2	77	80	87	80	62	82	82	87	85	87	809
3	74	78	78	75	69	84	91	83	79	80	791
4	65	65	62	20	26	60	55	68	55	50	526
5	68	82	78	82	64	88	85	86	78	80	791
6	94	87	93	87	83	77	89	88	88	89	875
7	88	90	95	87	79	85	96	91	87	89	887
8	80	84	73	79	72	83	85	91	77	76	800
9	70	70	68	50	44	65	75	81	79	79	681
10	93	92	85	92	81	83	92	89	84	85	876
$\Sigma(g)$	794	814	807	737	655	787	838	851	797	802	7882

Both of these results, in which an attempt was made to correct for the personal equation of the instructors in determining the correlation between the estimates of different instructors on the same paper, or to correct for the differences in merit of the papers in testing the individuality of the instructors, are higher than the raw values given above, which are clearly spurious. Similar results‡ are obtained from Jacoby's astronomical grades§.

* *Science*, loc. cit.

† *Science*, N. S. Vol. xxxviii. p. 630, 1913.

‡ Personally, I can attach little pedagogical significance to series as short as those of either Starch or Jacoby. They serve here as illustrations of method merely because I know of no more extensive series.

§ *Science*, N. S. Vol. xxxi. p. 819, 1910.

The essentials of this note may be summarized as follows:

In using Intra-class coefficients care must be taken to guard against spurious values arising through differentiation among the individuals of the class.

Besides the orderly differentiation (due to age of individuals, position of organs on axis, etc.) for which Pearson has determined corrective formulae in terms of correlation coefficients, a disorderly differentiation for which such corrective formulae have not as yet been found sometimes obtains. Illustrations of such cases are here given.

Probably the empirical methods used here in correcting for this disorderly differentiation should be replaced by formulae with a sounder theoretical foundation. This I have not as yet been able to do.

The purpose of this note will have been served if it directs attention to a source of danger which may sometimes be encountered in the use of serviceable formulae, and indicates a method by which in the absence of more perfect methods practical results may be secured.

COLD SPRING HARBOR, N.Y.

February 3, 1914.

II. On an Extension of the Method of Correlation by Grades or Ranks.

BY KARL PEARSON, F.R.S.

In a memoir published in 1907* I have shown how, on the hypothesis of normal distribution, the true correlation of variates r may be ascertained from the correlation ρ of grades. If g_1 and g_2 be the two grades, ν_1 and ν_2 the corresponding ranks, x and y the corresponding variates with means $\bar{x}$ and $\bar{y}$, and standard-deviations σ_1 and σ_2 , while

$$z = \frac{N}{2\pi\sigma_1\sigma_2} \frac{1}{\sqrt{1-r^2}} e^{-\frac{1}{2} \frac{1}{1-r^2} \left(\frac{x^2}{\sigma_1^2} - \frac{2rxy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right)}$$

is the normal frequency surface of the variates, then

$$\begin{aligned} \bar{g}_1 &= \frac{1}{2}N = \bar{g}_2, & \sigma_{g_1}^2 &= \sigma_{g_2}^2 = \frac{1}{12}N, \\ g_1 - \bar{g}_1 &= i_1 = \frac{N}{\sqrt{2\pi}\sigma_1} \int_0^x e^{-\frac{1}{2} \frac{x^2}{\sigma_1^2}} dx, \\ g_2 - \bar{g}_2 &= i_2 = \frac{N}{\sqrt{2\pi}\sigma_2} \int_0^y e^{-\frac{1}{2} \frac{y^2}{\sigma_2^2}} dy, \\ \nu_1 &= \nu_2 = g + \frac{1}{2}, & \bar{\nu}_1 &= \bar{\nu}_2 = \frac{1}{2}(N+1), \\ \sigma_{\nu_1}^2 &= \sigma_{\nu_2}^2 = \frac{1}{12}(N^2-1). \end{aligned}$$

Further I showed in the memoir just cited that

$$r = 2 \sin \left(\frac{\pi}{6} \rho \right)$$

* "On Further Methods of Determining Correlation," *Drapers' Company Research Memoirs* (Dulau and Co.), pp. 11, 12.

where a convenient method of finding ρ was by the formula

$$\rho = 1 - \frac{6S(g_1 - g_2)^2}{N^3}$$

or again by

$$= 1 - \frac{6S(\nu_1 - \nu_2)^2}{N(N^2 - 1)}.$$

The problem has recently occurred of dealing with data where :

(i) One variate is given quantitatively, the other variate is given by ranks.

For example, place in school-class has to be considered in relation to marks in examination, or the rank in a teacher's general appreciation has to be considered in relation to marks in examination.

(ii) One variate is given by broad categories, the other by ranks.

For example, five or six categories of general intelligence are given as the basis of the teacher's classification of intelligence, and this has to be considered with regard to rank in, say, class or examination, possibly with regard to a special subject.

We require in both cases to deduce from the data the true variate correlation.

Case (i). Let x be the character measured by its grade, y the character given quantitatively. Then with the notation above, if ρ' equal the correlation of grade and of variate, r the correlation of the two variates :

$$\rho' = \frac{p_{g,y}}{N\sigma_g\sigma_y},$$

where

$$p_{g,y} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z(y - \bar{y})(g_1 - \bar{g}_1) dx dy,$$

$$\frac{dp_{g,y}}{dr} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (y - \bar{y}) i_1 \frac{dz}{dr} dx dy.$$

Integrating by parts after putting $\bar{y}=0$ and writing

$$\frac{dz}{dr} = \sigma_1\sigma_2 \frac{d^2z}{dx dy}, *$$

$$\frac{dp_{g,y}}{dr} = - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_1\sigma_2 y \frac{di_1}{dx} \frac{dz}{dy} dx dy.$$

Integrating again by parts :

$$\begin{aligned} \frac{dp_{g,y}}{dr} &= \sigma_1\sigma_2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{di_1}{dx} z dx dy \\ &= \sigma_1\sigma_2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{N}{\sqrt{2\pi}\sigma_1} e^{-\frac{1}{2}\frac{x^2}{\sigma_1^2}} z dx dy \\ &= \frac{\sigma_2 N^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{2\pi} \frac{1}{\sqrt{1-r^2}} e^{-\frac{1}{2}\left(\frac{2-r^2}{1-r^2}x'^2 - \frac{2rx'y'}{1-r^2} + \frac{y'^2}{1-r^2}\right)} dx' dy' \\ &= \frac{\sigma_2 N^2}{\sqrt{2\pi}} \frac{1}{\sqrt{1-r^2}} \frac{1}{\sqrt{\frac{2-r^2}{(1-r^2)^2} - \frac{r^2}{(1-r^2)^2}}} = \frac{\sigma_2 N^2}{2\sqrt{\pi}}. \end{aligned}$$

Hence

$$\frac{d\rho'}{dr} = \frac{d}{dr} \left(\frac{p_{g,y}}{N\sigma_2\sigma_{g_1}} \right) = \frac{N}{2\sqrt{\pi}\sigma_{g_1}} = \sqrt{\quad}.$$

* *Phil. Trans. A.*, Vol. 195, p. 25.

Thus since ρ' vanishes with r ,

$$\rho' = \sqrt{\frac{3}{\pi}} r.$$

If we used ranks instead of grades, we should have σ_{v_1} for σ_{g_1} and consequently

$$\rho'' = \sqrt{\frac{3}{\pi}} \frac{N}{\sqrt{N^2-1}} r.$$

Thus finally

$$r = \sqrt{\frac{\pi}{3}} \rho' = 1.0233 \rho',$$

$$\text{or} \quad r = \frac{\sqrt{N^2-1}}{N} 1.0233 \rho''.$$

It will be clear from this that the correlation ρ' between rank and quantitative variate can never be "perfect," for it cannot exceed the value .9772, otherwise the correlation r would exceed unity. It will be seen that for practical purposes r is very close to ρ' , but still from the theoretical standpoint, it is not without interest to discover that the correlation between ranks and a quantitative variate can never be perfect. For example, it is impossible to have perfect correlation between place in class and examination test, even if the boys were in the same order in class and examination. The defect, however, will be very slight.

Case (ii). Let the subscript C refer to any "broad" class and let η be found from either of the formulae

$$\eta^2 = \frac{12}{N^3} S \{n_c (\bar{g}_c - \bar{g})^2\},$$

$$\text{or} \quad \eta'^2 = \frac{12}{N(N^2-1)} S \{n_c (\bar{v}_c - \bar{v})^2\},$$

the first applying to grades and the second to ranks; then

$$r = 1.0233 \sqrt{\frac{12}{N^3} S \{n_c (\bar{g}_c - \bar{g})^2\}},$$

$$\text{or} \quad = 1.0233 \sqrt{\frac{12}{N^3} S \{n_c (\bar{v}_c - \bar{v})^2\}},$$

according as grades or ranks are used. In actual practice the values of η' or η'' should be correct for number of classes and for "broad" categories. See *Biometrika*, Vol. VIII. p. 256 and Vol. IX. p. 118.

Numerical illustrations will be provided later.

III. Correction of a Misstatement by Mr Major Greenwood, Junior.

In a recent paper by Mr Major Greenwood and Mrs Frances Wood "On changes in the Recorded Mortality from Cancer and their Possible Interpretation*" occur the following words: "The case is evidently analogous to that studied by Professor Karl Pearson in his pamphlet, *The Fight against Tuberculosis and the Death-rate from Phthisis* (Dulau and Co.). Professor Pearson published three diagrams: (a) the general death-rate of England and Wales; (b) the phthisis death-rate; (c) the ratio of phthisis deaths to all deaths. The original figures seem to have been the crude rate for males and females separately from 1835 onwards." The "evident analogy" with what appears to me the wholly fallacious treatment of the authors in their paper above cited I do not now stay to discuss, but I wish to draw attention to the words: "The

* Royal Society of Medicine, *Proceedings*, Vol. VII. Section of Epidemiology, pp. 79-170. March 27, 1914.

original figures seem to have been the crude rate for males and females separately from 1835 onwards." Why the writer of these words should have assumed them without any inquiry of me, or any examination of the values of the crude death-rates (which are accessible to everybody) to be "crude death-rates," I do not know, but they illustrate his readiness to form a biased judgment when his feelings are stirred by unfavourable criticism. As a matter of fact the rates were *standardised* rates reduced to the population of 1901, and most kindly provided at my special request by the General Register Office. It is of interest to observe that Dr Weinberg of Stuttgart—recently made precisely the same charge as Mr Major Greenwood with the same over-hasty assumption that the reality must be the desired, if undemonstrated, error*. With the German as with other foes, it is well to leave ample opportunity for their assuming you to be foolish; their assumption may lead them to run against hard reality.

K. P.

IV. Note on Reproductive Selection.

By DAVID HERON, D.Sc.

The fact that in the case of man† fifty per cent. of one generation comes from twenty-five per cent. of the preceding one was first noted by Professor Karl Pearson in the *Chances of Death* (Vol. I. p. 80) and in dealing more fully with this important generalisation in the *Groundwork of Eugenics*, p. 27, he said: "It is very difficult from any English statistics to determine how many adults never marry. No information on this point is asked in the death schedule for males; it is asked but imperfectly answered in the case of the schedule for females." In a footnote he adds: "The Registrar-General informs me that the record of civil condition in the case of female deaths is worthless and that no useful return can be made from it." He found that in the Argentine and in Scotland 60 per cent. died unmarried, in the United States 51 per cent., and from the last two English Censuses and the Annual Reports 48 per cent., and added "This indirect method of reaching the result is, however, not very satisfactory. We may, I think, conclude in round numbers that 40 per cent. of the population dies before it reaches the age of 21 and that probably another 20 per cent. are never married." On this assumption Professor Pearson proceeds to show that "about 12 per cent. of all the individuals born in the last generation provide half the next generation."

Some data published in *Bulletin of Population and Vital Statistics No. 30 for the Commonwealth of Australia* (Tables 48 and 84 a and b) prove that the assumptions made lie very close to the facts. The data are shown in the following table which gives the conjugal condition and issue of the males and females who died in Australia in 1912. From this we find that half the total number of children came from 3337 of the parents (all those who had at least 9 children and part of those who had each 8 children). It thus appears that of the males 17,404 out of 30,285 = 57.5% died unmarried while half the total offspring came from 25.9% of those who married and 11.0% of the whole number of males, so that approximately three-fifths of the males born die unmarried and one-half of one generation comes from one-quarter of the married population or from one-ninth of all the males born in the preceding generation. The diagram gives a graphical illustration of the argument.

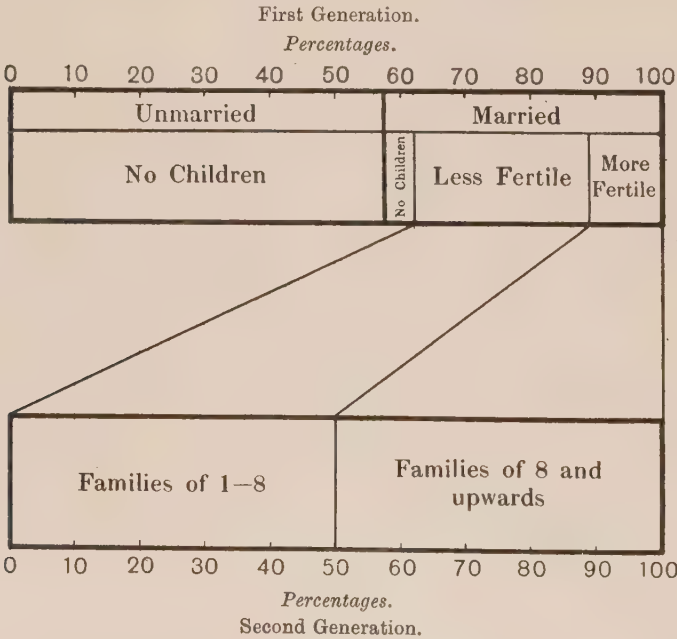
In exactly the same way we find that nearly one-half of the females born in Australia die unmarried and that one-half of one generation comes from one-quarter of the married and from one-seventh of all the females born in the preceding generation.

* *Archiv für Rassen- und Gesellschafts-Biologie*, ix. Jahrgang, S. 87. Leipzig, 1912.

† It has also been dealt with in various mammals. See the *Groundwork of Eugenics*, Eugenics Lecture Series II (Dulau and Co.), p. 29.

Conjugal Condition	Children in Each Family	Deaths of Males	Total Children	Deaths of Females	Total Children
Single	0	17404	—	10011	—
Married	0	1422	—	1317	—
"	1	1036	1036	1083	1083
"	2	1098	2196	992	1984
"	3	1127	3381	1050	3150
"	4	1147	4588	1001	4004
"	5	1070	5350	976	4880
"	6	1058	6348	1013	6078
"	7	1040	7280	974	6818
"	8	992	7936	881	7048
"	9	819	7371	799	7191
"	10	801	8010	622	6220
"	11	473	5203	469	5159
"	12	394	4728	314	3768
"	13	196	2548	193	2509
"	14	109	1526	101	1414
"	15	50	750	57	855
"	16	27	432	22	352
"	17	7	119	8	136
"	18	5	90	3	54
"	19	6	114	2	38
"	20	3	60	2	40
"	21	—	—	1	21
"	22	—	—	1	22
"	23	1	23	—	—
Totals		30285	69089	21892	62824

Diagram to illustrate the fact that three-fifths of those born die unmarried and that one-ninth of one generation produce one-half of the next. (Deduced from records of males.)



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